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MCNP5 Validation for Criticality Calculations: Homogeneous and Heterogeneous Systems

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Title:	MCNP5 Validation for Criticality Calculations: Homogeneous and Heterogeneous Systems
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ABSTRACT

The purpose of this work is to validate the Monte Carlo neutron transport code MCNP5 for criticality calculations at Westinghouse Sweden (WSE). The validation is performed against critical experiments, where 57 experiments are homogeneous and 90 are heterogeneous. From the criticality calculations the precision of the MCNP5 can be obtained, and a criticality safety margin called the upper subcritical limit (USL) can be determined. In the WSE fuel factory addition of a B term is used instead as the USL. The B term is calculated as: B = 0.95 – USL.

Two sets of cross-section libraries have been used. The first set is based on ENDF/B-VI release 6 and the second set is based on the latest libraries available, which are ENDF/B-VI release 6 and 8, and pre ENDF/B-VII. Since they only give a small difference and since set 1 only is based on one cross-section library, set 1 is preferred.

For WSE factory applications with uranium enriched to 5.00 wt% and an optimum moderation ratio (H/X) of 200 to 400, the results from the homogeneous and the heterogeneous systems differ very little, whereby a general B term can be used:

B = 0.013

For investigations at other H/X ratios and at other enrichments, the range for homogeneous systems is B = 0.013 to 0.022 and for heterogeneous systems B = 0.012 to 0.021.

CONTENTS

APPENDICES

ABBREVIATIONS

CSEWG	Cross Section Evaluation Working Group
CSNI	Committee on the Safety of Nuclear Installations
ENDF	Evaluated Nuclear Data Files
H/U	Moderation ratio for Hydrogen to Uranium
H/X	Moderation ratio for Hydrogen to Uranium-235
IHECSBE	International Handbook of Evaluated Criticality Safety Benchmark Experiments
k_{eff}	Effective neutron multiplication factor
LANL	Los Alamos National Laboratory
MCNP5	Monte Carlo N-Particle version 5
USL	Upper Subcritical Limit

1 INTRODUCTION

The purpose of this work is to validate the Monte Carlo neutron transport code MCNP5 [1] for criticality calculations at WSE. The validation is performed against critical experiments. From the criticality calculations the precision of the MCNP5 can be obtained, and a criticality safety margin can be determined. This validation is necessary to minimize the uncertainties in the use of MCNP5 in future calculations and to ensure that risk of reaching criticality during transport and production of the fuel is minimized.

From the critical experiments investigated, 57 are homogeneous and 90 are heterogeneous. Homogeneous denotes that the fissile material and the moderator are evenly distributed in a uniform compound; whereas heterogeneous means that the fissile material and the moderator are separate compounds. Based on previous Westinghouse validation reports the studied experiments are presented in seven different groups, where the three first are homogeneous and the following four are heterogeneous:

1. Homogeneous Systems with Low Moderator Concentration [3][4]
2. Homogeneous Experiments used for validation at Westinghouse Columbia [5]
3. CSNI Homogeneous Experiments [6][7][8]
4. Underwater Close-Packed Storage of LWR Fuel Experiments [9][10][11]
5. KRITZ-experiments [12][13][15]
6. Storage Experiments [16]
7. Heterogeneous Experiments used for validation at Westinghouse Columbia [17]

The handbook IHECSBE [2] is a large collection of benchmarked experiments. IHECSBE has been extensively used as a reference in this work since many of the experiments investigated in this work are included in there.

MCNP use cross-section libraries to describe the nuclides. In this work have two setups of cross-section libraries been used, where set 1 uses ENDF/B-VI release 6 for all the nuclides. Set 2 uses the latest accessible libraries at the time of the performed calculations, which are ENDF/B-VI release 6 and 8, and pre ENDF/B-VII.

A statistical evaluation of the results from the calculations is performed to determine a safety margin named the upper subcritical limit (USL). The USL states the maximum allowed k_{eff} for a certain fissile configuration when making future calculations with MCNP5. If the MCNP5-output is below the USL it is ensured that criticality is avoided. In the WSE fuel factory addition of a B term is used instead as the USL. The B term is calculated as: B = 0.95 – USL.

Structure of the report:

- Description of experiments
- Modeling conditions with information about MCNP and the Monte Carlo method
- Results from the MCNP5-calculations
- Statistical evaluation of the results
- Area and range of applicability of this work
- Conclusion

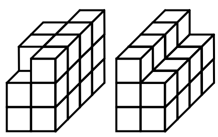
2 DESCRIPTION OF EXPERIMENTS

2.1 HOMOGENEOUS SYSTEMS

2.1.1 Homogeneous Systems with Low Moderator Concentration

A series of critical experiments using low-moderated uranium oxide (U_3O_8) was performed at the Rocky Flats Critical Experimental Facility in 1978. The experiments were performed to provide benchmarks for low-moderated and low-enriched uranium oxide. The moderation ratio, H/U (atom percent hydrogen over atom percent uranium) varied in the experiments from 0.77 to 2.03. The experiments evaluated in this report are the systems with an H/U of 0.77 and 1.25. Ref. [3] describes the system with H/U of 0.77, whereas Ref. [4] deals with the systems with H/U of 1.25. Both references are found in IHECSBE. The uranium oxide was contained in 15 cm cubical aluminum cans. The density of the oxide was 4.7 g/cm^3, which filled each box with approximately 15 kg of material. Moderating material between the cans (interstitial moderation) was necessary to be able to achieve criticality. The experiments were performed at room temperature.

The experiments were constructed using an array of the cans with varying amounts of Plexiglas moderator between the cans. The cans were put on two tables, and were reflected by either Plexiglas or concrete. The tables were the then brought together to achieve criticality. The number of cans varies from 38 to 100. Figure 1 shows the can configuration for H/U=0.77 case 1.



Case 1 for
H/U=0.77

Figure 1 Isometric view of case 1 in the homogeneous experiment with H/U = 0.77

For the homogeneous systems, there were 21 experiments with very low-moderated uranium oxide and had an H/U of 0.77. The uranium oxide was enriched to 4.46 wt% ^{235}U. The number of cans was adjusted to produce the smallest critical table separation possible, which in these experiments ranged from 0.23 to 1.83 cm. For the interstitial Plexiglas, the thickness varied from approximately 1.0 to 2.5 cm. To determine the effect of having moderate neutron absorbers, some of the experiments also included thin sheets of steel (Cases 7, 8, 9, 20 and 21) or polyvinyl chloride (PVC) (Cases 6, 18 and 19) surrounding each can. The cases 1 to 12 used Plexiglas as a reflector and 13 to 21 used concrete.

All of the 21 experiments were judged as acceptable as criticality-safety benchmark experiments (IHECSBE). However, the cases 13 and 14 are very similar, with the same configuration of cans but with the cans shuffled within the core. Therefore, they should not be considered as independent experiments and thereby not be used for purposes of validation. In this report case 14 is excluded in the statistical evaluation and the conclusion, although the MCNP5-output for case 14 is still presented in the results.

In Appendix 1, the can configurations for all the cases are graphically presented. Case 5 and 17 are attempts to simulate larger cans, which are constructed by grouping ordinary cans. There is only moderator material between each large can and not inside them. For a more detailed description of each case see Ref [3]. The characteristics of the experiments are described in Table 1 and Table 2. Illustrations of the can and the reflector can be found in Figure 2 and Figure 3.

Three of the experiments with low-moderated uranium oxide had an H/U of 1.25, which are described in Ref. [4]. The uranium oxide was enriched to 4.48 wt% ^{235}U. The critical table separation ranged from 0.80 to 1.56 cm and the Plexiglas thickness is 0.93 to 2.43 cm. The experimental arrangements are illustrated in Appendix 1. Plexiglas was used as a reflector. The characteristics for the can, moderator etc. are the same as for the "H/U = 0.77"-experiments. The three cases are referred to as case 22, 23 and 24.

Table 1 Characteristics of the homogeneous experiments with H/U = 0.77 and 1.25

Fuel	Material	Powder of primarily U_3O_8
	Powder density	4.7 ± 0.3 g/cm^3
	H/U	0.77
	Uranium enrichment	4.46 wt% ^{235}U
	H/U	1.25
	Uranium enrichment	4.48 wt% ^{235}U
Al can	Cube side	15.24 cm
	Thickness	0.16 cm

Table 2 Materials in the homogeneous experiments with H/U = 0.77 and 1.25

Refl. shell	Ordinary methyl methacrylate composition (inner pieces) Fire retardant methyl methacrylate composition (Cases 1-12 and 22-24) Concrete (Cases 13-21)
Moderator	Ordinary methyl methacrylate composition
Absorbers	Steel (Cases 7-9, 20, 21) PVC (Cases 6, 18, 19)

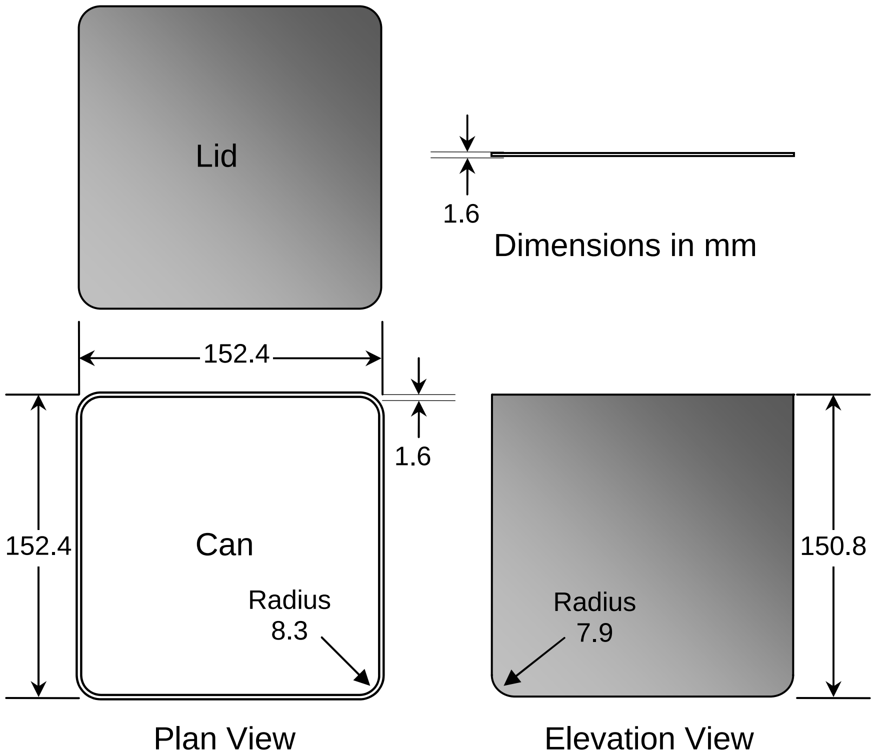


Figure 2 Oxide can dimensions for the homogeneous experiments with low moderator concentration

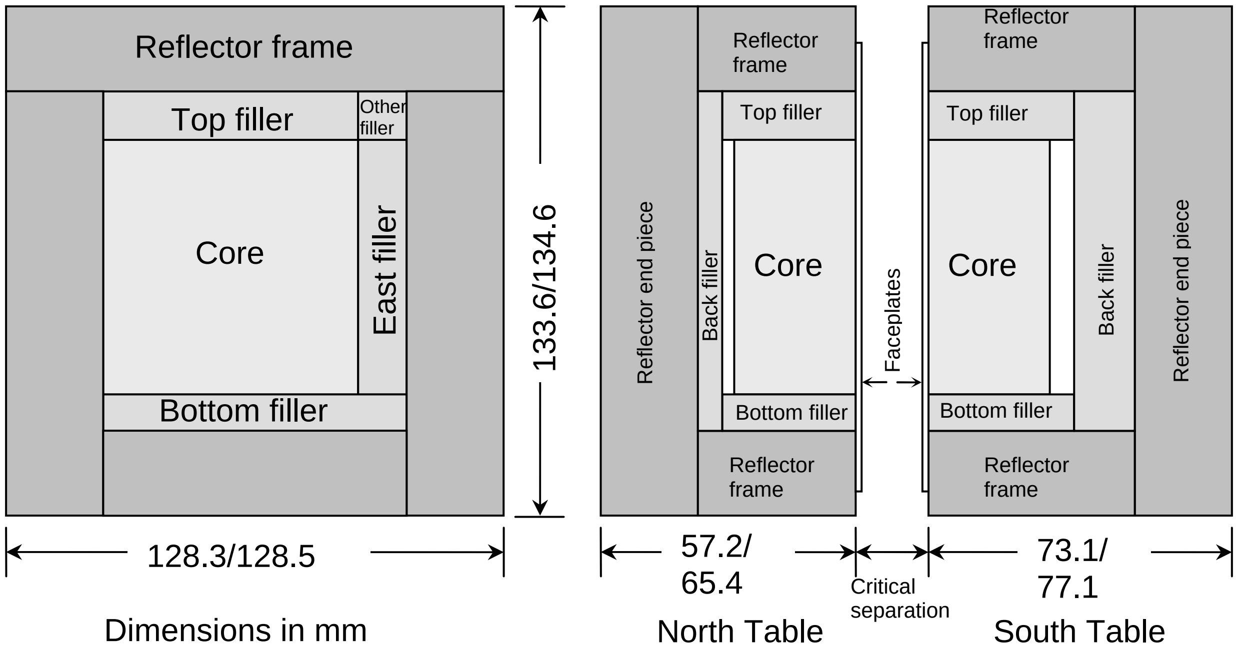


Figure 3 Front view and side view of reflectors in the homogeneous experiments with low moderator concentration (Values given as minimum/maximum)

2.1.2 Homogeneous Experiments used for Validation at Westinghouse Columbia

The Westinghouse Columbia homogeneous experiments were chosen by Westinghouse Columbia and are of two types: paraffin block cases (cases 1-16), where the uranium is homogenized with paraffin, and solution cases (cases 17-34), where the uranium is homogenized with water. They describe similar systems that exist at the Columbia Fuel Fabrication Facility (CFFF). These systems include homogeneous materials such as liquids, powders or mixtures. The material is found in tanks, pipes, equipment, and other containers. The experiments have been evaluated with MCNP4B [5]. This report uses the same input-file from the MCNP4B-evaluation for MCNP5. The input-file is updated with the new cross-section libraries. In the handbook IHECSBE some of the cases have been evaluated, as shown in Table 3. The MCNP-input files for those cases have been revised to match the handbook. Table 3 lists characteristic data for the experiments. In Figure 4 to Figure 6 the three types of configurations; cuboid, cylinder and sphere are illustrated. The core sizes vary, where the biggest cuboid is 81.45×86.70×88.22 cm, the biggest cylinder has the radius 38.05 cm and height 44.8 cm, and the biggest sphere has the radius 34.50 cm. Some of the cases use no reflecting material. All experiments were performed at room temperature.

Table 3 Characteristics of the Westinghouse Columbia homogeneous experiments

Case	Material	Enrichment	Refl. Mat.	Shape	H/X	Report label in the handbook IHECSBE
1	UF_4	2%	paraffin	cuboid	196	LEU-COMP-THERM-033-003
2	UF_4	2%	paraffin	cuboid	294	LEU-COMP-THERM-033-006
3	UF_4	2%	paraffin	cuboid	407	LEU-COMP-THERM-033-009
4	UF_4	2%	paraffin	cuboid	496	LEU-COMP-THERM-033-011
5	UF_4	2%	poly	cuboid	613	LEU-COMP-THERM-033-013
6	UF_4	2%	poly	cuboid	973	LEU-COMP-THERM-033-015
7	UF_4	3%	paraffin	cuboid	133	LEU-COMP-THERM-033-019
8	UF_4	3%	poly	cuboid	277	LEU-COMP-THERM-033-022
9	UF_4	2%	-	cuboid	196	LEU-COMP-THERM-033-024
10	UF_4	2%	-	cuboid	294	LEU-COMP-THERM-033-028
11	UF_4	2%	-	cuboid	407	LEU-COMP-THERM-033-033
12	UF_4	2%	-	cuboid	496	LEU-COMP-THERM-033-039
13	UF_4	2%	-	cuboid	613	LEU-COMP-THERM-033-042
14	UF_4	2%	-	cuboid	973	LEU-COMP-THERM-033-045
15	UF_4	3%	-	cuboid	133	LEU-COMP-THERM-033-049
16	UF_4	3%	-	cuboid	277	LEU-COMP-THERM-033-051
17	UO_2F_2	5%	water	cylinder	501	-
18	UO_2F_2	4.89%	water	cylinder	526	-
19	UO_2F_2	4.89%	water	cylinder	646	-
20	UO_2F_2	4.89%	water	cylinder	734	-
21	UO_2F_2	4.89%	water	cylinder	989	-
22	UO_2F_2	4.89%	water	sphere	1098	LEU-SOL-THERM-002-001
23	UO_2F_2	4.89%	-	cylinder	526	-
24	UO_2F_2	4.89%	-	cylinder	645	-
25	UO_2F_2	4.89%	-	cylinder	734	-
26	UO_2F_2	4.89%	-	cylinder	989	-
27	UO_2F_2	4.89%	-	sphere	1001	LEU-SOL-THERM-002-002
28	UO_2F_2	5%	water	cylinder	489	-
29	UO_2F_2	5%	water	cylinder	489	-
30	UO_2F_2	5%	water	cylinder	489	-
31	UO_2F_2	5%	water	cylinder	489	-
32	UO_2F_2	5%	water	cylinder	489	-
33	UO_2F_2	5%	water	cylinder	489	-
34	UO_2F_2	5%	-	sphere	488	-

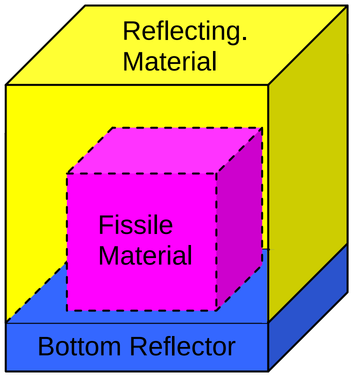


Figure 4 Cuboid, homogeneous experiments

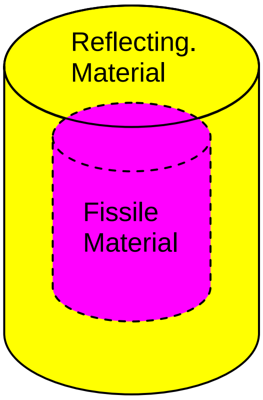


Figure 5 Cylinder, homogeneous experiments

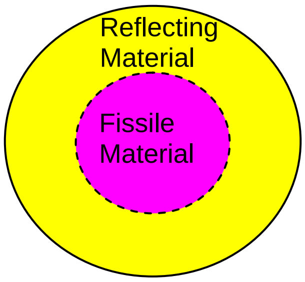


Figure 6 Sphere, homogeneous experiments

2.1.3 CSNI Homogeneous Experiments

Homogeneous experiments have been assessed in the CSNI Report No. 71 and 78. The CSNI reports contain simulated experiments and one 'real' experiment, where several of them have been evaluated with the programs, PHOENIX-4 and KENO-V.A [6]. The CSNI report no. 71 deals with fuel transport containers and no. 78 with large finite and infinite arrays of fissile units. The CSNI reports can be found in Ref. [7] and [8]. All the CSNI-cases are excluded from the statistical evaluation since the simulated experiments are not critical and the experimental case is similar to case 3 in the H/U = 0.77-series. The experiments were performed at room temperature.

The notation of the CSNI-based calculations corresponds to the one used in Ref. [6] and is abbreviated as; a-b-c, where
a = number of CSNI report (71 or 78)
b = EXP/SIM (experimental or simulated)
c = number of the problem.

The results of the calculations are compared with the following reference k_{eff} values (Table 17), which are given by:

- The results of the experiments (EXP) (in general critical experiments).

- A value agreed among the participants of these exercises for some of the simulated cases, (SIM).

- The range of values including the average value obtained by the different calculation methods for those simulated cases in which no agreement was reached, (SIM).

2.1.3.1 71-SIM-4 A and B

The 71-SIM-4A system is a simulated PWR spent fuel shipping cask, containing seven fuel elements consisting of 2.35% enriched UO_2 fuel rods (17×17 pins), surrounded by Boral plates (0.25 cm) and flooded with water. In this model the fuel is homogenized. The cask is cylindrical with two shells of steel encompassing a biological shield of thick lead.

The optional configuration 71-SIM-4A (o) is the same as above but with water added between the rectangular box containing the fuel and the circular biological shield.

In 71-SIM-4B the system is same as for 71-SIM-4A but with a biological shield of thick steel instead of lead. The geometrical details are displayed in Figure 7.

2.1.3.2 71-SIM-5

71-SIM-5 is the same as for 71-SIM-4B, but with 4.75% enriched UO_2 rods and water of reduced density (0.16 g/cm^3) between the seven fuel elements. The pins are flooded with water with a density of 1.0 g/cm^3.

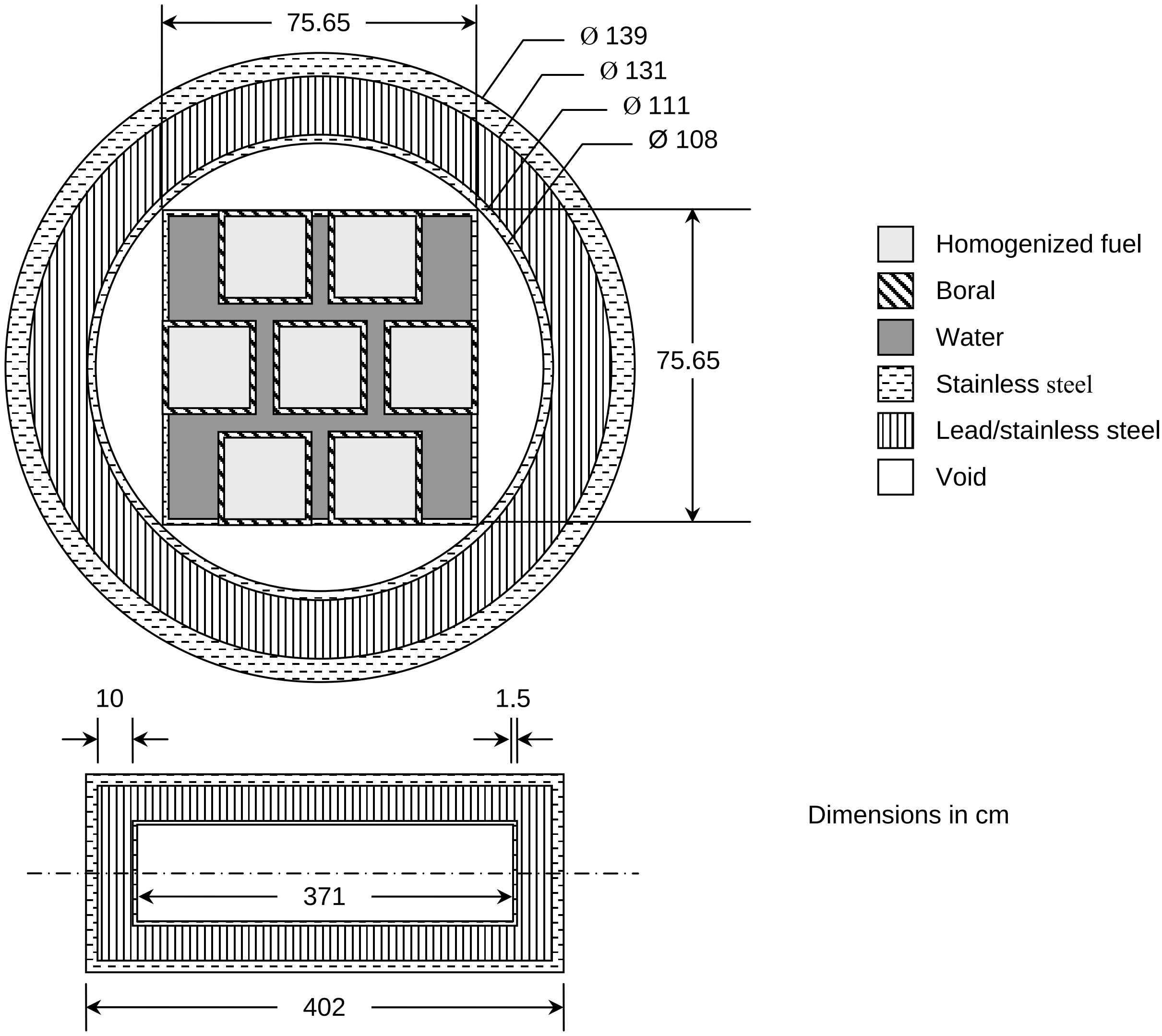


Figure 7 Side views of containment and fuel in 71-SIM-4 A and B, and 5.

2.1.3.3 78-SIM-4 A and B

78-SIM is a simulated array of 8×8×8 cubic transport packages (cube side = 60 cm). Each package contains a sphere (R = 14 cm radius) of a mixture of 5% enriched UO_2 and water (H/U = 20, density 2.2 g/cm^3). The array is reflected on all sides by 20 cm of water. Each fuel sphere is surrounded by a water shell of thickness (T);
A) 0.0 cm,
B) 4.0 cm.
The remaining space in the cube is filled with air. Figure 8 displays the geometry details for one unit package.

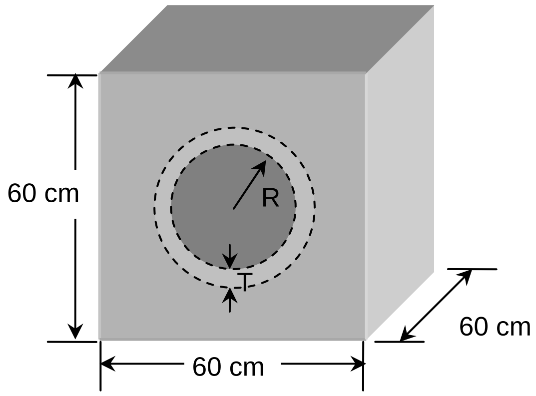


Figure 8 One unit package in 78-SIM-4

2.1.3.4 78-EXP-3

78-EXP-3 is a system consisting of a 5×4×5 array of cubes containing fissile powder. The cube sides are 15.3 cm, including a 0.15 cm thick aluminum box wall. The cubes are separated by 0.923 cm thick Plexiglas reflector plates on all sides. The fissile material is U_3O_8 with water injected and has a moderation ratio of H/U = 0.77 and a density of 4.7 g/cm^3. The ^{235}U enrichment is 4.46%. This experiment is actually the same as Case 3 from the series of homogeneous experiments with H/U = 0.77. The reasons for evaluating it again are the following:

- To reinvestigate the same evaluated experiments as for the reference report [6].

- The description of the experiment is less detailed in CSNI-report [8] than in the IHECSBE [3]. But to make a good comparison with previous result of this experiment is the description from CSNI-report chosen.

An extra calculation was performed considering an infinite array of these core boxes separated by Plexiglas plates, 78-EXP-3(inf). Figure 9 shows the construction of the core and reflectors.

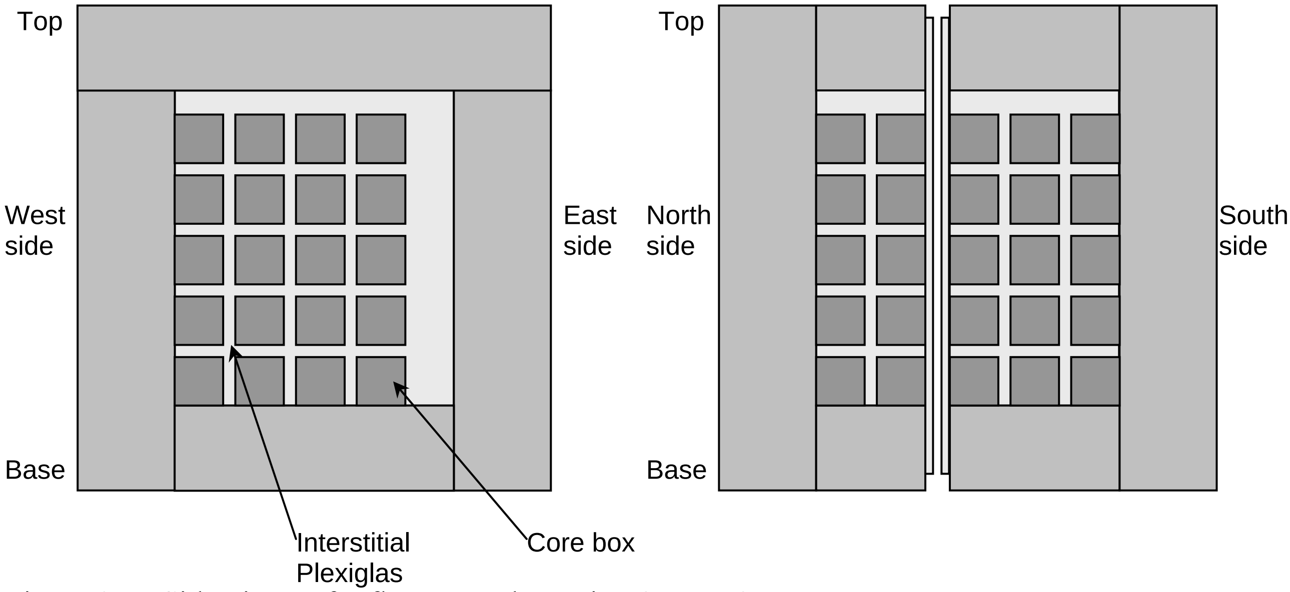


Figure 9 Side views of reflectors and core in 78-EXP-3

2.2 HETEROGENEOUS SYSTEMS

2.2.1 Underwater Close-Packed Storage of LWR Fuel

Underwater closed-packed storage of LWR fuel is evaluated. To achieve an optimum use of the spent fuel storage capacity, the fuel must be stored with the minimum of spacing consistent with nuclear criticality safety. This requires that neutron absorbing materials, in the form of pins or plates, are placed between the assemblies. Between 1977 and 1979, at the CX-10 criticality facility of Babcock-Wilcox (B&W) Lynchburg Research Center, experiments were made to provide benchmark criticality data.

The assemblies in the experiments consisted of low-enriched UO_2 fuel rods in a water-moderated lattice. This was used to simulate a variety of close-packed LWR fuel storage configurations. The critical assemblies consisted of nine LWR-type 14×14 pin-clusters grouped in a 3×3 array. The spacing and material between the clusters could be varied to provide numerous critical configurations. Twenty-one benchmark cores were loaded and brought critical by increasing the water level. All cores were constructed inside a tank-type facility. The water inside the tank provided full reflection in the radial direction. All experiments were performed at room temperature. The benchmark cores are divided into two groups and are here entitled with roman numbers, where the code for the cores; I-IX are from Ref. [10], and the following cores, X-XI, are from Ref. [11]. Both references are found in IHECSBE.

Core I was a reference cylindrical core loaded with 438 fuel rods arranged in a homogeneous square lattice. Cores II-XXI were constructed as nine 14×14 fuel rod clusters grouped in a 3×3 array. All fuel rods were separated by a square pitch of 1.636 cm.

The following materials were used in the water gap separating the fuel rod clusters in cores II-XXI: B_4C-filled pins; stainless steel sheets; borated aluminum sheets; in core II, III and X-XXI there were boron in different concentrations in the moderator.

The first group includes cores I through IX. The second group includes cores X through XXI and used metal isolation sheets in the gap between the fuel clusters. The four corner rods in each cluster were replaced by solid aluminum rods with a 1.270 cm diameter. To position the metal sheets, aluminum locating bars were used at the bottom between the fuel clusters. The fuel rests on a 5.08 cm thick aluminum base plate, located 2.54 cm above the tank bottom. Moderator heights are referenced from the top of the aluminum base plate. Fuel rods are aluminum tubes filled with 2.46% enriched UO_2 pellets. Table 4 gives the properties of the fuel rods. Figure 10 to Figure 13 illustrate the different cores.

Table 4. Characteristics of the fuel elements for the underwater close-packed storage of LWR fuel.

Fuel material	UO_2 pellets
Enrichment	2.459 % ^{235}U
Pellet diameter	1.030 cm
Pellet density	10.29 g/cm^3
Active fuel length	153.34 cm
Cladding material	6061
Clad outer diameter	1.206 cm
Wall thickness	0.081 cm
Total length of a rod	100 cm
Rod pitch	1.636 cm

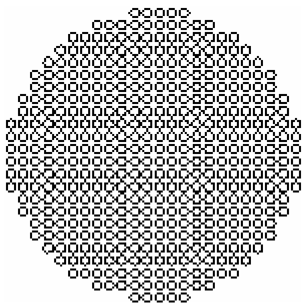


Figure 10 Core I loading diagram – cylindrical base case

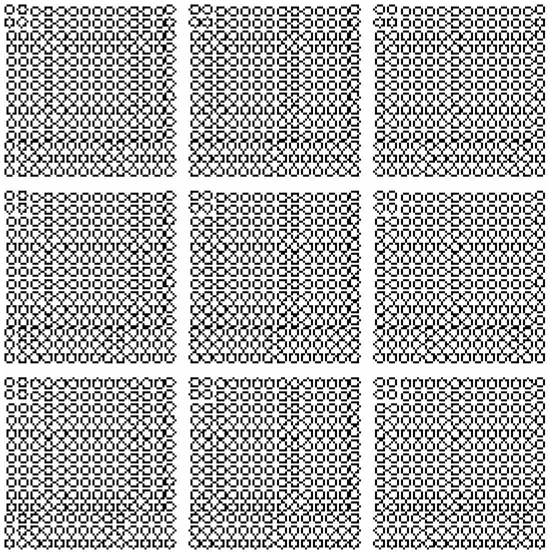


Figure 11 Core II, III, X and IX loading diagram – nine arrays separated by 0-4 pin pitch

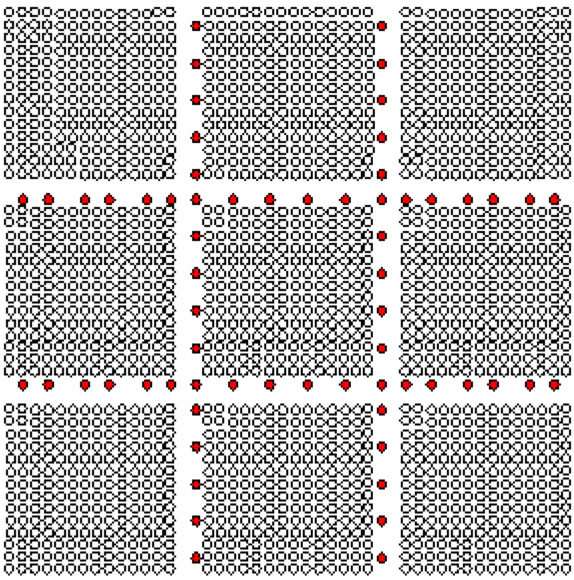


Figure 12 Cores IV, V, VI, VII and VIII - nine assemblies separated by different number of pin pitches and B_4C pins.

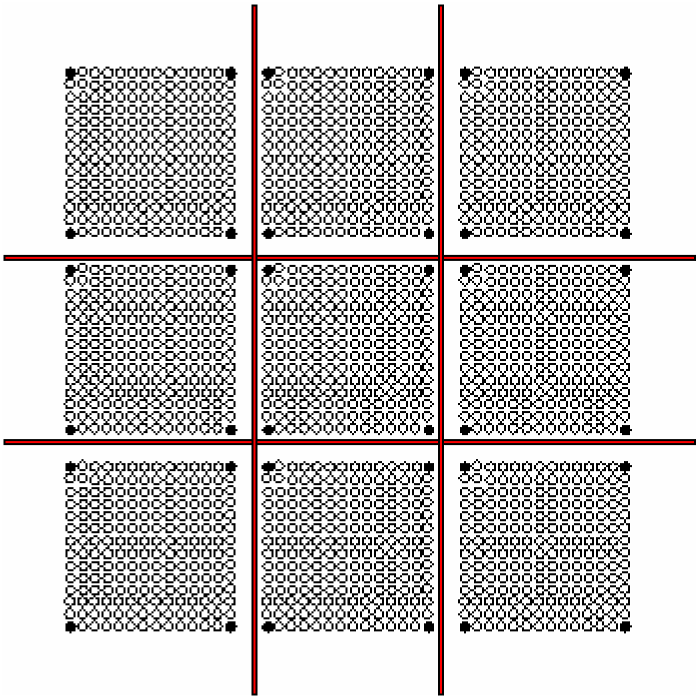


Figure 13 Cores XI–XXI - nine assemblies separated by different number of pin pitches and metal isolation sheets.

2.2.2 KRITZ-experiments

2.2.2.1 KRITZ-experiments with Burnable Absorbers

The KRITZ-experiments with burnable absorbers (KRITZ-BA) begun in 1975 when AB Atomenergi performed a series of experiments on BWR-assemblies as a cooperation project together with ASEA-ATOM, Statens Vattenfallsverk and AKK [12]. The experiments have been evaluated with MCNP4C2 [13]. The approach for this report is to rerun the MCNP4C2-input files in MCNP5 and with updated cross-section libraries. The input files are also revised due to new information about the enrichment in some of the fuel rods [14].

One assembly consists of 8×8 rods, where the corner rods have small displacements from the centre of the assembly. The cores are either constructed with 16 assemblies (cases 1-6) or with 21 assemblies (cases 7-10). There are 10 cases in this series that have been recalculated from Ref. [13]. The cases 2, 4 and 6 are hot cases and have a water temperature of about 240 °C. The hot cases are evaluated here but have been excluded from the statistical evaluation, since they differ too much from the other experiments in this report. The cases left to be statistically evaluated are 1, 3, 5 and 7-10, where all of them were performed at room temperature. Case 8 and 10 use no water in the central assembly. Table 5 displays the characteristics of the assemblies, the boxes and the spacers, whereas Table 6 displays the characteristics of the fuel elements.

Table 5 Characteristics of the assemblies, the boxes and the spacers for KRITZ-BA experiments

Rods per assembly		8×8
Rod pitch	normal rod –normal rod	1.630 cm
	normal rod – corner rod	1.605 cm
	corner rod – corner rod	15.80 cm
Box	Material	Zircalloy-4
	Wall thickness	0.23 cm
	Horizontal inner side	13.4 cm
	Corner radius	1.02 cm
Spacers	Material	Inconel-X 750
	Weight	135 g
	Vertical separation	56.1 cm

In the hot cases other thermal $S(\alpha,\beta)$ cross-section libraries have been utilized. As libraries only exist for 400 and 600 K, and not for the exact 513 K (240 °C), calculations at each temperature has been performed. By using interpolation is the k_{eff} obtained.

The names of the assemblies are labeled: e.g. *3X* where the *3* is the number of BA-rods and *X* is the poisoning level per BA-rod ($X = 1.0$, $Y \approx 0.8$). These labels are used in Figure 20 to Figure 25. Figure 14 explains how a standard assembly is constructed. There are also two more types of assemblies; *a* and *b*, which are shown in Figure 15 and Figure 16. They are similar to the standard assembly except for the presence of 1.860 % enriched rods. The reason for having the two extra assemblies is explained in Ref. [14]. The assembly configurations where 2, 3 or 5 BA-rods are present are illustrated in Figure 17, Figure 18 and Figure 19.

Figure 20 to Figure 25 show the evaluated cores. An unlabeled square, in the core figures, means a standard assembly. The Ref. corner is also marked in the figures, where one can see the assembly rotation. There are 12 corner rods in each assembly with decreased rod diameter and they are situated three and three in each corner. For the assemblies *a* and *b*, with 1.860 % enriched rods, the new rods use the same diameter as the displaced rod.

Table 6 Characteristics for the fuel elements for the KRITZ-BA experiments. (* type Y has two values, with and without Gd_2O_3.)

		Corner rods		Normal rods			BA-rods	
Rod type		3	6H	6N	A	C	X	Y*
Fuel material		UO_2	UO_2	UO_2	UO_2	UO_2	UO_2	UO_2
^{235}U in U, wt%		1.409	2.086	2.086	2.813	3.074	3.069	3.069 3.089
Fuel	diameter, mm	10.08	10.08	10.58	10.58	10.58	10.58	10.58 10.58
	density, g/cm^3	10.38	10.35	10.41	10.35	10.38	10.20	10.20 10.28
	Gd_2O_3 wt%	-	-	-	-	-	1.975	1.970 0
	mass/rod, g	3008		3311			2382	2386
	Length, mm	3650		3650			2650	
Clad	material Zircalloy-4							
	thickness, mm	0.74		0.74				
	outer diameter, mm	11.75		12.25				

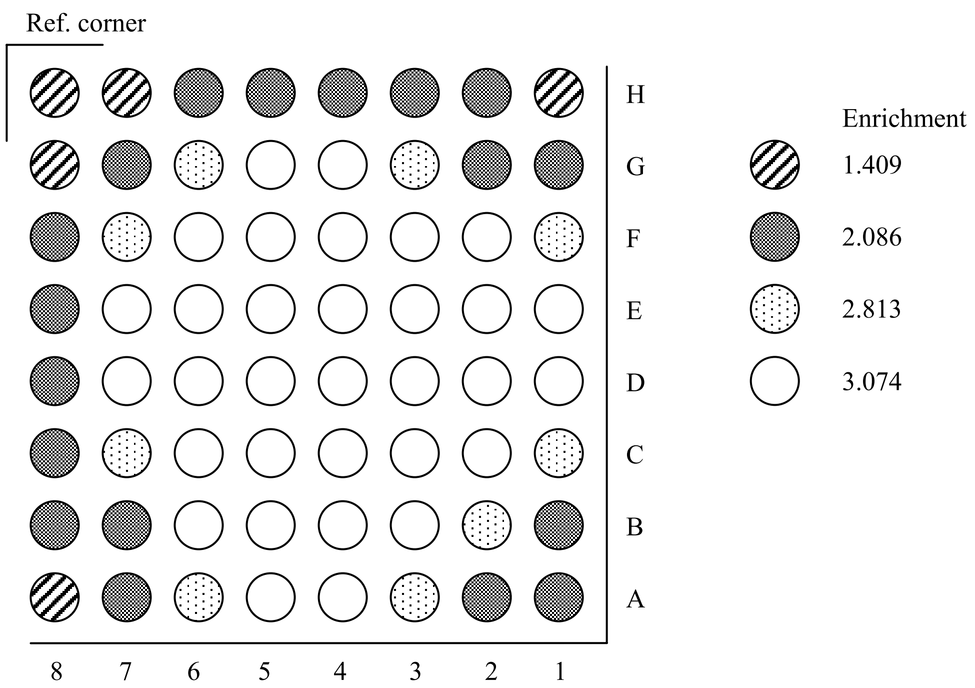


Figure 14 Standard assembly, KRITZ-BA

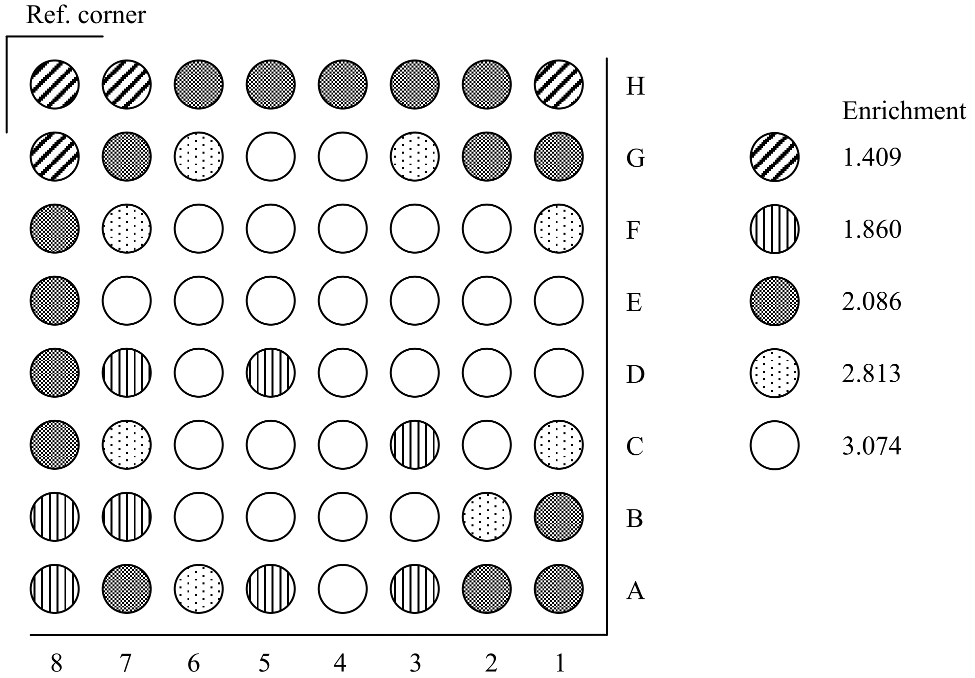


Figure 15 Assembly *a*, addition of 1.860 %-enriched rods, KRITZ-BA

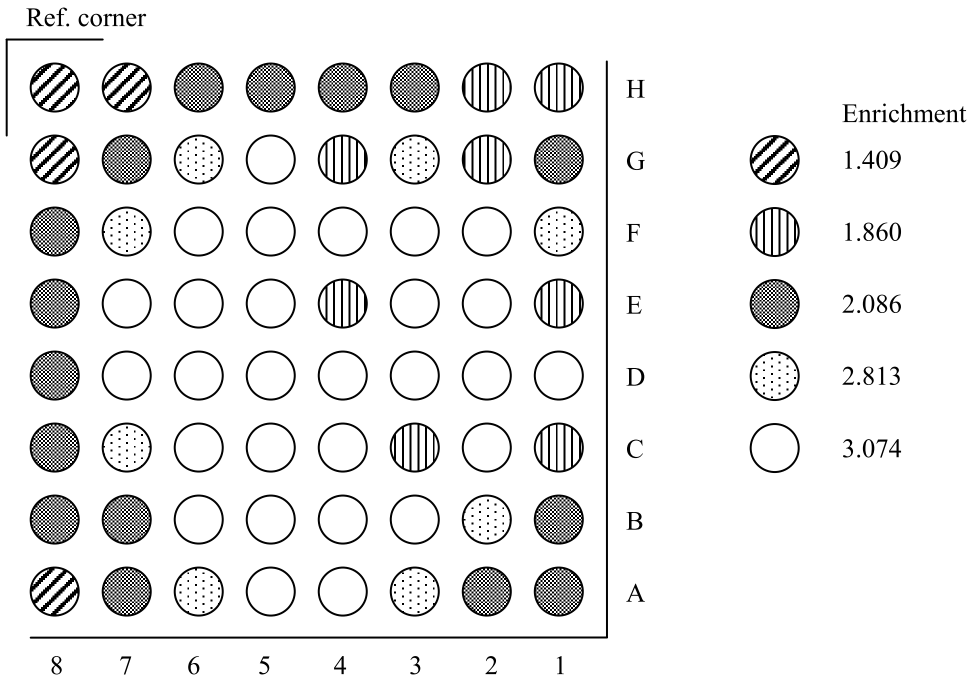


Figure 16 Assembly *b*, addition of 1.860 %-enriched rods, KRITZ-BA

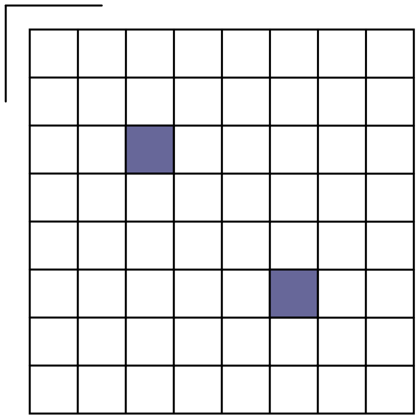


Figure 17 2 BA-rods, KRITZ-BA

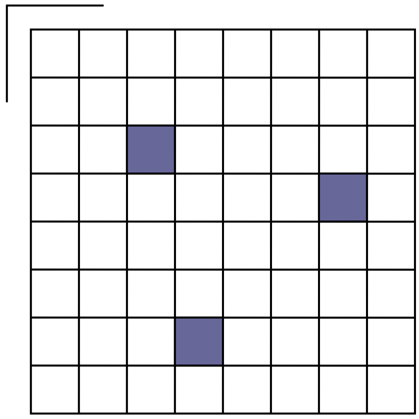


Figure 18 3 BA-rods, KRITZ-BA

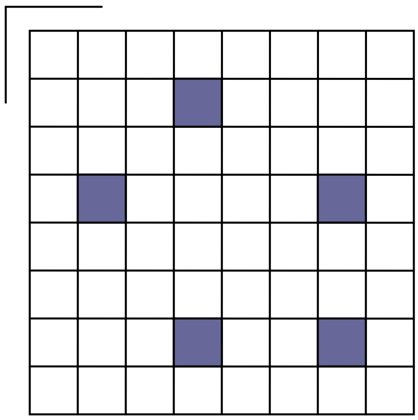


Figure 19 5 BA-rods, KRITZ-BA

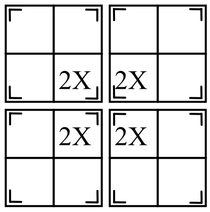


Figure 20 Core 2:3
in case 1 and 2

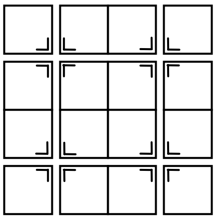


Figure 21 Core 3:1
in case 3 and 4

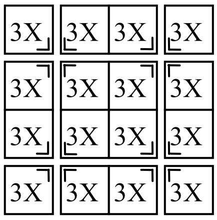


Figure 22 Core 5
(cold) in case 5

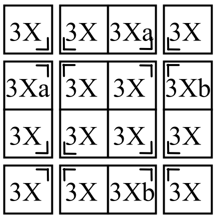


Figure 23 Core 5 (hot) in case 6

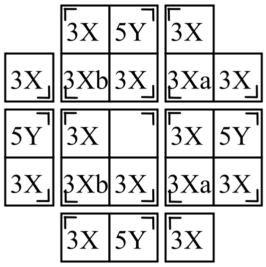


Figure 24 Core 6:1 in case 7 and 8

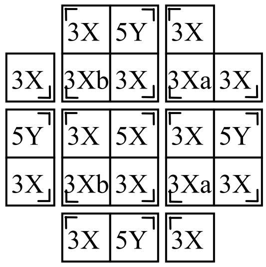


Figure 25 Core 6:2 in case 9 and 10

2.2.2.2 KRITZ-experiments with MOX fuel

The KRITZ-experiments with MOX-fuel were performed 1972 by AB Atomenergi as a cooperation project together with ASEA-ATOM, Statens Vattenfallsverk and AKK [15].

One assembly consists of 8×8 rods, where the corner rods have small displacement from the centre of the assembly. The cores are constructed with 16 assemblies. There are 4 cases in this series that have been named case 11 to 14. The cases 12 and 14 are hot cases and have a water temperature of 239 and 88 °C. All four cases are evaluated here but have been excluded from the statistical evaluation, since they differ too much from the other experiments in this report. In the hot cases other thermal *S(*α,β*)* cross-section libraries have been utilized. As libraries only exist for 400 and 600 K, and not for the exact 512 K (239 °C), calculations at each temperature has been performed. By using interpolation is the k_{eff} obtained. Table 7displays the characteristics of the assemblies, the boxes and the spacers.

Table 7 Characteristics of the assemblies, the boxes and the spacers for KRITZ-MOX experiments

Rods per assembly		8×8
Rod pitch	normal rod –normal rod	1.630 cm
	normal rod – corner rod	1.605 cm
	corner rod – corner rod	15.80 cm
Box	Material	Zircalloy-4
	Wall thickness	0.20 cm
	Horizontal inner side	13.4 cm
	Corner radius	1.04 cm
Spacers	Material	Inconel-X
	Weight	135 g
	Vertical separation	54.9 cm

As the KRITZ-experiments with MOX fuel are very similar to the experiments with burnable absorbers, therefore are the MCNP-input files from the burnable absorbers used as basis for the MOX-input files.

Figure 26 and Figure 27 displays the two types of assemblies, which are used in the core 1:4 and 1:5 as in Figure 28 and Figure 29. Table 8 lists the characteristics of the fuel element.

Table 8 Characteristics of the fuel elements for the KRITZ-MOX experiments

		Pu-rod	U-rod
Fuel material		PuO_2/UO_2	UO_2
^{235}U in U, wt%		0.16	1.86
PuO_2 in UO_2, wt%		1.50	-
Pu-composition, at%			
	^{239}Pu	91.41	-
	^{240}Pu	7.83	-
	^{241}Pu	0.73	-
	^{242}Pu	0.03	-
Fuel	diameter, mm	9.45	10.58
	Length, mm	1232	3650
	mass/rod, g	828	3260
	Density, g/cm^3	9.58	10.3
Clad	material	Zircalloy-4	Zircalloy-4
	thickness, mm	0.68	0.74
	outer diameter, mm	10.79	12.25

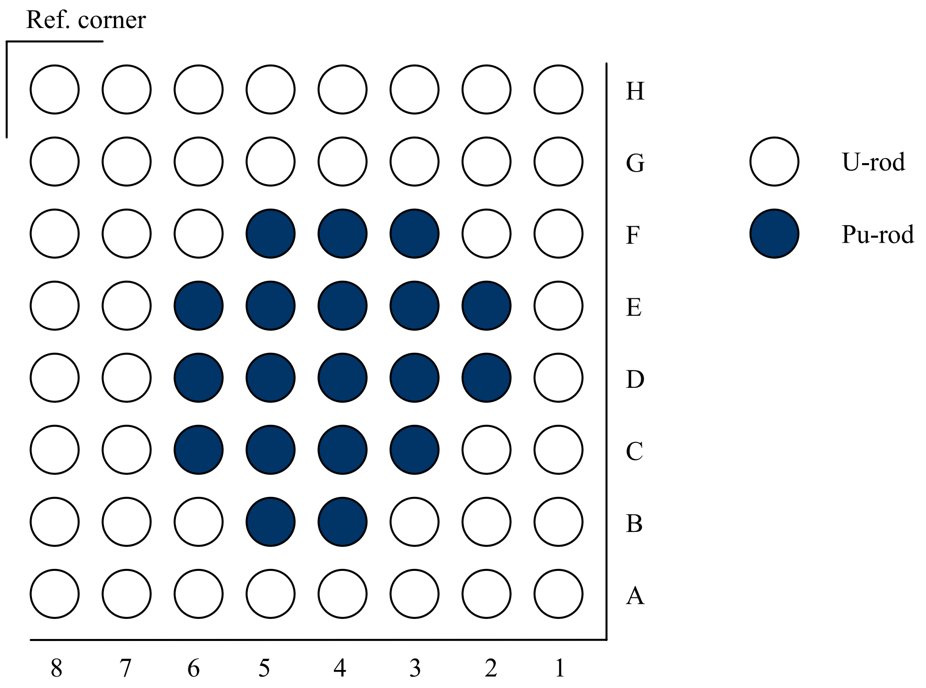


Figure 26 Assembly named PU-island, KRITZ-MOX

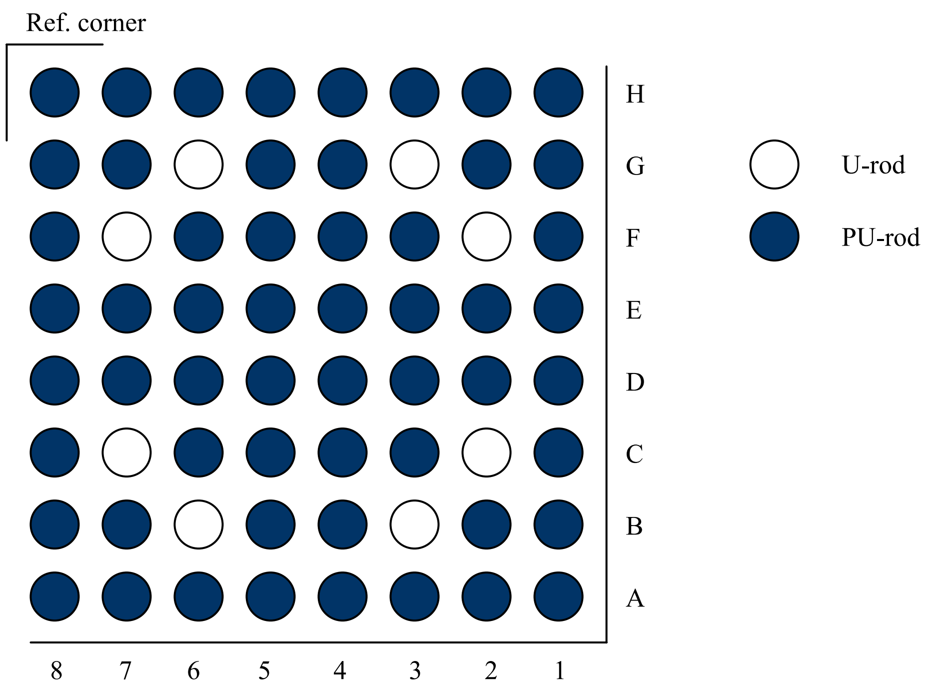
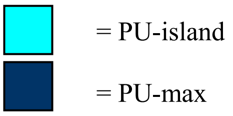


Figure 27 Assembly named PU-max, KRITZ-MOX



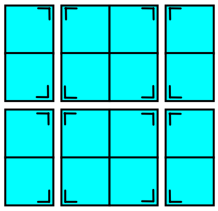


Figure 28 Core 1:4
in case 11 and 12

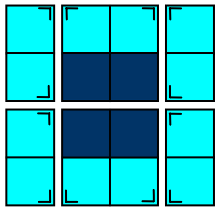


Figure 29 Core 1:5
in case 13 and 14

2.2.3 Storage Experiments

The storage experiments are benchmark calculations based on experiments performed in 1979 at the Nuclear Safety Department of the French Commissariat à l'Energie Atomique. The experiments are described in Ref. [16].

The experimental arrangement consists of four assemblies of 18×18 rods, placed in an experimental tank. The assemblies are separated by a cross-shaped aluminum container filled with hydrogenous materials, Figure 30 and Figure 31. All experiments were performed at a temperature of 22 °C. The hydrogenous materials are:

- Air
- Expanded polystyrene $(C_8H_8)_n$
- Polyethylene powder $(CH_2)_n$
- Polyethylene balls $(CH_2)_n$
- Water

The criticality has been achieved by increased water level and the different heights, together with other system data are showed in Table 10. Characteristics of the fuel element can be found in Table 9.

Table 9 Characteristics of the fuel elements in the storage experiments

Fuel material	UO_2 pellets
Enrichment	4.472 ± 0.001 % ^{235}U
Pellet diameter	0.790 ± 0.002 cm
Pellet height	1.495 ± 0.007 cm
Average weight of UO_2 per rod	457.5 g
Pellet density	10.38 ± 0.04 g/cm^3
Sum of the impurities on boron equivalent	5.19×10^{-6} g/cm^3
Total length of fuel in rod	90 cm
Cladding material	aluminum alloy (AG 5)
Composition wt%	Al: 98.85, Mg: 0.47, Si: 0.43, Fe: 0.22 and Zn: 0.03
Clad inner diameter	0.82 cm
Clad outer diameter	0.94 cm
Total length of a rod	100 cm

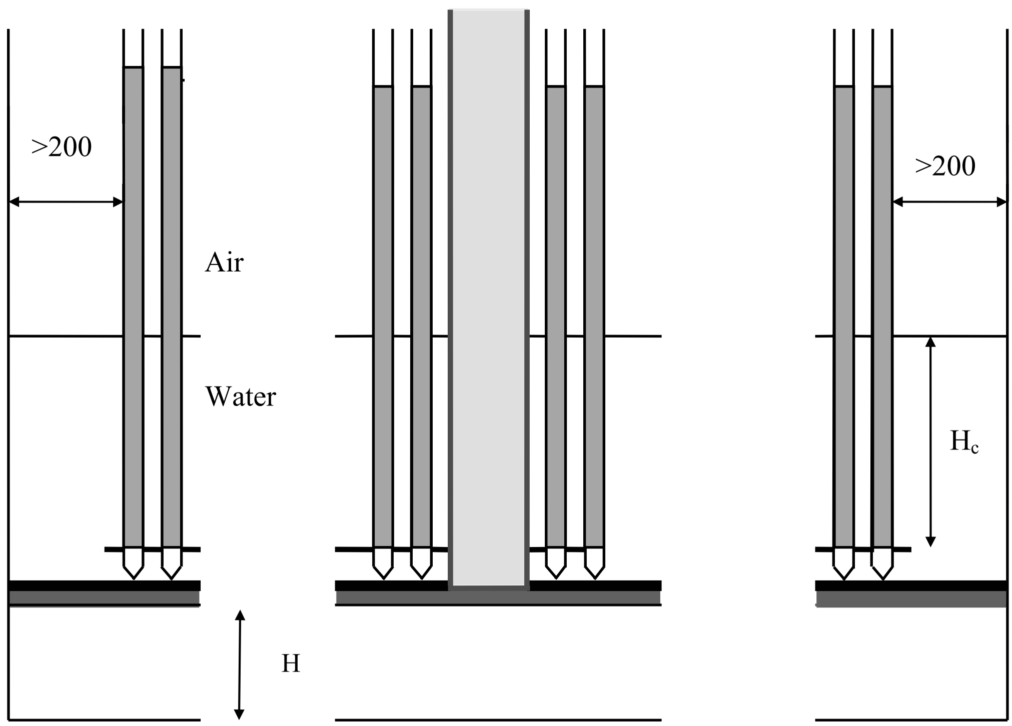


Figure 30 Experimental setup, side view, in the storage experiments

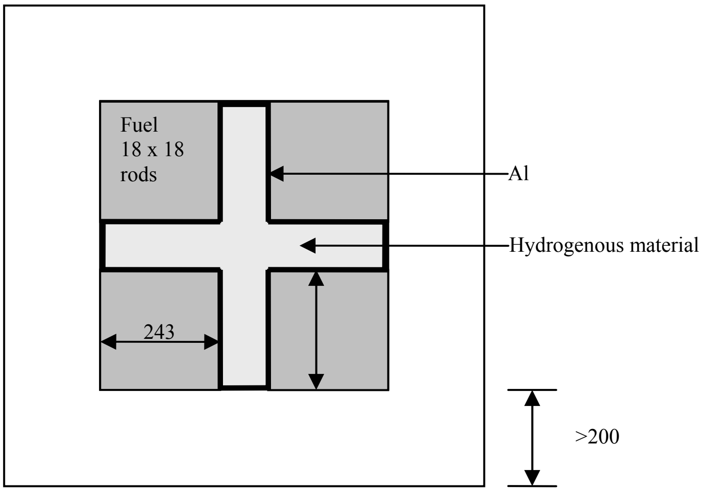


Figure 31 Experimental setup, top view, of the storage experiments

Table 10 Characteristics of the systems of the storage experiments

Container width [mm]	Type of system	Density [g/cm^3]	Hydrogen conc. [g/cm^3]	Water critical height [mm]
0	Water	1.0	0.1119	238 ± 0.6
25	Box + air	0	0	290.3 ± 0.9
	Box + $(C_8H_8)_n$	0.0323	0.0025	286.1 ± 0.8
	Box + $(CH_2)_n$ powder	0.2879	0.0414	269.8 ± 0.6
	Box + $(CH_2)_n$ balls	0.5540	0.0800	255.4 ± 0.6
	Box + water	1.0	0.1119	256.6 ± 0.7
	Water	1.0	0.1119	244.8 ± 0.6
50	Box + air	0	0	344.8 ± 0.7
	Box + $(C_8H_8)_n$	0.0262	0.0020	343.9 ± 0.8
	Box + $(CH_2)_n$ powder	0.3335	0.0480	301.6 ± 0.6
	Box + $(CH_2)_n$ balls	0.5796	0.0833	307.3 ± 0.8
	Box + water	1.0	0.1119	327.8 ± 0.8
	Water	1.0	0.1119	314.7 ± 0.6
100	Box + air	0	0	460.8 ± 0.7
	Box + $(C_8H_8)_n$	0.0288	0.0022	456.2 ± 0.8
	Box + $(CH_2)_n$ powder	0.3216	0.0464	420.5 ± 0.6
	Box + $(CH_2)_n$ balls	0.5680	0.0816	499.4 ± 0.6
	Box + water	1.0	0.1119	641.2 ± 0.9
	Water	1.0	0.1119	643.4 ± 0.8

2.2.4 Heterogeneous Experiments used for Validation at Westinghouse Columbia

The Westinghouse Columbia heterogeneous experiments are benchmarks for light-water-reactor (LWR) fuel applications. The experiments have been evaluated with MCNP4B [17]. They were selected based on materials chosen for the new Westinghouse shipping container and the fuel assemblies to be loaded into the container. The fuel assemblies considered were undamaged LWR fuel with zircalloy cladding.

The input files from the MCNP4B-evaluation [17] are revised due to new data from the IHECSBE. Table 11 and Table 12 show which of the experiments that have been revised. Other experiments in the table, marked *N/A*, have not been revised and the input files from MCNP4B [17] have been retained.

Criticality has been achieved by varying the water height for the Hexcrit- and the nse71-experiments, Table 13. For all other experiments the criticality is obtained by varying the distance between the clusters. Figure 32 to Figure 35 display the four different types of configurations for the experiments.

Table 11 Experiments from IHECSBE (2005). (Length dimensions in cm) included in the Westinghouse Columbia heterogeneous experiments

Experiment	Cluster dimensions (number of rods)	Enrich-ment	Pitch	Plate Mat	B (wt%)	Plate Thick	Separation between clusters	Wall	Distance between reflecting walls and fuel clusters	Report identity in IHECSBE (LEU-COMP-THERM-…)
Hexcrit1	3267 (circular)	5.06%	1.300	-	-	-	-	SS	-	020-001
Hexcrit2	1305 (circular)	5.06%	1.300	-	-	-	-	SS	-	020-002
Hexcrit3	1051 (circular)	5.06%	1.300	-	-	-	-	SS	-	020-003
Hexcrit4	952 (circular)	5.06%	1.300	-	-	-	-	SS	-	020-004
Hexcrit5	842 (circular)	5.06%	1.300	-	-	-	-	SS	-	020-005
Hexcrit6	785 (circular)	5.06%	1.300	-	-	-	-	SS	-	020-006
nse71sq	22×22 (484)	4.74%	1.260	-	-	-	-	-	-	007-001
nse71w1	459	4.74%	1.260	-	-	-	-	-	-	039-001
nse71w2	420	4.74%	1.260	-	-	-	-	-	-	039-003
p3602n11	25×18 (center) 20×18 (two outer)	2.35%	1.684	-	-	-	8.566	SS	0.0	017-015
p3602n12	25×18 (center) 20×18 (two outer)	2.35%	1.684	-	-	-	9.166	SS	0.66	017-016
p3602n13	25×18 (center) 20×18 (two outer)	2.35%	1.684	-	-	-	9.246	SS	1.684	017-018
p3602n14	25×18 (center) 20×18 (two outer)	2.35%	1.684	-	-	-	8.126	SS	3.912	017-021
p3602n21	19×16	2.35%	2.032	-	-	-	9.598	SS	2.616	017-013
p3602n22	19×16	2.35%	2.032	-	-	-	10.438	SS	0.66	017-011
p3602n31	12×16	4.31%	1.892	-	-	-	14.393	SS	0.0	010-014
p3602n32	12×16	4.31%	1.892	-	-	-	15.263	SS	0.660	010-015
p3602n33	12×16	4.31%	1.892	-	-	-	15.393	SS	1.321	010-016

Table 12 Continuing of Table 11

Experiment	Cluster dimensions (number of rods)	Enrich-ment	Pitch	Plate Mat	B (wt%)	Plate Thick	Separation between clusters	Wall	Distance between reflecting walls and fuel clusters	Report identity in IHECSBE (LEU-COMP-THERM-…)
p3602n34	12×16	4.31%	1.892	-	-	-	15.363	SS	1.956	010-017
p3602n35	12×16	4.31%	1.892	-	-	-	14.973	SS	2.616	010-018
p3602n36	12×16	4.31%	1.892	-	-	-	13.343	SS	5.405	010-019
p3602n41	13×8	4.31%	2.540	-	-	-	11.765	SS	0.0	010-009
p3602n42	13×8	4.31%	2.540	-	-	-	12.995	SS	1.321	010-011
p3602n43	13×8	4.31%	2.540	-	-	-	11.315	SS	2.616	010-012
p3314ba	11×16 (upper) 11×14 (lower)	4.31%	1.892	B	28.7	0.713	2.823 (x) 4.793 (y)	-	-	N/A
p3602bb	12×16	4.31%	1.892	B	30.4	0.292	83.0	SS	1.956	013-003
p3314bc	11×14	4.31%	1.892	B	31.9	0.231	2.831 (x) 3.521 (y)	-	-	N/A
p3314bs1	25×20 (center) 17×20 (two outer)	2.35%	1.684	SS	1.1	0.298	3.86	-	-	012-002
p3314bs3	12×16	4.31%	1.892	SS	1.1	0.298	7.228	-	-	N/A
p3602bs1	25×18 (center) 20×18 (two outer)	2.35%	1.684	SS	1.1	0.298	4.80	SS	1.321	042-002
p3602bs2	12×16	4.31%	1.892	SS	1.1	0.298	9.83	SS	1.956	013-002
p3314bs2	25×20 (center) 17×20 (two outer)	2.35%	1.684	SS	1.6	0.298	3.46	-	-	012-003
p3314bs4	12×16	4.31%	1.892	SS	1.6	0.298	6.628	-	-	N/A
p2438ss	20×16	2.35%	2.032	SS	-	0.485	6.88	-	-	016-001
p2615ss	15×8	4.31%	2.540	SS	-	0.485	8.58	-	-	009-001
p3314ss1	9×2 (upper) 9×12 (lower)	4.31%	1.892	SS	-	0.302	2.834 (x) 3.382 (y)	-	-	N/A
p3314ss2	9×13 (upper) 9×12 (lower)	4.31%	1.892	SS	-	0.302	2.834 (x) 11.552 (y)	-	-	N/A
p3314ss3	9×5 (upper) 9×12 (lower)	4.31%	1.892	SS	-	0.485	2.835 (x) 4.475 (y)	-	-	N/A
p3314ss4	9×12	4.31%	1.892	SS	-	0.485	2.835 (x) 8.175 (y)	-		N/A
p3314ss5	25×20 (center) 17×20 (two outer)	2.35%	1.684	SS	-	0.302	7.80	-	-	012-001
p3314ss6	12×16	4.31%	1.892	SS	-	0.302	10.522	-	-	N/A
p3602ss1	25×18 (center) 20×18 (two outer)	2.35%	1.684	SS	-	0.302	8.28	SS	1.321	042-001
p3602ss2	12×16	4.31%	1.892	SS	-	0.302	13.75	SS	1.956	013-001

Table 13 Water critical height for water-level-varying experiments in the Westinghouse Columbia heterogeneous experiments

Experiment	Water critical height [mm]
Hexcrit1	213.8± 0.1
Hexcrit2	300.7 ± 0.1
Hexcrit3	347.4 ± 0.1
Hexcrit4	375.5 ± 0.1
Hexcrit5	423.6 ± 0.1
Hexcrit6	461.5 ± 0.1
nse71sq	90.69 ± 0.1
nse71w1	81.36 ± 0.07
nse71w2	73.05 ± 0.06

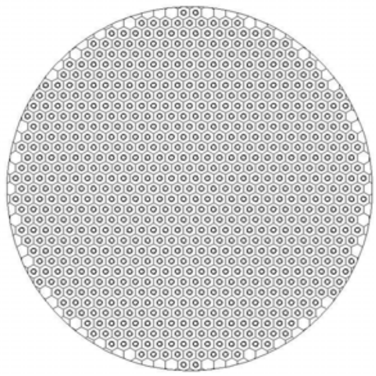


Figure 32 Case hexrit6

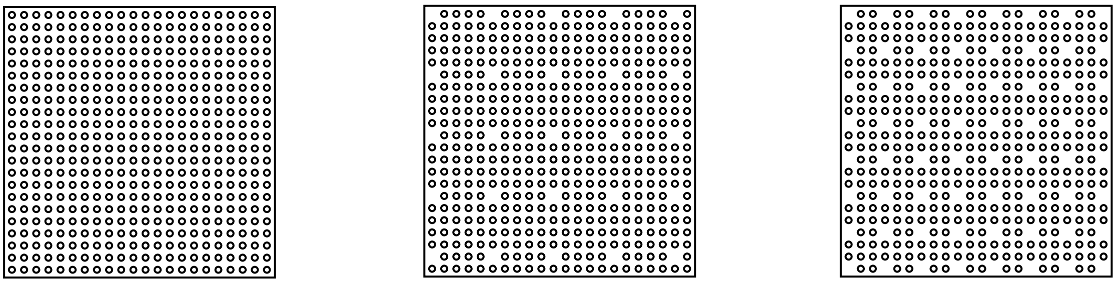


Figure 33 Case nse71sq nse71w1 nse71w2

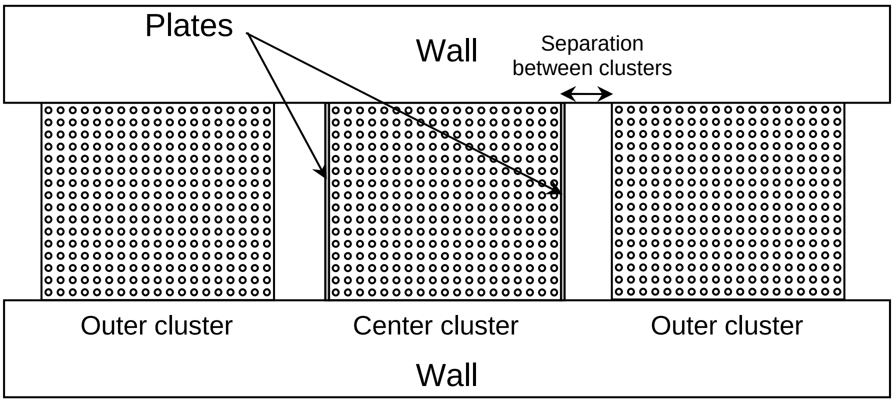


Figure 34 Cases with three clusters

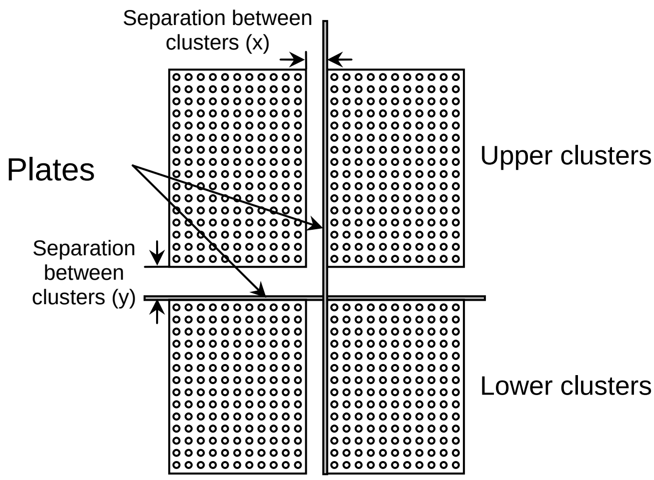


Figure 35 Cases with four clusters; p3314ba, p3314bc, p3314ss1, p3314ss2, p3314ss3 and p3314ss4

3 MODELING CONDITIONS

3.1 MCNP AND THE MONTE CARLO METHOD

3.1.1 MCNP

MCNP is a general Monte Carlo N-Particle transport code. It has the capability of transporting neutrons, electrons and photons. In this work the transporting of neutrons is considered. These features make it possible for MCNP to simulate different types of fissile systems, e.g. a nuclear reactor or a nuclear fuel container, and obtain information such as k_{eff}. The neutron energy range is up to 20 MeV for all isotopes and up to 150 MeV for some isotopes.

The user creates an input file for MCNP. This file contains information about the problem such as:

the geometry specifications,
the material specifications and cross-section selection,
the location and the characteristics of the neutron, photon, or electron source,
the type of answer desired, and
different techniques used to improve calculation efficiency and for variance reduction.

The geometry specification is made by constructing surfaces and cells. The surfaces are implemented as planes, cylinders, spheres, etc. From these planes are cells defined by intersections, unions and complements of the regions bounded by the surfaces.

Each cell is filled with a material or void. The material is described using nuclear data libraries, such as ENDF/B. Nuclear data tables exist for neutron interactions, neutron-induced photons, photon interactions, and thermal particle scattering $S(\alpha,\beta)$. Unique identifiers, called ZAIDs, are used to select specific data tables. The general form for the ZAID is ZZZAAA.nnX, where ZZZ is the atomic number, AAA is the mass number, nn is the unique evaluation identifier, and X indicates the class of data. Thermal data tables with $S(\alpha,\beta)$ scattering treatment at various temperatures are available for light and heavy water, beryllium metal, beryllium oxide, benzene, graphite, polyethylene, zirconium and hydrogen in zirconium hydride.

The location and the characteristics of the neutron, photon, or electron source can also be described in the input file. The source of a criticality calculation is normally set inside the fissile material. If the fissile material is heterogeneously spread, more than one source can be implemented.

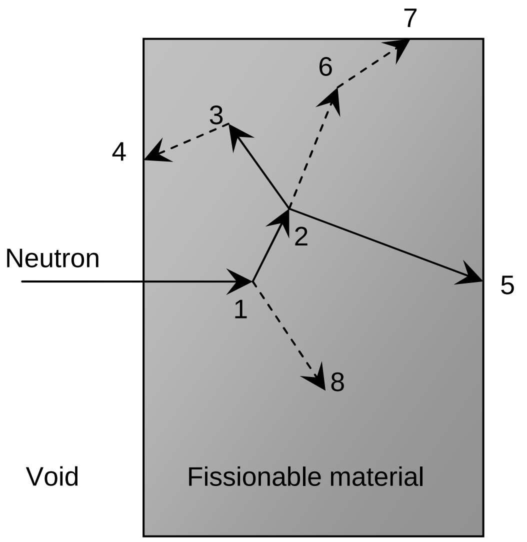
The type of answer desired from the calculation can be set in the input file. The output file for a criticality calculation contains information about: k_{eff} with an estimated standard deviation, the energy corresponding to the average neutron lethargy causing fission, percentage of fissions caused by thermal, intermediate and fast neutrons, average number of neutrons produced per fission.

Variance reduction techniques are primarily used for radiation transport problems and include methods such as truncation and population control methods. Truncation is that MCNP can operate energy or time cut-offs, which mean that the particle is killed below an energy threshold or if it exceeds a time cut-off. This saves computer time, but is dangerous because low-energy particles can produce fissions and in the end give error in the results. Population control methods use particle splitting or Russian roulette to control the number of particles in various regions. A better focus on interesting particles is hereby obtained, and computer time is saved. Unfortunately these variance reduction techniques are not efficient for k_{eff}, and therefore usually not used for criticality calculations.

3.1.2 The Monte Carlo Method

The Monte Carlo method is a stochastic process based on the selection of random numbers, analogous to throwing a dice in a gambling casino, and thereby the name “Monte Carlo”. The Monte Carlo method solves the transport equation by simulating individual particles and recording some aspects of their average behavior. This method is very different from the deterministic transport methods, where the most common is the discrete ordinates method, which solves the transport equation for the average particle behavior. The basic advantage of Monte Carlo codes over deterministic codes is that they require no average approximations in space, energy and time. This means that Monte Carlo codes are very well suited for complex problems that cannot be modeled by computer codes using deterministic methods. One disadvantage with Monte Carlo, however, is that the solution contains statistical errors. All results are estimations with uncertainties and the calculations can be pretty time consuming as the precision of the results increases.

In particle transport, the Monte Carlo technique is very realistic. It follows each of many particles from a source throughout its life to its death, with e.g. absorption and escape. The outcome at each step of its life is determined from a probability distribution, which is randomly sampled using transport data. Figure 36 is an example of a random history of a neutron incident.



Event log

1. Neutron scatter, photon production
2. Fission, photon production
3. Neutron capture
4. Photon leakage
5. Neutron leakage
6. Photon scatter
7. Photon leakage
8. Photon capture

Figure 36 A random history of a neutron incident.

Monte Carlo results represent an average of contribution from many histories sampled during a run. The statistical error (uncertainty) is related to the result, where more histories give less statistical error. The number of histories and the statistical method used give the precision of the calculations. The accuracy is a systematic error, which depend on three factors: the code, problem modeling, and the user. Code factors include: the physics features and the mathematical model in the calculations; uncertainties in data, such as atomic weights and cross-sections; and bugs in MCNP. The problem-modeling factors do very often contribute to a decrease in accuracy of a calculation. The geometrical description and characteristics of the material can sometimes be hard to describe and it contains uncertainties. Another problem-modeling factor is the model of the energy and angular distribution of the radiation source. The last factor affecting the accuracy is the user. An advanced user can understand results, create an advanced model and make an efficient code, whereas a less good user can not use all the tools in MCNP and can even make fatal mistakes, due to insufficient understanding of the calculation.

3.1.3 Physics in MCNP

As already said, a particle is followed from its birth (e.g. the source), throughout its life, to its death (e.g. absorption), see Figure 36. When following a neutron, MCNP starts to verify if it interacts or not in a medium. The probability for a first collision to occur between s and $s+ds$ along its line of flight is

$$p(s)ds = e^{-\Sigma_T s}\Sigma_T ds \tag{1}$$

where Σ_T is the macroscopic total cross-section. Setting ξ to be a random number, uniformly distributed on [0,1), with the line of flight to l, to be

$$\xi = \int_0^l p(s)ds = 1 - e^{-\Sigma_T l} \tag{2}$$

which can be written as

$$l = -\frac{1}{\Sigma_T}\ln(1-\xi) \tag{3}$$

Because 1- ξ is distributed in the same manner as ξ, the equation can be rewritten and an expression for the distance to collision is obtained,

$$l = -\frac{1}{\Sigma_T}\ln(\xi) \tag{4}$$

If the neutron collides with a nucleus, the following sequence occurs:

1. The collision nuclide is identified. If the material is composed of N different nuclei, the interaction occurs on nucleus k if

$$\sum_{i=1}^{k-1}\Sigma_{T,i} < \xi\sum_{i=1}^{N}\Sigma_{T,i} \le \sum_{i=1}^{k}\Sigma_{T,i} \tag{5}$$

For every new collision a new ξ on the interval [0,1) is created.

2. Either the *S(*α,β*)* treatment is used or the velocity of the target nucleus is sampled for low-energy neutrons.

3. Neutron capture is modeled. There are two ways to handle capture: Analogue capture, which is normal capturing, when the neutron is absorbed with probability of σ_a/σ_T. Implicit capture (default in MCNP5), which is when the neutron has a weight W_n that is decreased to

$$W_n' = \left(1 - \frac{\sigma_a}{\sigma_T}\right) W_n \tag{6}$$

   by the capture. (The neutron is still alive after the capture, but its contribution is smaller)

4. Either elastic scattering or inelastic reactions is selected when the *S(*α,β*)* treatment is not used. The new energy and direction of the outgoing track is determined. The probability that the reaction is elastic scattering is

$$\frac{\sigma_{el}}{\sigma_{inel} + \sigma_{el}} = \frac{\sigma_{el}}{\sigma_T - \sigma_a} \tag{7}$$

   and if the inelastic reaction occurs, the j^{th} reaction is chosen among M according to

$$\sum_{i=1}^{j-1} \sigma_i < \xi \sum_{i=1}^{M} \sigma_i \le \sum_{i=1}^{j} \sigma_i \tag{8}$$

5. The *S(*α,β*)* treatment is used instead of step 4 if the energy of the neutron is low enough and the appropriate *S(*α,β*)* table is present.

3.2 MODEL AND MCNP INPUT SETTINGS

All experiments were modeled with MCNP5. Two sets of cross-section libraries were used with source data based on ENDF/B-VI release 6 and 8, and pre ENDF/B-VII evaluations. ENDF is an abbreviation for Evaluated Nuclear Data Files, and is produced by the Cross Section Evaluation Working Group (CSEWG) based in USA and Canada [20]. The different source data libraries are named as follow;

- *endf66c* for ENDF/B-VI Release 6
- *actia* and *actib* for ENDF/B-VI Release 8
- *t16_2003* for pre ENDF/B-VII evaluations from LANL Group T-16

Set 1 uses the cross-section libraries ENDF/B-VI release 6. For the nuclides bromine and tin this library was not available, this made it necessary to use other cross-section libraries. Set 2 uses the latest ENDF/B version available at the time of the performed MCNP5-calculations, which turned out to be ENDF/B-VI release 6 and 8, and pre ENDF/B-VII. The reason for making two sets of calculations is to evaluate different source data and to see which of them that is preferable. The $S(\alpha,\beta)$ data libraries, e.g. *lwtr* and *poly*, which describe the hydrogen behavior in light water and in Polyethylene at different temperatures. In Appendix 2 all the cross-section and $S(\alpha,\beta)$ libraries used in the calculations are displayed. All the nuclides displayed in the table are not necessarily used in every experiment.

The level of precision in the results is dependent on the number of histories and the number of neutrons per history. As k_{eff} is calculated for each history more histories will give a better mean k_{eff}. A better precision is also obtained with more neutrons per cycle. The number of histories and neutrons per history in the calculations can be found in Table 14.

The neutrons start from an arbitrary distribution, causing a generally very large variance of results from the first history in comparison with the following histories. Therefore, the results from the first ~50 histories were skipped when calculating the average k_{eff}.

Table 14 Number of histories and number of neutrons per history.

Experimental model	History	Neutrons per history
Homogeneous experiments	1030	10000
Underwater close-packed storage of LWR fuel	1000	50000
KRITZ-experiments with burnable absorbers	1000	5000
Storage experiments	1015	10000
Westinghouse Columbia heterogeneous exp.	400	10000

4 RESULTS

Table 15 to Table 22 show the results of the Monte Carlo calculations expressed as effective multiplication factor (k_{eff}) ± 1σ. The benchmark k_{eff} and uncertainty are also listed. The cases with no benchmark data available are marked with a star *. These experiments have instead been given assumed values for experimental uncertainty and the assumptions are further explained in section 5.1 and in Table 24.

4.1 HOMOGENEOUS SYSTEMS

The results for the homogeneous systems are displayed followed by an illustrating diagram in Figure 37.

Table 15 Homogeneous experiments with H/U = 0.77 (case 1-21) and 1.25 (22-24)

Case	Benchmark k_{eff}	Benchmark 1σ	Set 1 (Calculated) k_{eff}	Set 1 (Calculated) 1σ	Set 2 (Calculated) k_{eff}	Set 2 (Calculated) 1σ
1	1.0017	0.0025	1.01141	0.00024	1.01373	0.00025
2	1.0017	0.0025	1.00613	0.00025	1.00905	0.00025
3	1.0029	0.0029	0.99081	0.00023	0.99441	0.00024
4	1.0029	0.0029	0.99834	0.00025	1.00165	0.00024
5	1.0006	0.0031	1.00177	0.00024	1.00469	0.00025
6	1.0039	0.0026	1.01278	0.00025	1.01474	0.00025
7	0.9996	0.0024	1.01238	0.00023	1.01496	0.00023
8	0.9996	0.0024	1.01235	0.00023	1.01407	0.00023
9	1.0014	0.0030	1.01044	0.00024	1.01312	0.00024
10	0.9989	0.0029	1.00282	0.00024	1.00573	0.00023
11	1.0020	0.0028	1.01784	0.00024	1.02031	0.00025
12	1.0020	0.0028	1.00570	0.00025	1.00838	0.00025
13	1.0044	0.0035	1.02365	0.00024	1.02555	0.00024
14	1.0044	0.0035	1.02310	0.00023	1.02493	0.00024
15	1.0014	0.0040	1.01197	0.00024	1.01537	0.00024
16	1.0014	0.0040	1.01036	0.00023	1.01325	0.00023
17	1.0045	0.0036	1.01264	0.00024	1.01556	0.00023
18	1.0041	0.0031	1.02698	0.00022	1.02833	0.00023
19	1.0041	0.0031	1.02358	0.00024	1.02596	0.00025
20	1.0026	0.0031	1.02089	0.00023	1.02253	0.00023
21	1.0026	0.0031	1.02104	0.00023	1.02223	0.00023
22	1.0014	0.0028	1.00843	0.00025	1.01137	0.00025
23	1.0017	0.0042	1.00133	0.00025	1.00449	0.00025
24	1.0021	0.0021	1.00181	0.00025	1.00528	0.00026

Table 16 Westinghouse Columbia homogeneous experiments (* assumed values for the benchmark k_{eff} and uncertainty)

Case	Benchmark k_{eff}	Benchmark 1σ	Set 1 (Calculated) k_{eff}	Set 1 (Calculated) 1σ	Set 2 (Calculated) k_{eff}	Set 2 (Calculated) 1σ
1	1.0000	0.0038	1.00029	0.00022	1.00180	0.00023
2	1.0000	0.0039	1.00207	0.00022	1.00390	0.00022
3	1.0000	0.0040	0.99973	0.00022	1.00107	0.00021
4	1.0000	0.0039	0.99846	0.00021	0.99903	0.00022
5	1.0000	0.0041	0.99766	0.00019	0.99850	0.00019
6	1.0000	0.0051	0.99211	0.00016	0.99224	0.00017
7	1.0000	0.0038	1.00964	0.00024	1.01173	0.00024
8	1.0000	0.0039	1.01075	0.00025	1.01218	0.00025
9	1.0000	0.0040	0.99977	0.00024	1.00096	0.00024
10	1.0000	0.0039	1.00293	0.00024	1.00370	0.00024
11	1.0000	0.0039	1.00026	0.00022	1.00114	0.00023
12	1.0000	0.0040	0.99847	0.00022	0.99954	0.00022
13	1.0000	0.0041	0.99845	0.0002	0.99900	0.00021
14	1.0000	0.0050	0.99166	0.00017	0.99216	0.00017
15	1.0000	0.0042	1.01354	0.00026	1.01520	0.00026
16	1.0000	0.0041	1.01616	0.00027	1.01704	0.00027
17*	1.0000	0.005	0.99854	0.00024	0.99738	0.00023
18*	1.0000	0.005	1.00474	0.00024	1.00434	0.00023
19*	1.0000	0.005	0.99936	0.00023	0.99835	0.00022
20*	1.0000	0.005	1.00099	0.00021	1.00008	0.00021
21*	1.0000	0.005	0.99988	0.00020	0.99922	0.00019
22	1.0038	0.0040	0.99904	0.00018	0.99844	0.00018
23*	1.0000	0.005	0.99191	0.00025	0.99168	0.00026
24*	1.0000	0.005	0.99298	0.00024	0.99221	0.00025
25*	1.0000	0.005	0.99426	0.00023	0.99386	0.00023
26*	1.0000	0.005	0.99054	0.00021	0.98949	0.00021
27	1.0024	0.0037	0.99558	0.00020	0.99482	0.00020
28*	1.0000	0.005	1.00094	0.00025	0.99983	0.00024
29*	1.0000	0.005	1.00378	0.00024	1.00237	0.00024
30*	1.0000	0.005	1.00489	0.00024	1.00393	0.00025
31*	1.0000	0.005	0.99633	0.00024	0.99548	0.00025
32*	1.0000	0.005	0.99750	0.00024	0.99667	0.00025
33*	1.0000	0.005	0.99710	0.00026	0.99667	0.00025
34*	1.0000	0.005	1.00173	0.00027	1.00148	0.00026

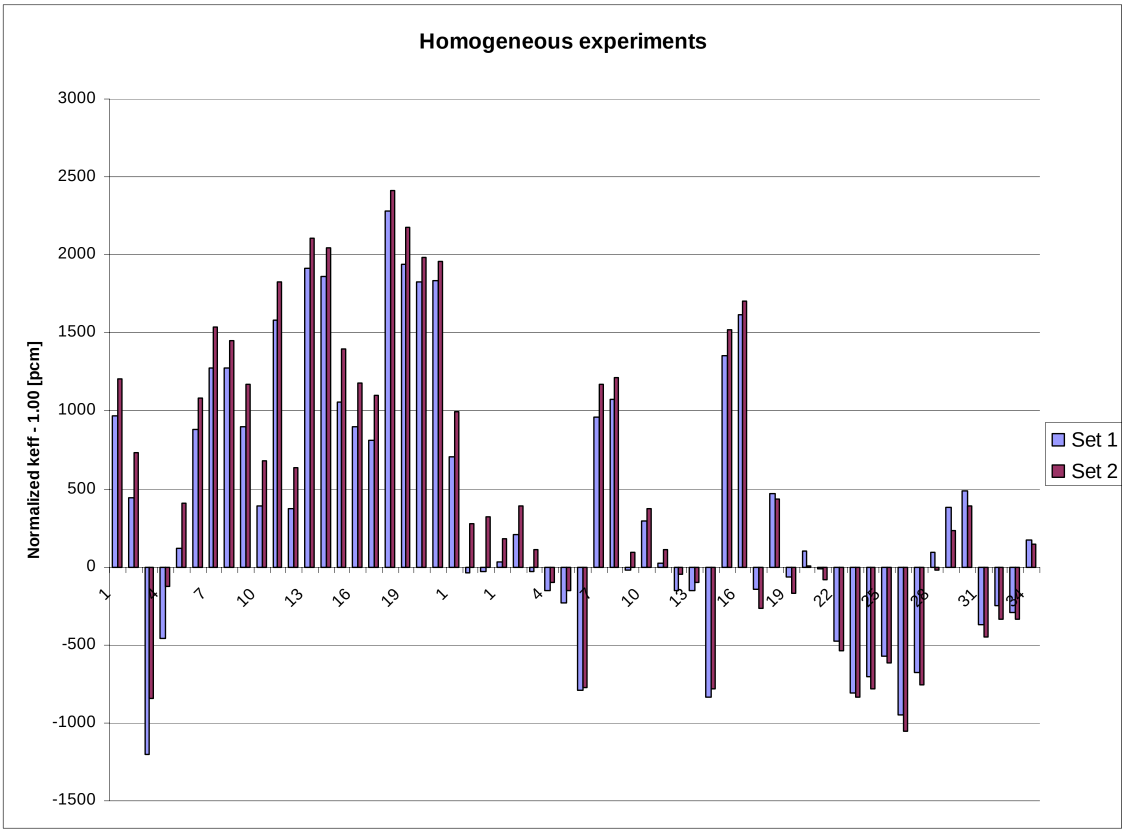


Figure 37 The difference between the normalized k_{eff} and the k_{eff}=1.00 for homogeneous experiments

Table 17 Benchmark Results of CSNI Report No. 71 and 78.
k_{eff}(ref) is the reference k_{eff} or a k_{eff} range.
(inf) denotes an infinite system.
(o) stands for optional.

Case	Reference value k_{eff}(ref)	Set 1 (Calculated) k_{eff}	Set 1 (Calculated) 1σ	Set 2 (Calculated) k_{eff}	Set 2 (Calculated) 1σ
71-SIM-4A	0.645-0.700	0.66388	0.00022	0.66574	0.00023
71-SIM-4A(o)	0.643-0.690	0.65463	0.00022	0.65718	0.00023
71-SIM-4B	0.645-0.701	0.66357	0.00023	0.66568	0.00023
71-SIM-5	0.900-0.955	0.92166	0.00025	0.92338	0.00025
78-EXP-3	1.014	0.99706	0.00025	1.00099	0.00024
78-EXP-3(inf)	1.288-1.355	1.30242	0.00019	1.30238	0.00020
78-SIM-4A	0.969	0.96662	0.00022	0.96620	0.00022
78-SIM-4B	-	0.90518	0.00023	0.90446	0.00024

4.2 HETEROGENEOUS SYSTEMS

The results for the heterogeneous systems are displayed followed by an illustrating diagram in Figure 38.

Table 18 Underwater close-packed storage of LWR-fuel

	Benchmark		Set 1 (Calculated)		Set 2 (Calculated)	
Core	k_{eff}	1σ	k_{eff}	1σ	k_{eff}	1σ
1	1.0010	0.0018	0.99264	0.00024	0.99447	0.00021
2	1.0009	0.0032	0.99510	0.00010	0.99651	0.00010
3	1.0009	0.0032	0.99465	0.00009	0.99605	0.00009
4	1.0010	0.0017	0.99158	0.00010	0.99292	0.00010
5	1.0010	0.0017	0.99087	0.00010	0.99223	0.00009
6	1.0010	0.0017	0.99114	0.00010	0.99247	0.00010
7	1.0010	0.0017	0.98988	0.00010	0.99158	0.00009
8	1.0010	0.0017	0.99006	0.00009	0.99148	0.00010
9	1.0010	0.0018	0.98942	0.00009	0.99108	0.00009
10	1.0010	0.0020	0.99112	0.00009	0.99273	0.00009
11	1.0010	0.0024	0.99674	0.00010	0.99820	0.00010
12	1.0010	0.0019	0.99438	0.00010	0.99554	0.00009
13	1.0010	0.0019	0.99622	0.00010	0.99744	0.00010
14	1.0010	0.0019	0.99237	0.00010	0.99385	0.00010
15	1.0010	0.0022	0.98781	0.00010	0.98909	0.00010
16	1.0010	0.0019	0.98705	0.00010	0.98853	0.00010
17	1.0010	0.0024	0.99119	0.00010	0.99257	0.00010
18	1.0010	0.0020	0.99001	0.00009	0.99128	0.00010
19	1.0010	0.0027	0.99239	0.00010	0.99373	0.00010
20	1.0010	0.0021	0.99075	0.00010	0.99240	0.00010
21	1.0010	0.0019	0.98941	0.00010	0.99087	0.00009

Table 19 KRITZ-experiments with burnable absorbers (* assumed values for the benchmark k_{eff} and uncertainty)

	Benchmark		Set 1 (Calculated)		Set 2 (Calculated)	
Core	k_{eff}	1σ	k_{eff}	1σ	k_{eff}	1σ
1*	1.0000	0.005	0.99521	0.00032	0.99636	0.00033
3*	1.0000	0.005	0.99451	0.00033	0.99602	0.00033
5*	1.0000	0.005	0.99716	0.00031	0.99854	0.00031
7*	1.0000	0.005	0.99507	0.00030	0.99556	0.00032
8*	1.0000	0.005	0.99445	0.00031	0.99467	0.00032
9*	1.0000	0.005	0.99525	0.00032	0.99703	0.00030
10*	1.0000	0.005	0.99561	0.00031	0.99674	0.00031

Table 20 KRITZ-experiments with burnable absorbers and with MOX fuel (* assumed values for the benchmark k_{eff} and uncertainty)

Core	Special conditions	Benchmark k_{eff}	Benchmark 1σ	Set 1 (Calculated) k_{eff}	Set 1 (Calculated) 1σ	Set 2 (Calculated) k_{eff}	Set 2 (Calculated) 1σ
2*	Warm	1.0000	0.005	0.99966	0.00033	1.00047	0.00033
4*	Warm	1.0000	0.005	0.99908	0.00032	1.00053	0.00032
6*	Warm	1.0000	0.005	0.99940	0.00032	1.00087	0.00031
11*	MOX	1.0000	0.005	0.99543	0.00029	0.99649	0.00028
12*	Warm/MOX	1.0000	0.005	0.99716	0.00030	0.99867	0.00030
13*	MOX	1.0000	0.005	0.99624	0.00029	0.99777	0.00030
14*	Warm/MOX	1.0000	0.005	0.99726	0.00029	0.99889	0.00029

Table 21 Storage experiments (* assumed values for the experimental k_{eff} and uncertainty)

Container width [mm]	Core	Benchmark k_{eff}	Benchmark 1σ	Set 1 (Calculated) k_{eff}	Set 1 (Calculated) 1σ	Set 2 (Calculated) k_{eff}	Set 2 (Calculated) 1σ
0	Water*	1.0000	0.005	0.99602	0.00027	0.99662	0.00026
25	Box, air*	1.0000	0.005	0.99223	0.00026	0.99371	0.00026
	Box, $(C_8H_8)_n$*	1.0000	0.005	0.98976	0.00026	0.99145	0.00025
	Box, $(CH_2)_n$ powder*	1.0000	0.005	1.00629	0.00026	1.00737	0.00027
	Box, $(CH_2)_n$ balls*	1.0000	0.005	1.00140	0.00027	1.00312	0.00025
	Box, water*	1.0000	0.005	1.01663	0.00025	1.01807	0.00025
	Water*	1.0000	0.005	0.99369	0.00026	0.99535	0.00026
50	Box, air*	1.0000	0.005	0.99177	0.00025	0.99394	0.00025
	Box, $(C_8H_8)_{n*}$	1.0000	0.005	0.99391	0.00025	0.99509	0.00025
	Box, $(CH_2)_n$ powder*	1.0000	0.005	1.02156	0.00025	1.02303	0.00025
	Box, $(CH_2)_n$ balls*	1.0000	0.005	1.01211	0.00025	1.01326	0.00025
	Box, water*	1.0000	0.005	1.02123	0.00025	1.02224	0.00024
	Water*	1.0000	0.005	0.99369	0.00026	0.99535	0.00026
100	Box, air*	1.0000	0.005	0.99456	0.00025	0.99648	0.00025
	Box, $(C_8H_8)_{n*}$	1.0000	0.005	0.99684	0.00025	0.99871	0.00024
	Box, $(CH_2)_n$ powder*	1.0000	0.005	1.00438	0.00024	1.00620	0.00024
	Box, $(CH_2)_n$ balls*	1.0000	0.005	1.01752	0.00025	1.01893	0.00024
	Box, water*	1.0000	0.005	0.99982	0.00025	1.00144	0.00024
	Water*	1.0000	0.005	0.99504	0.00025	0.99596	0.00024

Table 22 Westinghouse Columbia heterogeneous experiments (* assumed values for the experimental k_{eff} and uncertainty)

Core	Benchmark k_{eff}	Benchmark 1σ	Set 1 (Calculated) k_{eff}	Set 1 (Calculated) 1σ	Set 2 (Calculated) k_{eff}	Set 2 (Calculated) 1σ
Hexcrit1	1.0000	0.0061	0.99806	0.00038	0.99792	0.00039
Hexcrit2	1.0000	0.0061	1.00224	0.00038	1.00156	0.00042
Hexcrit3	1.0000	0.0061	1.00364	0.00040	1.00298	0.00038
Hexcrit4	1.0000	0.0061	1.00317	0.00038	1.00241	0.00042
Hexcrit5	1.0000	0.0061	1.00296	0.00038	1.00161	0.00037
Hexcrit6	1.0000	0.0061	1.00272	0.00043	1.00267	0.00037
nse71sq	1.0000	0.0016	0.99125	0.00044	0.99166	0.00043
nse71w1	1.0000	0.0014	0.98905	0.00045	0.99174	0.00043
nse71w2	1.0000	0.0014	0.98902	0.00042	0.99093	0.00045
p3602n11	1.0000	0.0028	0.99239	0.00038	0.99336	0.00036
p3602n12	1.0000	0.0028	0.99300	0.00035	0.99464	0.00037
p3602n13	1.0000	0.0028	0.99397	0.00038	0.99477	0.00037
p3602n14	1.0000	0.0028	0.99186	0.00036	0.99431	0.00038
p3602n21	1.0000	0.0031	0.99429	0.00035	0.99528	0.00034
p3602n22	1.0000	0.0031	0.99065	0.00033	0.9923	0.00036
p3602n31	1.0000	0.0028	0.99483	0.00044	0.99804	0.00041
p3602n32	1.0000	0.0028	0.99612	0.00043	0.99769	0.00041
p3602n33	1.0000	0.0028	0.99625	0.00043	0.99872	0.00041
p3602n34	1.0000	0.0028	0.99594	0.00044	0.99805	0.00042
p3602n35	1.0000	0.0028	0.99582	0.00042	0.99748	0.00042
p3602n36	1.0000	0.0028	0.99428	0.00044	0.99590	0.00040
p3602n41	1.0000	0.0021	0.99677	0.00041	0.99703	0.00040
p3602n42	1.0000	0.0021	0.99711	0.00040	0.99714	0.00038
p3602n43	1.0000	0.0021	0.99595	0.00041	0.99504	0.00038
p3314ba*	1.0000	0.0040	0.99757	0.00042	0.99992	0.00040
p3602bb	1.0000	0.0018	0.99222	0.00041	0.99421	0.00040
p3314bc*	1.0000	0.0040	0.99862	0.00040	1.00009	0.00043
p3314bs1	1.0000	0.0034	0.98354	0.00037	0.98538	0.00039
p3314bs3*	1.0000	0.0040	0.99582	0.00039	0.99813	0.00045
p3602bs1	1.0000	0.0016	0.99325	0.00036	0.99428	0.00037
p3602bs2	1.0000	0.0018	0.99417	0.00040	0.99674	0.00043
p3314bs2	1.0000	0.0034	0.98395	0.00038	0.98516	0.00036
p3314bs4*	1.0000	0.0040	0.99791	0.00043	1.00024	0.00040
p2438ss	1.0000	0.0031	0.99409	0.00036	0.99453	0.00036
p2615ss	1.0000	0.0021	0.99541	0.00041	0.99579	0.00038
p3314ss1*	1.0000	0.0040	0.99773	0.00039	1.00059	0.00042
p3314ss2*	1.0000	0.0040	0.99793	0.00041	1.00098	0.00038
p3314ss3*	1.0000	0.0040	0.99679	0.00041	0.99764	0.00039
p3314ss4*	1.0000	0.0040	0.99828	0.00041	0.99958	0.00043
p3314ss5	1.0000	0.0034	0.98219	0.00036	0.98451	0.00038
p3314ss6*	1.0000	0.0040	0.99705	0.00038	0.99955	0.00041
p3602ss1	1.0000	0.0016	0.99255	0.00035	0.99374	0.00036
p3602ss2	1.0000	0.0018	0.99425	0.00042	0.99641	0.00042

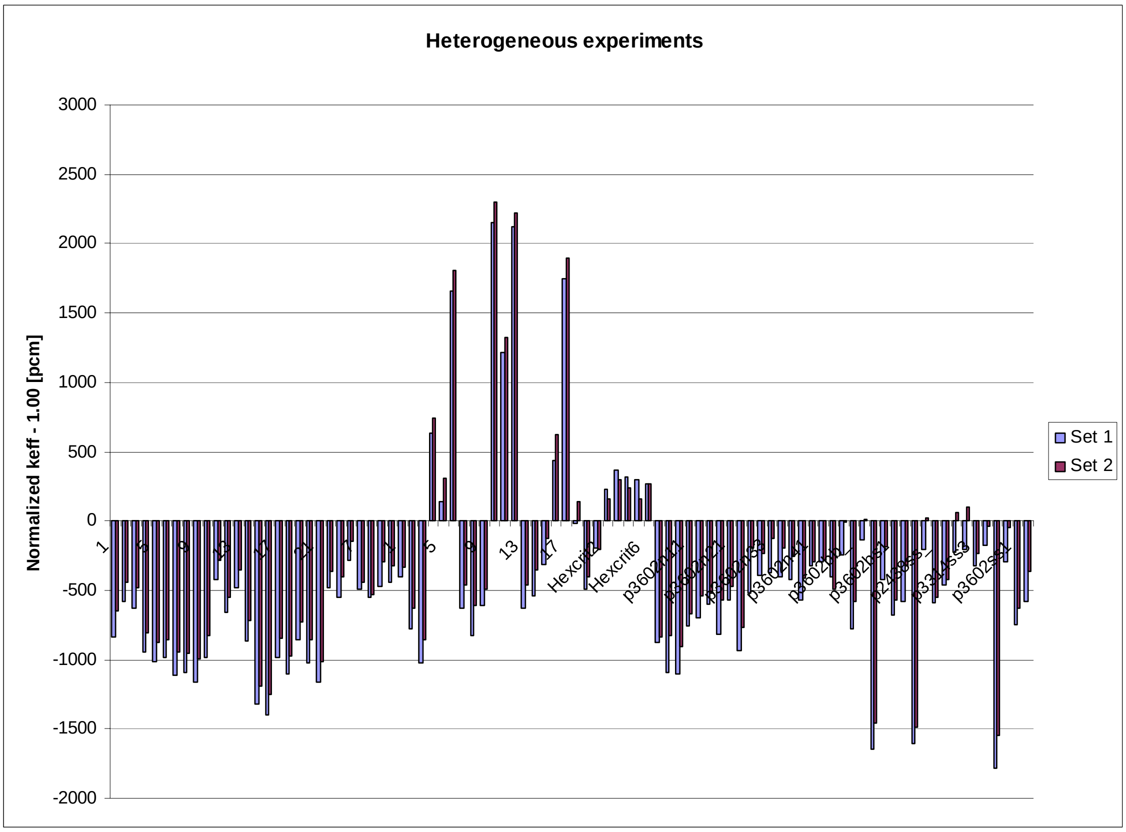


Figure 38 The difference between the normalized k_{eff} and the k_{eff}=1.00 for heterogeneous experiments in pcm

4.3 SUMMARY

The average k_{eff} resulting from the calculations are presented in Table 23 (results from Table 17 excluded). The k_{eff}'s have first been normalized as in Equation 11.

Table 23 Unweighted k_{eff} with uncertainty (Normalized)

		k_{eff}	uncertainty
Homogeneous	set 1	1.00339	0.00828
	set 2	1.00449	0.00900
Heterogeneous	set 1	0.99549	0.00709
	set 2	0.99685	0.00695

Note that above values for average k_{eff} are the simple unweighted averages, i.e. they do not take into account the fact that uncertainties may be different for different experiments. The uncertainty is the estimated standard deviation on the k_{eff}. In the next section 5 the weighted averages will be calculated.

5 STATISTICAL EVALUATION

A statistical evaluation of the results is performed. It is done to determine the accuracy of MCNP5 and decrease the uncertainties of the program precision. To ensure conservative results from the program a criticality safety margin is stated. The criticality safety margin gives the k_{eff} upper limit for a fissile configuration when making future calculations with MCNP5. The criticality safety margin is named the upper subcritical limit (USL). The statistical evaluation from the homogeneous and the heterogeneous experiments' results is done separately.

Two methods were used to determine the upper subcritical limits. The first method uses a linear regression analysis of the data. This method displays the USL as graphs, which very well exhibit trends. The second method uses an estimation of a lower tolerance limit and gives a value for the USL. If the data exhibit significant trends the first method is recommended. Otherwise the second method should be used.

5.1 DETERMINATION OF UPPER SUBCRITICAL LIMIT BY USLSTATS

In the determination of the upper subcritical limit (USL) the trends are examined. Graphs are plotted with the k_{eff} versus H/U, H/X, enrichment, etc. (Appendix 3). The fact that all experiments are critical means that if the average result deviates from $k_{eff} = 1$ it is a bias. The investigation of bias, bias uncertainty and trends has here been performed with the computer program USLSTATS version 1.4.2 [18]. The USLSTATS output files can be found in Appendix 4. Input to USLSTATS is given to have a confidence on the fit to 95%. The USLSTATS' so called USL Method 1 (USL-1) does not take credit of any positive bias. By definition,

$$USL\text{-}1(x) = 1 - \Delta k_m - W + min\{0, B(x)\} \tag{9}$$

where
Δk_m = additional margin to ensure subcriticality, here = 0.05,
W = the confidence band width obtained from a statistical analysis of the spread in the data,
B(x) = bias resulting from a fit to the data.

The USL-1 limit will always be less than 1 - Δk_m, if W is greater than zero. With more spread results and with bigger uncertainties the greater will W be. In addition, only negative (non-conservative) biases are taken into account.

The uncertainty of the experimental results (benchmark), σ_e, and the uncertainty of the calculated results, σ_c, are combined statistically to give a combined uncertainty, called the sample uncertainty, σ_s:

$$\sigma_s = \sqrt{\sigma_e{}^2 + \sigma_c{}^2} \tag{10}$$

The sample uncertainty is required as input to USLSTATS in order to estimate W.

The calculated k_{eff} is normalized to the benchmark k_{eff}, according to

$$k_{eff}(normalized) = k_{eff}(calculated) / k_{eff}(benchmark) \tag{11}$$

The normalized k_{eff} is then used as input in USLSTATS.

The experimental k_{eff} and the experimental uncertainty are found for the experiments found in IHECSBE, and are presented with the results. For the experiments with no such data, assumed values have been given. The assumed k_{eff} and uncertainty are set from similar experiments with known values. The assumed values are shown in Table 24.

Table 24 Assumed experimental k_{eff} and experimental uncertainty.

Experimental model	k_{eff}	σ_e
Westinghouse Columbia homogeneous exp.	1.0000	0.005
KRITZ-experiments with burnable absorbers	1.0000	0.005
Storage experiments	1.0000	0.005
Westinghouse Columbia heterogeneous exp.	1.0000	0.004

From the χ^2 (chi-square) goodness-of-fit test in the USLSTATS output it is decided if the combination of experiments is normally distributed. It is concluded normal for $\chi^2 \leq 9.49$. The USLs are presented in diagrams, where the diagram with the strongest correlation is of most interest. To decide which diagram that has the best correlation, the R^2-factor is used. The R^2-factor is a standard feature in MS Excel and can be obtained when treating diagrams. $R^2 = 1$ means perfect correlation, while a curve with $R^2 = 0$ has no correlation whatsoever. For the homogeneous systems the X=H/X has the best correlation and for the heterogeneous the X=Enrichment, Table 25.

Table 25 χ^2 goodness-of-fit, and the R^2-factor for different correlations

	χ^2	Normal	R^2-factor			
			H/U	H/X	Enrich.	EALF
Homogeneous set 1	6.07	Yes	0.3221	**0.4251**	0.0078	0.0496
set 2	2.21	Yes	0.4437	**0.5460**	0.0033	0.1119
Heterogeneous set 1	24.1	No	0.0988	0.0028	**0.3124**	0.0000
set 2	19.3	No	0.0554	0.0161	**0.3112**	0.0065

As none of the sets for the heterogeneous systems are normally distributed, new combinations of the heterogeneous systems are chosen from where the χ^2 goodness-of-fit test is checked, Table 26.

Table 26 χ^2 goodness-of-fit and the R^2-factor for heterogeneous systems and set 1

Heterogeneous systems, set 1	No. exp.	χ^2	Normal	R^2-factor			
				H/U	H/X	Enrich.	EALF
1. All	90	*24.1*	No	0.0988	0.0028	**0.3124**	0.0000
2. All IHECSBE	55	*5.27*	Yes	**0.4575**	0.0956	0.4384	0.0785
3. Underwater close-packed storage exp. Westinghouse Columbia het. exp.	64	*0.375*	Yes	0.2763	0.0087	**0.4901**	0.0041
4. Underwater close-packed storage exp. Westinghouse Columbia het. exp. KRITZ-BA exp.	71	*1.47*	Yes	0.2249	0.0028	**0.4192**	0.0057
5. Underwater close-packed storage exp. Westinghouse Columbia het. exp. KRITZ-BA exp. Storage exp., except box powder, box balls and box water	81	*4.49*	Yes	0.2195	0.0029	**0.3330**	0.0064

The 5th configuration in Table 26 includes the most experiments but still can fulfill the normal distribution criteria. However, since the 5th configuration allows a higher USL than the 1st configuration, the 1st can be used due to conservativeness even though it is not agreed normal. Therefore the following USL for heterogeneous systems is based on the configuration with 90 experiments.

In Table 27 the USL-1 is displayed for both X=H/X and X=Enrichment. A term calculated as B = 0.95 - USL-1 may be added to the calculated k_{eff} values while keeping the 0.95 limit. The range of B is also showed in Table 27, where the range of applicability for the homogeneous systems is 17.46 < H/U < 1098, and for the heterogeneous systems 2.35 % < Enrichment < 5.06 %. The B term reveal how big safety margin that is necessary. A high B term indicates that a big safety margin is necessary and a low B term means that the safety margin can be small. One can say that a low B term indicates that MCNP5 is an efficient tool. The B maxima are displayed in Table 28.

Table 27 USL-1 and B, based on X=H/X and X=Enrichment in USLSTATS

X=H/X	X range	USL-1	B	B range
Homogeneous set 1	X ≤ 541.33 X > 541.33	0.9367 0.9456 – 1.6339E-5*X	0.0133 0.0044 + 1.6339E-5*X	0.0133 < B < 0.0223
set 2	X ≤ 556.92 X > 556.92	0.9371 0.9483 – 2.0121E-5*X	0.0129 0.0017 + 2.0121E-5*X	0.0129 < B < 0.0238
X=Enrichment				
Heterogeneous set 1	X < 4.8772E-2 X ≥ 4.8772E-2	0.9204 + 3.6433E-1*X 0.9381	0.0296 - 3.6433E-1*X 0.0119	0.0119 < B < 0.0210
set 2	X < 4.5249E-2 X ≥ 4.5249E-2	0.9221 + 3.5645E-1*X 0.9383	0.0279 - 3.5645E-1*X 0.0117	0.0117 < B < 0.0195

Table 28 Boundary conditions for USL-1 and B, based on results from Table 27

		USL-1 minima	B maxima
Homogeneous	set 1	0.9277	0.0223
	set 2	0.9262	0.0238
Heterogeneous	set 1	0.9290	0.0210
	set 2	0.9305	0.0195

5.2 DETERMINATION OF UPPER SUBCRITICAL LIMIT USING LOWER TOLERANCE LIMIT

In section 4.3, the unweighted average of k_{eff} was calculated. Here is the weighted average calculated for the homogeneous and the heterogeneous systems. The sampled uncertainty, σ_s is obtained as described in Equation 10, and the k_{eff} is normalized as in Equation 11. First the variance about the mean, *s*, is calculated:

$$s^2 = \frac{\frac{1}{n-1}\sum_s \frac{1}{\sigma_s^2}(k_{eff\,s} - \overline{k_{eff}})^2}{\frac{1}{n}\sum_s \frac{1}{\sigma_s^2}} \tag{12}$$

Then the average uncertainty is determined:

$$\overline{\sigma}_s^2 = \frac{n}{\sum_s \frac{1}{\sigma_s^2}} \tag{13}$$

The weighted mean k_{eff} is obtained from equation 14.

$$\overline{k_{eff}} = \frac{\sum_s \frac{1}{\sigma_s^2} k_{eff\,s}}{\sum_s \frac{1}{\sigma_s^2}} \tag{14}$$

S_p is the pooled standard deviation, which is received from the square root from variance about the mean and the average uncertainty, as in Equation 15. Pooling means an average of two sample variances.

$$S_p = \sqrt{s^2 + \overline{\sigma}_s^{\,2}} \tag{15}$$

The weighted average, $\overline{k_{eff}}$, ± the pooled standard deviation, S_p are presented in Table 29.

Table 29 Weighted mean k_{eff} and S_p.

	mean k_{eff}	S_p
Homogeneous set 1	1.00514	0.00931
set 2	1.00682	0.00983
Heterogeneous set 1	0.99265	0.00622
set 2	0.99410	0.00610

A lower tolerance limit is calculated. That is, at least a proportion P of a normal distribution of calculated k_{eff}'s are above $\overline{k_{eff}} - KS_p$ with a probability of γ. Pr{} is a function to fit in a probability condition, and K is based on the number of observations and is determined satisfy the following equation.

$$\Pr\left\{\Pr(k_{eff} \geq \overline{k_{eff}} - KS_P) \geq P\right\} = \gamma \tag{16}$$

Choosing $P = \gamma = 95\%$ and with 57 homogeneous and 90 heterogeneous systems the value K can be obtained from tables in Ref. [19]:

Homogeneous: $K = 2.034$
Heterogeneous: $K = 1.944$

The lower tolerance limit is now calculated as:

$$K_L = \overline{k_{eff}} - KS_p \tag{17}$$

In analogy with section 5.1 the upper subcritical limits are calculated as $USL = K_L - 0.05$ and B as $B = 0.95 - USL$. Table 30 shows these values.

Table 30 K_L, USL and B.

	K_L	USL	B
Homogeneous set 1	0.98619	0.93619	0.01381
set 2	0.98682	0.93682	0.01318
Heterogeneous set 1	0.98055	0.93055	0.01945
set 2	0.98223	0.93223	0.01777

The B terms calculated by this method can now be compared with the B terms calculated in the previous, Table 27. It is noted that the B values in Table 30 is covered by the intervals computed in the section 5.1 based on the X=H/X (homogeneous systems) and X=Enrichment (heterogeneous systems) correlations.

6 AREA AND RANGE OF APPLICABILITY

6.1 GENERAL AREA AND RANGE OF APPLICABILITY

The area of applicability for this MCNP5 validation is:

- Low enriched uranium in a homogeneous solution with water or paraffin.
- Heterogeneous systems with low enriched uranium oxide fuel rod assemblies in array configurations moderated and reflected by water.

The range of applicability for a homogeneous system:

- H/X ratios between 17.46 and 1098.
- Low enriched uranium between 2.00 and 5.00 wt% U-235.

The range of applicability for a heterogeneous system:

- H/X ratios between 105 and 446.
- Low enriched uranium between 2.35 and 5.06 wt% U-235.

6.2 WESTINGHOUSE SPECIFIC AREA AND RANGE OF APPLICABILITY

In the Westinghouse fuel factory, there are applications that are of special interest to investigate. The factory handles uranium with maximum enrichment of 5.00 wt% ^{235}U which have an optimum moderation ratio in H/X of 200 to 400. Due to the specific factory conditions, the validation of MCNP5 in the previous noted enrichment and H/X range is central.

7 CONCLUSION

MCNP5 has been validated against 57 homogeneous and 90 heterogeneous critical experiments. In the validation two different setups of cross-section libraries have been used. The first setup named set 1 uses the ENDF/B-VI release 6 library, and set 2 uses the latest accessible libraries at the time of the performed calculations, which are ENDF/B-VI release 6 and 8, and pre ENDF/B-VII. The results from the both sets of cross-section libraries differ very little. Set 1 might be preferably used since only one cross-section library (ENDF/B-VI) needs to be handled.

From Table 29 with weighted k_{eff} one can see that for homogeneous systems the k_{eff} are around 500 to 700 pcm supercritical, while the heterogeneous systems give a lower value: around 600 to 700 pcm subcritical. These results make the heterogeneous systems' USL to be lower than the homogeneous.

The B and the USL terms obtained using the lower tolerance limit method can be used, but trends are observed with the USLSTATS method. In Appendix 3 diagrams are displayed, where trends are studied, and one can see that the H/X diagram for the homogeneous systems and the enrichment diagram for the heterogeneous systems exhibit the clearest trends.

For Westinghouse factory applications with uranium enriched to 5.00 wt% and an optimum moderation ratio (H/X) of 200 to 400, the results from the homogeneous and the heterogeneous systems differ very little, whereby a general the B and the USL terms can be decided. For both set 1 and 2, the diagrams give a B term to

- B = 0.013

The Upper Subcritical limits for k_{eff} to

- USL = 0.937

For investigations of applications at lower enrichments or other H/X ratios, the diagrams in Appendix 3 should be studied. The diagrams give a B-range for homogeneous systems using the set-1-cross-section libraries to 0.013 to 0.022 and for heterogeneous systems to 0.012 to 0.021.

8 REFERENCES

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APPENDIX 1. EXPERIMENTS WITH H/U = 0.77 AND 1.25

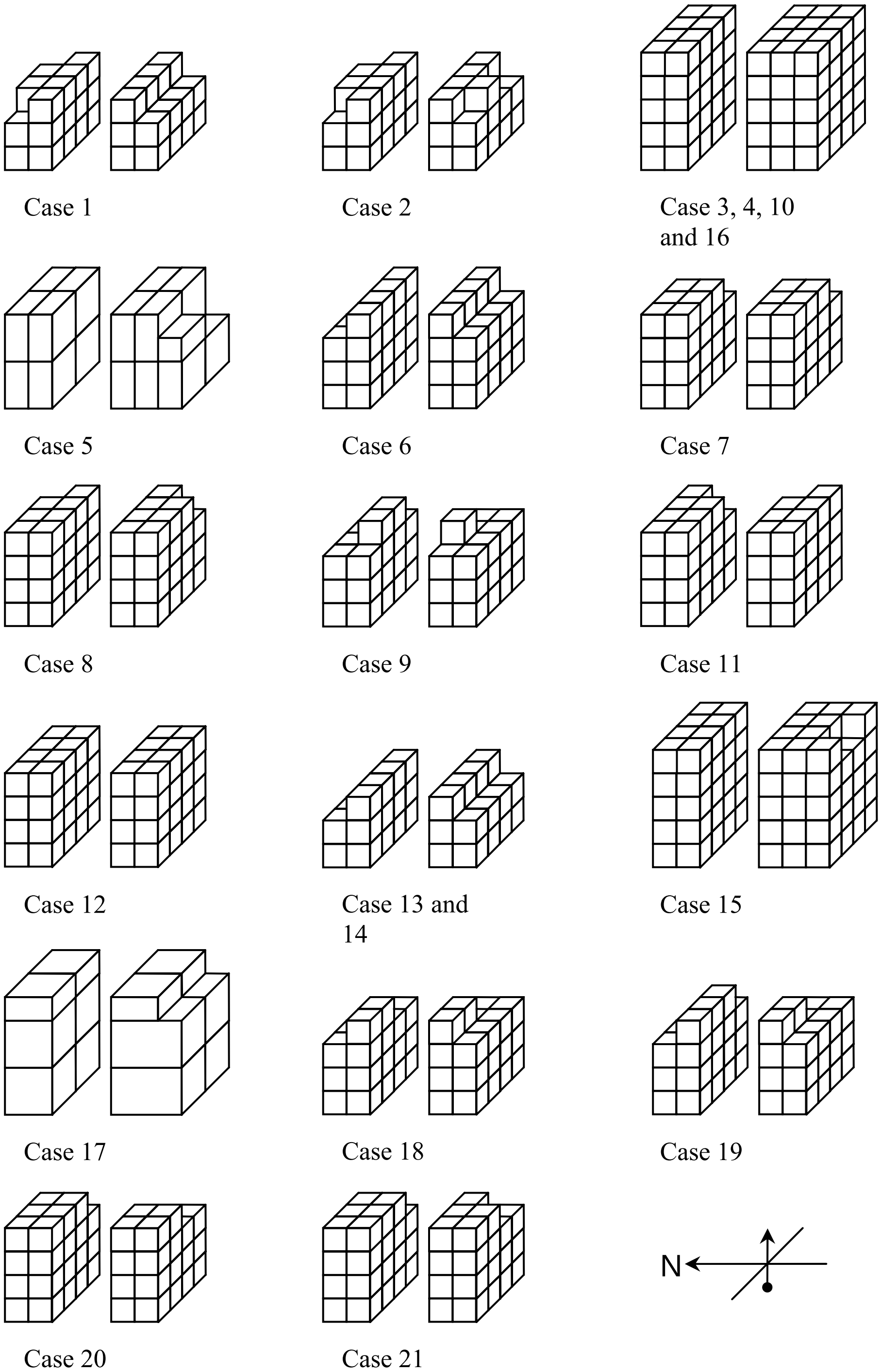

Figure 39 Homogeneous Experiments with H/U = 0.77. Isometric views of each core.

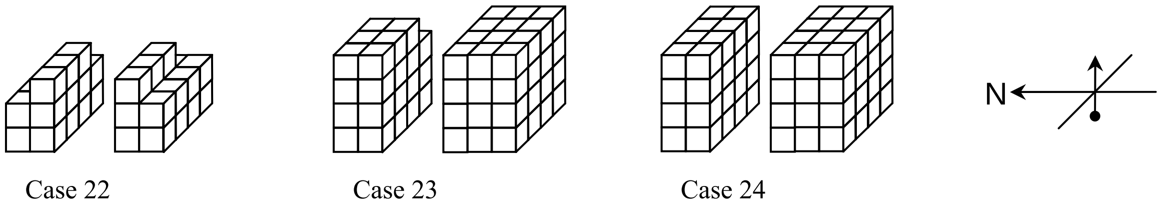

Figure 40   Homogeneous Experiments with H/U = 1.25. Isometric views of each core.

APPENDIX 2. CROSS-SECTION LIBRARIES FOR SET 1 AND 2

Table	Data file
1001.66c	Endf/b-vi.6:x
5010.66c	Endf/b-vi.1
5011.66c	Endf/b-vi.0:x
6000.66c	Endf/b-vi.6
7014.66c	Endf/b-vi.6
7015.66c	Endf/b-vi.0
8016.66c	Endf/b-vi.6
9019.66c	Endf/b-vi.0x
11023.66c	Endf/b-vi.1
12000.66c	Endf/b-vi.0
13027.66c	Endf/b-vi.6
14028.66c	Endf/b-vi.6
14029.66c	Endf/b-vi.6
14030.66c	Endf/b-vi.6
15031.66c	Endf/b-vi.6
16000.66c	Endf/b-vi.0
17000.66c	Endf/b-vi.0
19000.66c	Endf/b-vi.0
20000.66c	Endf/b-vi.6
22000.66c	Endf/b-vi.0
24050.66c	Endf/b-vi.6
24052.66c	Endf/b-vi.6
24053.66c	Endf/b-vi.6
24054.66c	Endf/b-vi.6
25055.66c	Endf/b-vi.5
26054.66c	Endf/b-vi.6
26056.66c	Endf/b-vi.6
26057.66c	Endf/b-vi.6
26058.66c	Endf/b-vi.5
28060.66c	Endf/b-vi.6
28061.66c	Endf/b-vi.6
28062.66c	Endf/b-vi.6
28064.66c	Endf/b-vi.6
29063.66c	Endf/b-vi.6
29065.66c	Endf/b-vi.6
30000.42c	Endl/Llnl:x
35079.55c	Lanl/t
35081.55c	Lanl/t
40000.66c	Endf/b-vi.1
41093.66c	Endf/b-vi.6
42000.66c	Endf/b-vi.0
50000.42c	Endl/Llnl:x
64152.66c	Endf/b-vi.4
64154.66c	Endf/b-vi.4
64155.66c	Endf/b-vi.0
64156.66c	Endf/b-vi.0
64157.66c	Endf/b-vi.0
64158.66c	Endf/b-vi.0

Table	Data file
64160.66c	Endf/b-vi.0
72174.66c	Endf/b-vi.2
72176.66c	Endf/b-vi.2
72177.66c	Endf/b-vi.2
72178.66c	Endf/b-vi.2
72179.66c	Endf/b-vi.2
72180.66c	Endf/b-vi.2
82206.66c	Endf/b-vi.6
82207.66c	Endf/b-vi.6
82208.66c	Endf/b-vi.6x
92234.66c	Endf/b-vi.0
92235.66c	Endf/b-vi.5
92236.66c	Endf/b-vi.0
92238.66c	Endf/b-vi.5
93237.66c	Endf/b-vi.1
94239.66c	Endf/b-vi.5
94240.66c	Endf/b-vi.2
94241.66c	Endf/b-vi.3
94242.66c	Endf/b-vi.0
95241.66c	Endf/b-vi.3:x
S(α,β) data	
Lwtr.60t	Endf/b-vi.3
Lwtr.61t	Endf/b-vi.3
Lwtr.62t	Endf/b-vi.3
Poly.60t	Endf/b-vi.3

Table 31. Cross-section data based on ENDF/B-VI used in set 1.

Table	Data file
1001.62c	Actia/b-vi.8
5010.66c	Endf/b-vi.1
5011.66c	Endf/b-vi.0:x
6000.66c	Endf/b-vi.6
7014.62c	Actia/b-vi.8
7015.66c	Endf/b-vi.0
8016.62c	Actia/b-vi.8
9019.62c	Actia/b-vi.8
11023.62c	Actia/b-vi.8
12000.62c	Actia/b-vi.8
13027.92c	Actia-vi.8
14028.62c	Actia/b-vi.6
14029.62c	Actia/b-vi.8
14030.62c	Actia/b-vi.6
15031.66c	Endf/b-vi.6
16000.62c	Actia/b-vi.8
17035.62c	Actia/b-vi.8
17037.62c	Actia/b-vi.8
19000.62c	Actia/b-vi.8
20000.62c	Actia/b-vi.8
22000.62c	Actia/b-vi.8
24050.62c	Actia/b-vi.8
24052.62c	Actia/b-vi.8
24053.62c	Actia/b-vi.8
24054.62c	Actia/b-vi.8
25055.62c	Actia/b-vi.8
26054.62c	Actia/b-vi.8
26056.62c	Actia/b-vi.8
26057.62c	Actia/b-vi.8
26058.62c	Actia/b-vi.8
28060.62c	Actia/b-vi.8
28061.62c	Actia/b-vi.8
28062.62c	Actia/b-vi.8
28064.62c	Actia/b-vi.8
29063.62c	Actia/b-vi.8
29065.62c	Actia/b-vi.8
30000.42c	Endl-Llnl:x
35079.55c	Lanl/t
35081.55c	Lanl/t
40000.66c	Endf/b-vi.1
41093.66c	Endf/b-vi.6
42000.66c	Endf/b-vi.0
50000.42c	Endl/Llnl:x
64152.66c	Endf/b-vi.4
64154.66c	Endf/b-vi.4
64155.66c	Endf/b-vi.0
64156.66c	Endf/b-vi.0
64157.66c	Endf/b-vi.0
64158.66c	Endf/b-vi.0

Table	Data file
64160.66c	Endf/b-vi.0
72174.66c	Endf/b-vi.2
72176.66c	Endf/b-vi.2
72177.66c	Endf/b-vi.2
72178.66c	Endf/b-vi.2
72179.66c	Endf/b-vi.2
72180.66c	Endf/b-vi.2
82206.66c	Endf/b-vi.6
82207.66c	Endf/b-vi.6
82208.66c	Endf/b-vi.6x
92234.69c	Lanl-t16
92235.69c	Lanl-t16
92236.69c	Lanl-t16
92238.69c	Lanl-t16
93237.69c	Lanl-t16
94239.69c	Lanl-t16
94240.69c	Lanl-t16
94241.69c	Lanl-t16
94242.69c	Lanl-t16
95241.69c	Lanl-t16
S(α,β) data	
Lwtr.60t	Endf/b-vi.3
Lwtr.61t	Endf/b-vi.3
Lwtr.62t	Endf/b-vi.3
Poly.60t	Endf/b-vi.3

Table 32. Cross-section data based on ENDF/B-VI, ACTIA/B-VI and LANL/T16 used in set 2.

APPENDIX 3. K_{EFF} DIAGRAMS FOR HOMOGENEOUS AND HETEROGENEOUS SYSTEMS

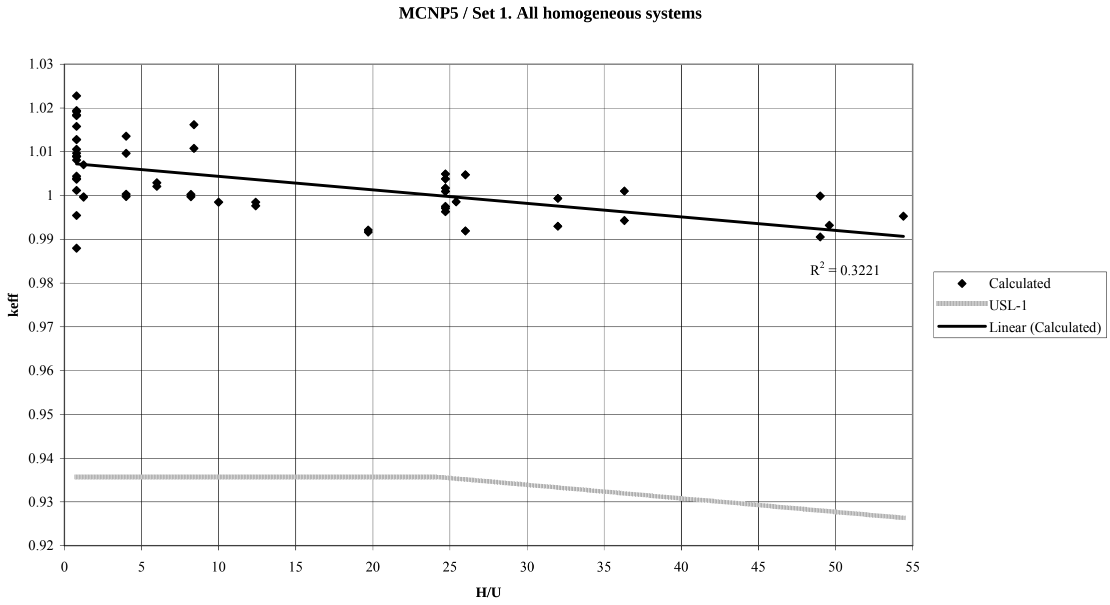

Figure 41 Homogeneous systems Set 1, k_{eff} vs. H/U

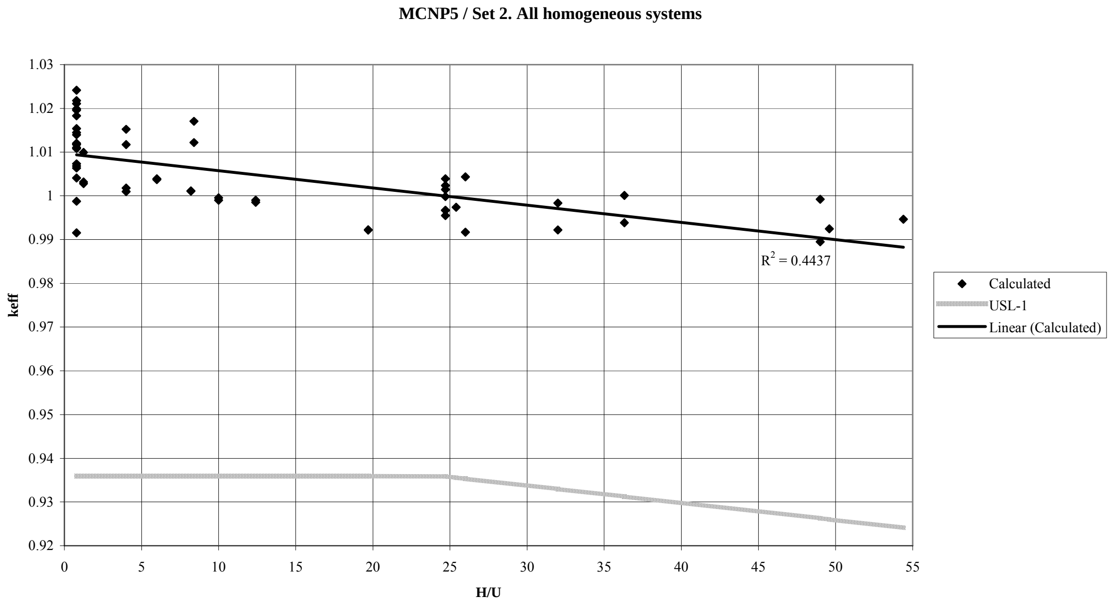

Figure 42 Homogeneous systems Set 2, k_{eff} vs. H/U

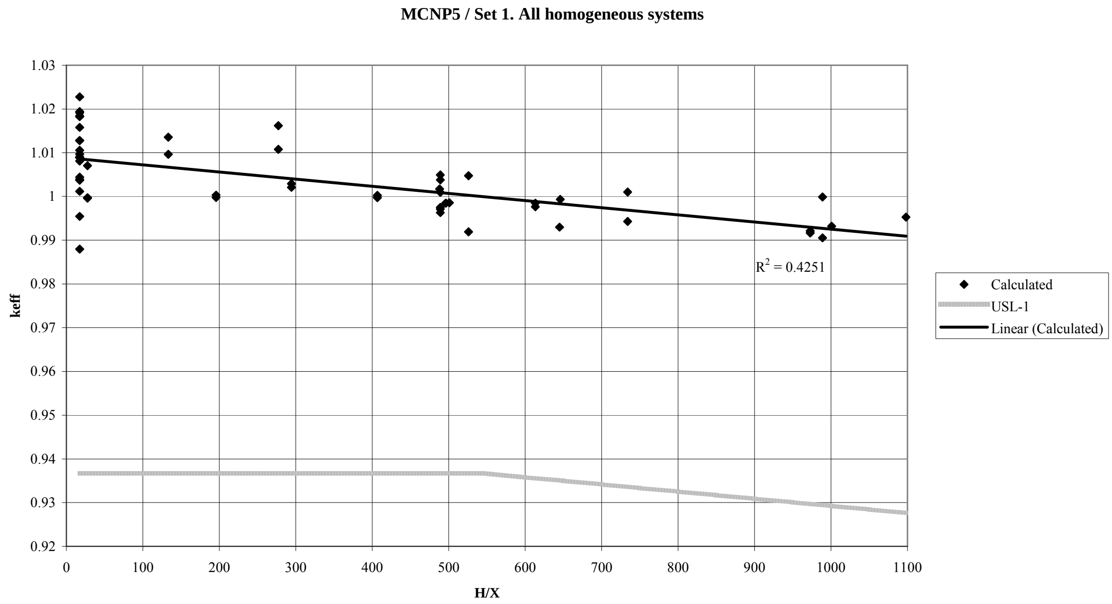

Figure 43 Homogeneous systems Set 1, k_{eff} vs. H/X

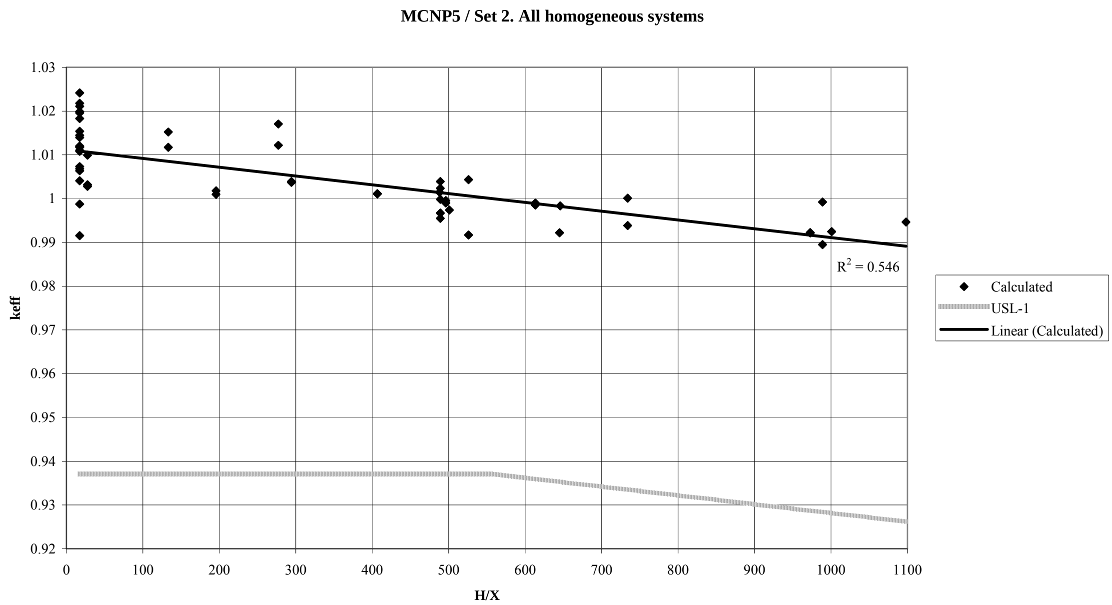

Figure 44 Homogeneous systems Set 2, k_{eff} vs. H/X

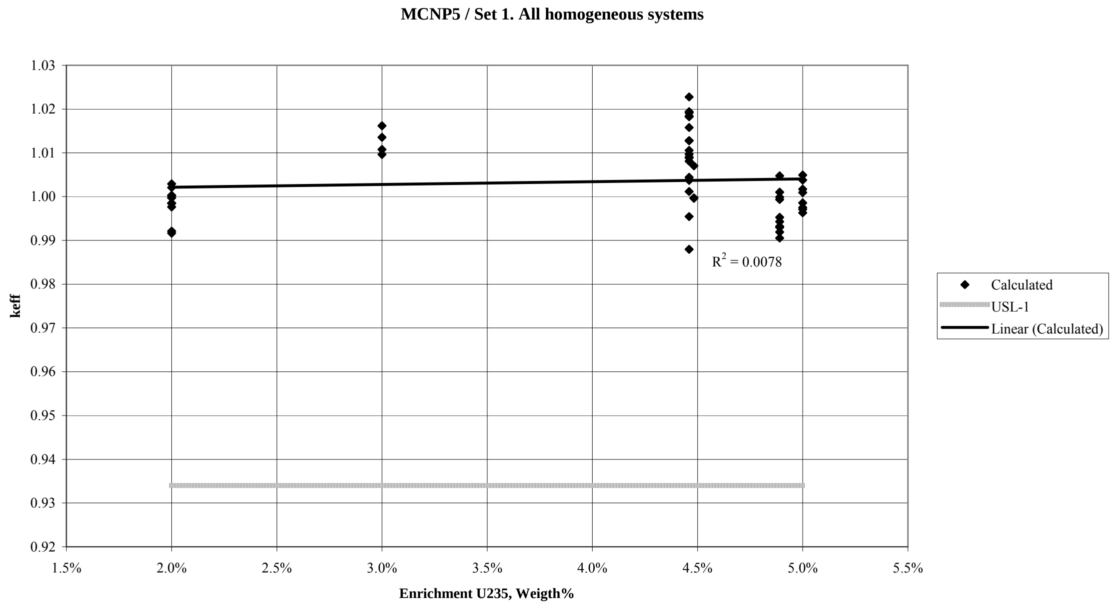

Figure 45 Homogeneous systems Set 1, k_{eff} vs. Enrichment

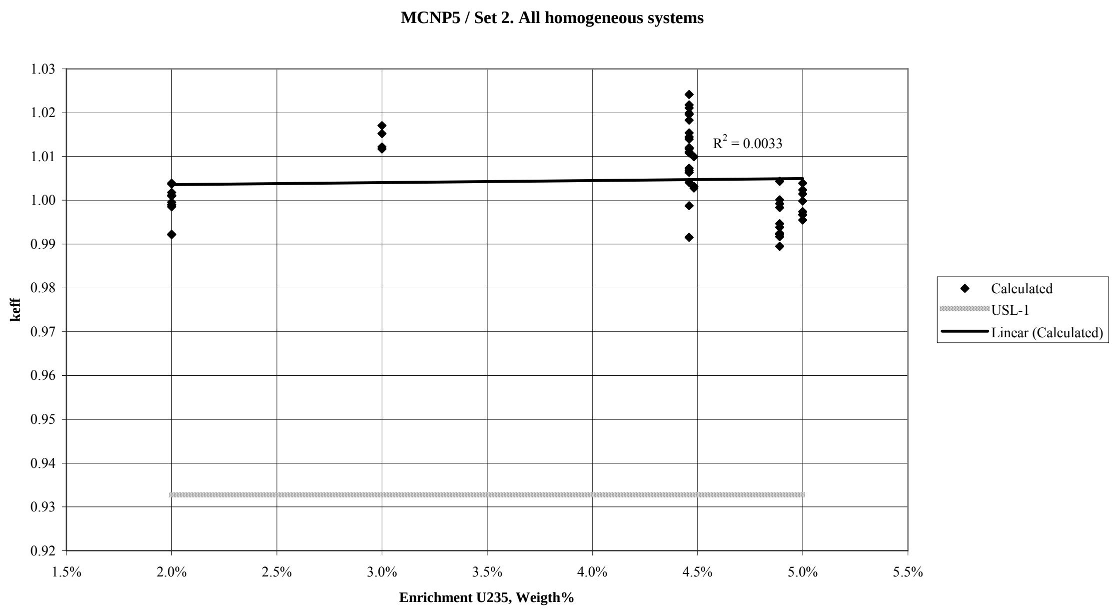

Figure 46 Homogeneous systems Set 2, k_{eff} vs. Enrichment

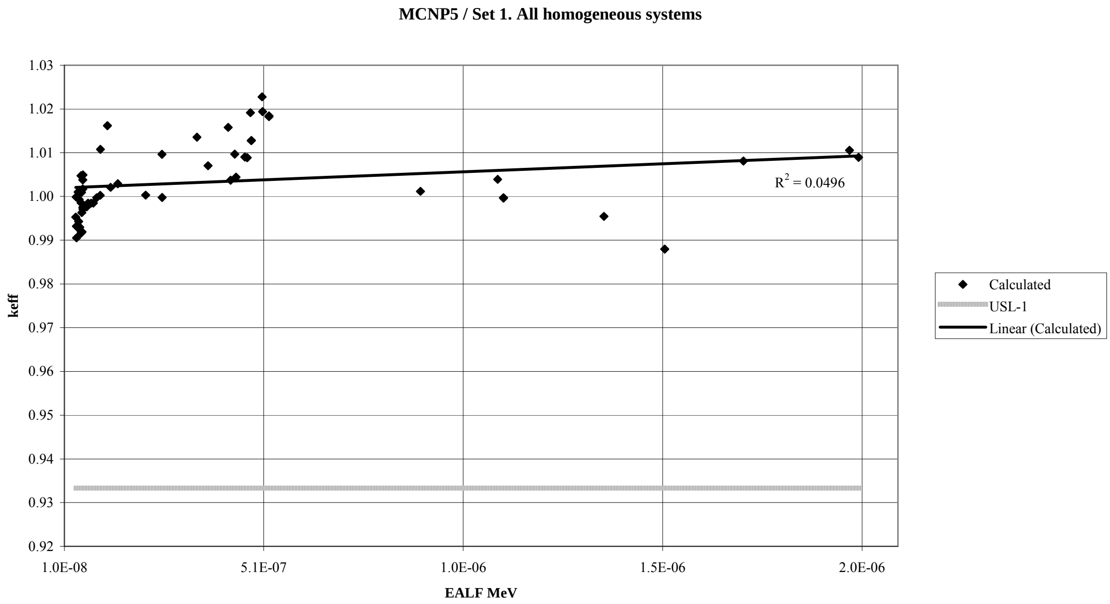

Figure 47 Homogeneous systems Set 1, k_{eff} vs. EALF (Lethargy)

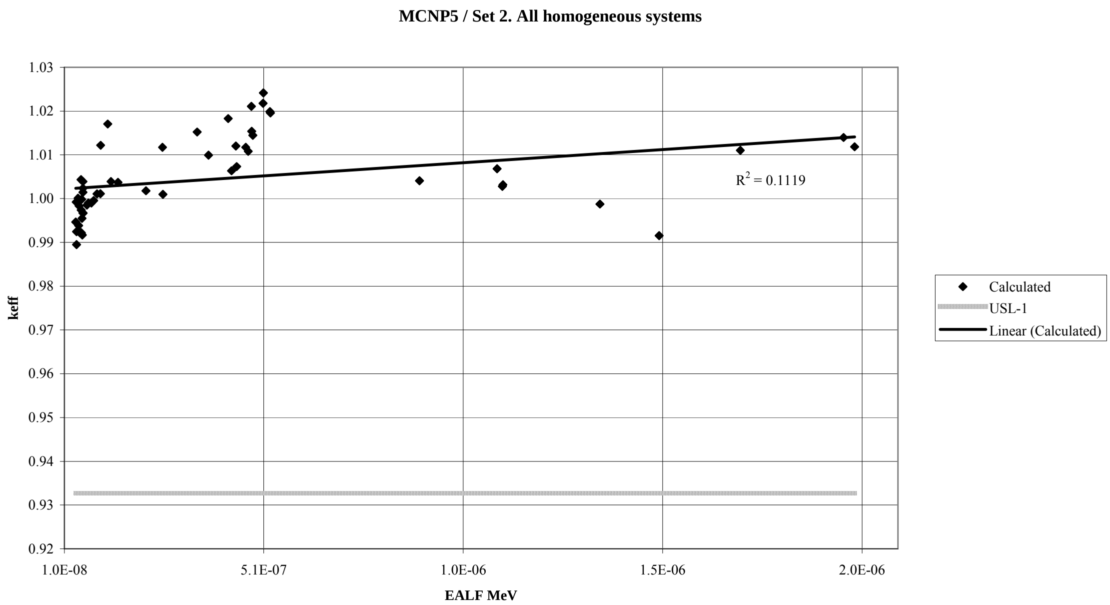

Figure 48 Homogeneous systems Set 2, k_{eff} vs. EALF (Lethargy)

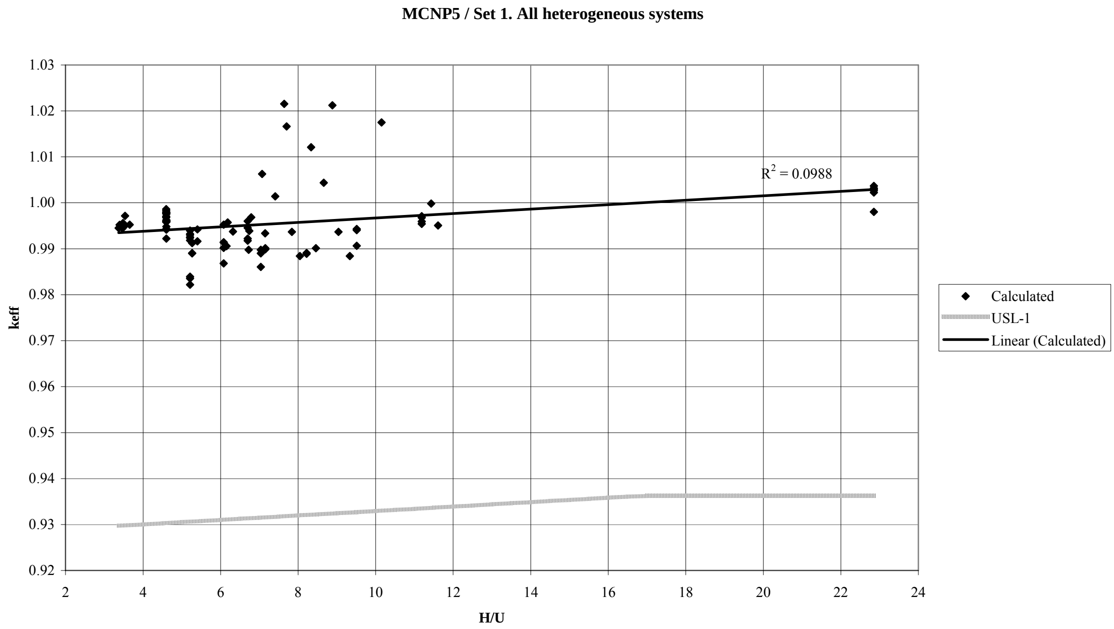

Figure 49 Heterogeneous systems Set 1, k_{eff} vs. H/U

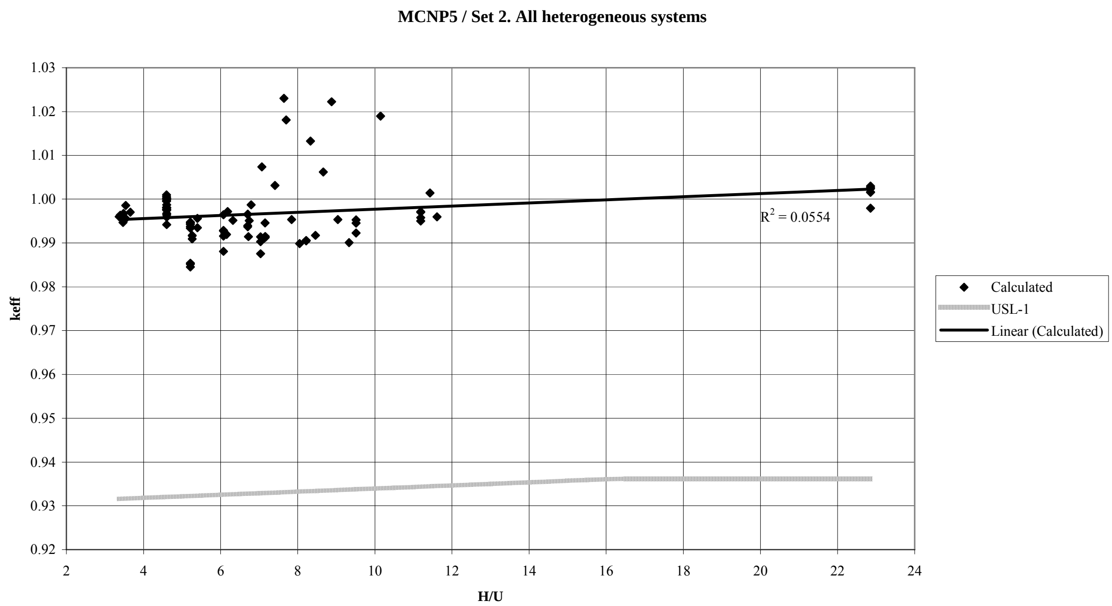

Figure 50 Heterogeneous systems Set 2, k_{eff} vs. H/U

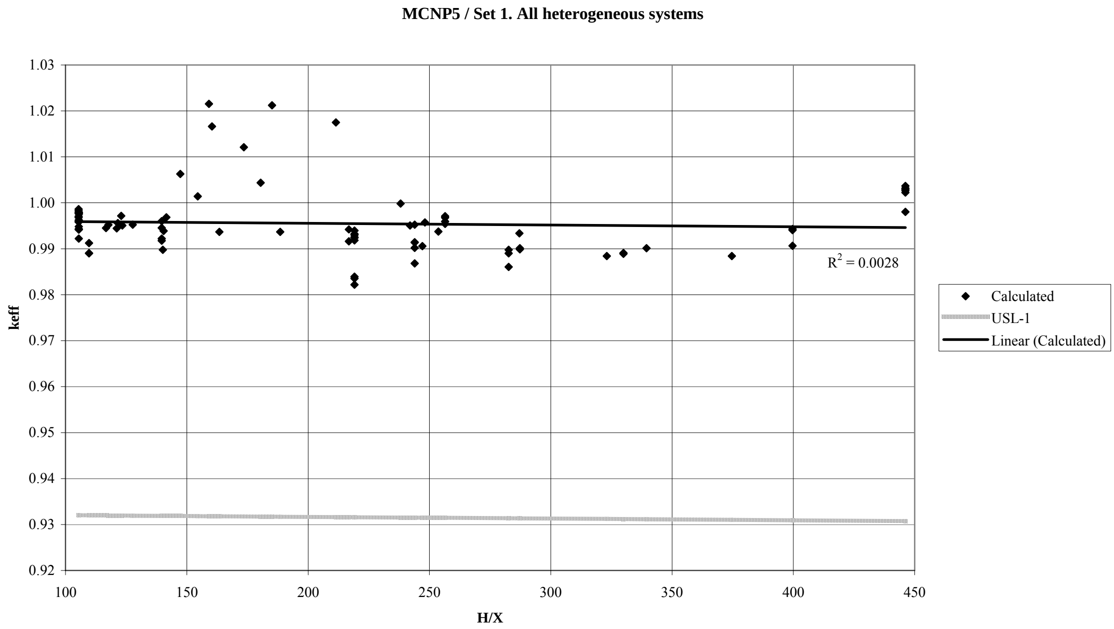

Figure 51   Heterogeneous systems Set 1, k_{eff} vs. H/X

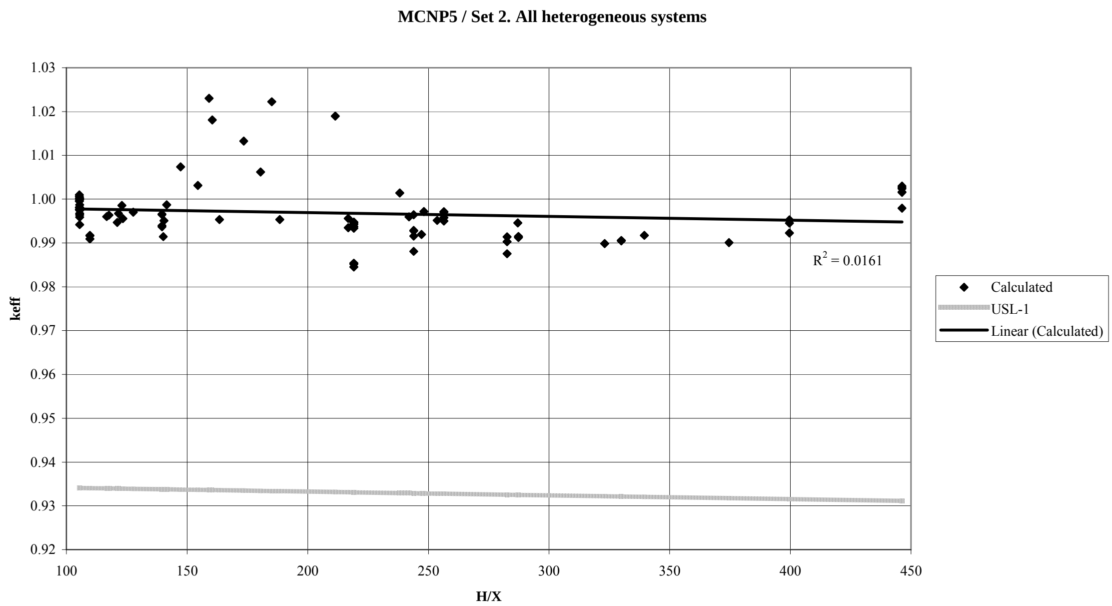

Figure 52   Heterogeneous systems Set 2, k_{eff} vs. H/X

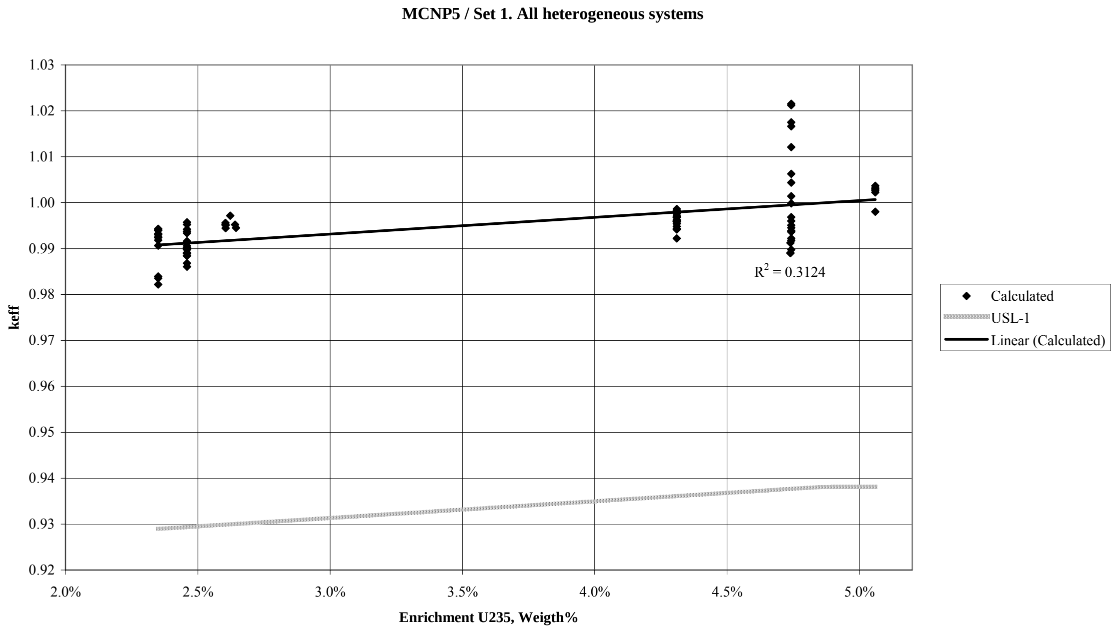

Figure 53 Heterogeneous systems Set 1, k_{eff} vs. Enrichment

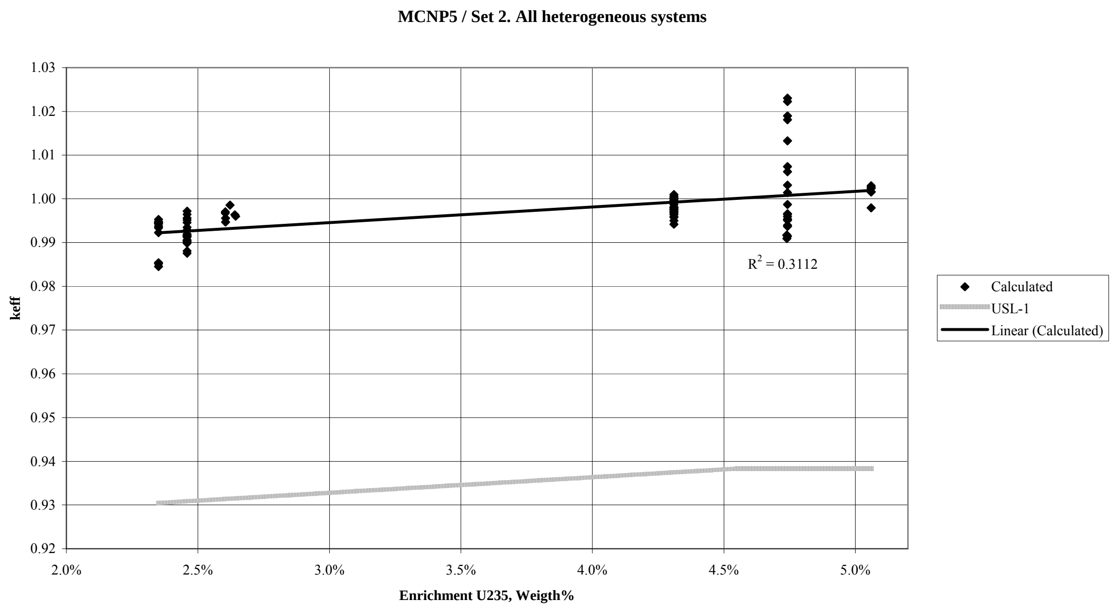

Figure 54 Heterogeneous systems Set 2, k_{eff} vs. Enrichment

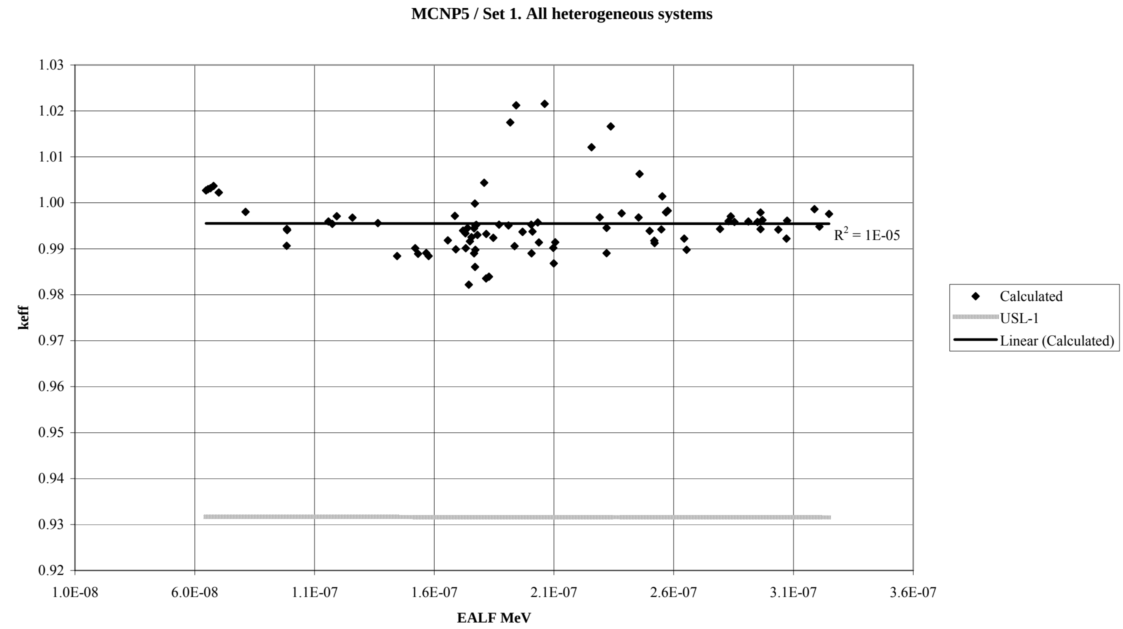

Figure 55 Heterogeneous systems Set 1, k_{eff} vs. EALF (Lethargy)

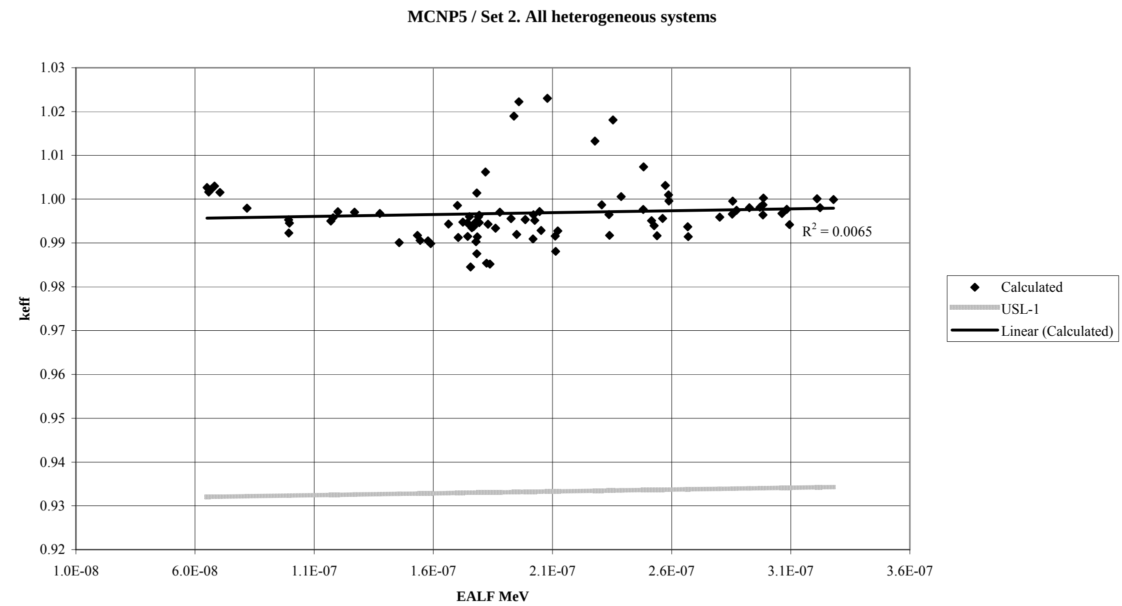

Figure 56 Heterogeneous systems Set 2, k_{eff} vs. EALF (Lethargy)

APPENDIX 4. USLSTATS OUTPUT FILES

USLSTATS output, Homogeneous systems Set 1, k_{eff} vs. H/U

```
             uslstats: a utility to calculate upper subcritical
                       limits for criticality safety applications

  ****************************************************************************
                         Version 1.4, April 23, 2003
                        Oak Ridge National Laboratory
  ****************************************************************************


   Input to statistical treatment from file:Hom-hu.in

Title: Homogeneous Experiments x=H/U

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   57
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y
7.88000E-01	1.00969E+00	2.51000E-03
7.88000E-01	1.00442E+00	2.51000E-03
7.88000E-01	9.87940E-01	2.91000E-03
7.88000E-01	9.95450E-01	2.91000E-03
7.88000E-01	1.00117E+00	3.11000E-03
7.88000E-01	1.00885E+00	2.61000E-03
7.88000E-01	1.01279E+00	2.41000E-03
7.88000E-01	1.01276E+00	2.41000E-03
7.88000E-01	1.00903E+00	3.01000E-03
7.88000E-01	1.00392E+00	2.91000E-03
7.88000E-01	1.01581E+00	2.81000E-03
7.88000E-01	1.00369E+00	2.81000E-03
7.88000E-01	1.01917E+00	3.51000E-03
7.88000E-01	1.01056E+00	4.01000E-03
7.88000E-01	1.00895E+00	4.01000E-03
7.88000E-01	1.00810E+00	3.61000E-03
7.88000E-01	1.02279E+00	3.11000E-03
7.88000E-01	1.01940E+00	3.11000E-03
7.88000E-01	1.01824E+00	3.11000E-03
7.88000E-01	1.01839E+00	3.11000E-03
1.24370E+00	1.00702E+00	2.81000E-03
1.24370E+00	9.99630E-01	4.21000E-03
1.24370E+00	9.99710E-01	2.11000E-03
4.00000E+00	1.00029E+00	3.81000E-03
6.00000E+00	1.00207E+00	3.91000E-03
8.20000E+00	9.99730E-01	4.01000E-03
1.00000E+01	9.98460E-01	3.91000E-03
1.24000E+01	9.97660E-01	4.10000E-03
1.97000E+01	9.92110E-01	5.10000E-03
4.00000E+00	1.00964E+00	3.81000E-03
8.40000E+00	1.01075E+00	3.91000E-03
4.00000E+00	9.99770E-01	4.01000E-03
6.00000E+00	1.00293E+00	3.91000E-03
8.20000E+00	1.00026E+00	3.91000E-03
1.00000E+01	9.98470E-01	4.01000E-03
1.24000E+01	9.98450E-01	4.10000E-03
1.97000E+01	9.91660E-01	5.00000E-03
4.00000E+00	1.01354E+00	4.21000E-03
8.40000E+00	1.01616E+00	4.11000E-03
2.54000E+01	9.98540E-01	5.01000E-03
2.60000E+01	1.00474E+00	5.01000E-03
3.20000E+01	9.99360E-01	5.01000E-03
3.63000E+01	1.00099E+00	5.00000E-03
4.90000E+01	9.99880E-01	5.00000E-03
5.44000E+01	9.95260E-01	4.00000E-03
2.60000E+01	9.91910E-01	5.01000E-03
3.20000E+01	9.92980E-01	5.01000E-03
3.63000E+01	9.94260E-01	5.01000E-03
4.90000E+01	9.90540E-01	5.00000E-03
4.96000E+01	9.93200E-01	3.71000E-03
2.47000E+01	1.00094E+00	5.01000E-03
2.47000E+01	1.00378E+00	5.01000E-03
2.47000E+01	1.00489E+00	5.01000E-03
2.47000E+01	9.96330E-01	5.01000E-03
2.47000E+01	9.97500E-01	5.01000E-03
2.47000E+01	9.97100E-01	5.01000E-03
2.47000E+01	1.00173E+00	5.01000E-03

```
   chi =  6.0702 (upper bound = 9.49). The data tests normal.


         Output from statistical treatment

   Homogeneous Experiments x=H/U

    Number of data points (n)                         57
    Linear regression, k(X)                          1.0075 + (-3.0956E-04)*X
    Confidence on fit (1-gamma) [input]               95.0%
    Confidence on proportion (alpha) [input]          95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]            99.5%
    Minimum value of X                                7.8800E-01
    Maximum value of X                                5.4400E+01
```

```
    Average value of X                                 1.3224E+01
    Average value of k                                  1.00339
    Minimum value of k                                  0.98794
    Variance of fit, s(k,X)^2                          4.7379E-05
    Within variance, s(w)^2                            1.6162E-05
    Pooled variance, s(p)^2                            6.3542E-05
    Pooled std. deviation, s(p)                        7.9713E-03
    C(alpha,rho)*s(p)                                  3.1362E-02
    student-t @ (n-2,1-gamma)                          1.67425E+00
    Confidence band width, W                           1.4305E-02
    Minimum margin of subcriticality, C*s(p)-W         1.7058E-02

    Upper subcritical limits: ( 0.78800     <= X <=   54.400    )
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)            USL1 = 0.9432 + (-3.0956E-04)*X (X >    24.183   )
                                           = 0.9357                   (X <=   24.183   )

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9761 + (-3.0956E-04)*X (X >  2.41827E+01)
                                           = 0.9686                   (X <= 2.41827E+01)


    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  7.88E-1  8.45E+0  1.61E+1  2.38E+1  3.14E+1  3.91E+1  4.67E+1  5.44E+1
       -----------------------------------------------------------------------
USL-1:    0.9357   0.9357   0.9357   0.9357   0.9335   0.9311   0.9287   0.9263
USL-2:    0.9686   0.9686   0.9686   0.9686   0.9664   0.9640   0.9617   0.9593
       -----------------------------------------------------------------------

  ****************************************************************************
                             Thus spake USLSTATS
                                    Finis.
         Plot file written to: Hom-hu.plt
```

USLSTATS output, Homogeneous systems Set 2, k_{eff} vs. H/U

```
             uslstats: a utility to calculate upper subcritical
                       limits for criticality safety applications

  ****************************************************************************
                        Version 1.4, April 23, 2003
                       Oak Ridge National Laboratory
  ****************************************************************************


   Input to statistical treatment from file:Hom-hu.in

Title: Homogeneous Experiments x=H/U

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   57
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
7.88000E-01	1.01201E+00	2.51000E-03	4.00000E+00	1.01173E+00	3.81000E-03
7.88000E-01	1.00734E+00	2.51000E-03	8.40000E+00	1.01218E+00	3.91000E-03
7.88000E-01	9.91530E-01	2.91000E-03	4.00000E+00	1.00096E+00	4.01000E-03
7.88000E-01	9.98750E-01	2.91000E-03	6.00000E+00	1.00370E+00	3.91000E-03
7.88000E-01	1.00409E+00	3.11000E-03	8.20000E+00	1.00114E+00	3.91000E-03
7.88000E-01	1.01080E+00	2.61000E-03	1.00000E+01	9.99540E-01	4.01000E-03
7.88000E-01	1.01537E+00	2.41000E-03	1.24000E+01	9.99000E-01	4.11000E-03
7.88000E-01	1.01448E+00	2.41000E-03	1.97000E+01	9.92160E-01	5.00000E-03
7.88000E-01	1.01170E+00	3.01000E-03	4.00000E+00	1.01520E+00	4.21000E-03
7.88000E-01	1.00684E+00	2.91000E-03	8.40000E+00	1.01704E+00	4.11000E-03
7.88000E-01	1.01827E+00	2.81000E-03	2.54000E+01	9.97380E-01	5.01000E-03
7.88000E-01	1.00637E+00	2.81000E-03	2.60000E+01	1.00434E+00	5.01000E-03
7.88000E-01	1.02106E+00	3.51000E-03	3.20000E+01	9.98350E-01	5.00000E-03
7.88000E-01	1.01395E+00	4.01000E-03	3.63000E+01	1.00008E+00	5.00000E-03
7.88000E-01	1.01183E+00	4.01000E-03	4.90000E+01	9.99220E-01	5.00000E-03
7.88000E-01	1.01101E+00	3.61000E-03	5.44000E+01	9.94660E-01	4.00000E-03
7.88000E-01	1.02413E+00	3.11000E-03	2.60000E+01	9.91680E-01	5.01000E-03
7.88000E-01	1.02177E+00	3.11000E-03	3.20000E+01	9.92210E-01	5.01000E-03
7.88000E-01	1.01988E+00	3.11000E-03	3.63000E+01	9.93860E-01	5.01000E-03
7.88000E-01	1.01958E+00	3.11000E-03	4.90000E+01	9.89490E-01	5.00000E-03
1.24370E+00	1.00996E+00	2.81000E-03	4.96000E+01	9.92440E-01	3.71000E-03
1.24370E+00	1.00279E+00	4.21000E-03	2.47000E+01	9.99830E-01	5.01000E-03
1.24370E+00	1.00317E+00	2.11000E-03	2.47000E+01	1.00237E+00	5.01000E-03
4.00000E+00	1.00180E+00	3.81000E-03	2.47000E+01	1.00393E+00	5.01000E-03
6.00000E+00	1.00390E+00	3.91000E-03	2.47000E+01	9.95480E-01	5.01000E-03
8.20000E+00	1.00107E+00	4.01000E-03	2.47000E+01	9.96670E-01	5.01000E-03
1.00000E+01	9.99030E-01	3.91000E-03	2.47000E+01	9.96670E-01	5.01000E-03
1.24000E+01	9.98500E-01	4.10000E-03	2.47000E+01	1.00148E+00	5.01000E-03
1.97000E+01	9.92240E-01	5.10000E-03			

```
   chi =  2.2105 (upper bound = 9.49). The data tests normal.


         Output from statistical treatment

   Homogeneous Experiments x=H/U

    Number of data points (n)                          57
    Linear regression, k(X)                           1.0097 + (-3.9482E-04)*X
    Confidence on fit (1-gamma) [input]                95.0%
    Confidence on proportion (alpha) [input]           95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]             99.5%
    Minimum value of X                                 7.8800E-01
    Maximum value of X                                 5.4400E+01
    Average value of X                                 1.3224E+01
    Average value of k                                  1.00449
    Minimum value of k                                  0.98949
```

```
    Variance of fit, s(k,X)^2                          4.5894E-05
    Within variance, s(w)^2                            1.6162E-05
    Pooled variance, s(p)^2                            6.2056E-05
    Pooled std. deviation, s(p)                        7.8776E-03
    C(alpha,rho)*s(p)                                  3.0993E-02
    student-t @ (n-2,1-gamma)                          1.67425E+00
    Confidence band width, W                           1.4136E-02
    Minimum margin of subcriticality, C*s(p)-W         1.6857E-02

    Upper subcritical limits: ( 0.78800     <= X <=   54.400    )
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)             USL1 = 0.9456 + (-3.9482E-04)*X (X >    24.600   )
                                            = 0.9359                   (X <=   24.600   )

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9787 + (-3.9482E-04)*X (X >  2.46003E+01)
                                            = 0.9690                   (X <= 2.46003E+01)


    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  7.88E-1  8.45E+0  1.61E+1  2.38E+1  3.14E+1  3.91E+1  4.67E+1  5.44E+1
       ------------------------------------------------------------------------
USL-1:    0.9359   0.9359   0.9359   0.9359   0.9332   0.9301   0.9271   0.9241
USL-2:    0.9690   0.9690   0.9690   0.9690   0.9663   0.9633   0.9603   0.9572
       ------------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
         Plot file written to: Hom-hu.plt
```

USLSTATS output, Homogeneous systems Set 1, k_{eff} vs. H/X

```
             uslstats: a utility to calculate upper subcritical
                       limits for criticality safety applications

 ***************************************************************************
                         Version 1.4, April 23, 2003
                         Oak Ridge National Laboratory
 ***************************************************************************


  Input to statistical treatment from file:Hom-hx.in

Title: Homogeneous Experiments x=H/X

  Proportion of the population = .995
  Confidence of fit            = .950
  Confidence on proportion     = .950
  Number of observations       =   57
  Minimum value of closed band =    0.00
  Maximum value of closed band =    0.00
  Administrative margin        =    0.05

  independent     dependent       deviation              independent     dependent       deviation
  variable - x    variable - y      in y                 variable - x    variable - y      in y

   1.74600E+01     1.00969E+00     2.51100E-03            1.33270E+02     1.00964E+00     3.80800E-03
   1.74600E+01     1.00442E+00     2.51200E-03            2.77210E+02     1.01075E+00     3.90800E-03
   1.74600E+01     9.87945E-01     2.90900E-03            1.95630E+02     9.99770E-01     4.00700E-03
   1.74600E+01     9.95453E-01     2.91100E-03            2.94360E+02     1.00293E+00     3.90700E-03
   1.74600E+01     1.00117E+00     3.10900E-03            4.06750E+02     1.00026E+00     3.90600E-03
   1.74600E+01     1.00885E+00     2.61200E-03            4.96210E+02     9.98470E-01     4.00600E-03
   1.74600E+01     1.01279E+00     2.41100E-03            6.13270E+02     9.98450E-01     4.10500E-03
   1.74600E+01     1.01276E+00     2.41100E-03            9.72770E+02     9.91660E-01     5.00300E-03
   1.74600E+01     1.00903E+00     3.01000E-03            1.33270E+02     1.01354E+00     4.20800E-03
   1.74600E+01     1.00392E+00     2.91000E-03            2.77210E+02     1.01616E+00     4.10900E-03
   1.74600E+01     1.01581E+00     2.81000E-03            5.01000E+02     9.98540E-01     5.00600E-03
   1.74600E+01     1.00369E+00     2.81100E-03            5.26000E+02     1.00474E+00     5.00600E-03
   1.74600E+01     1.01917E+00     3.50800E-03            6.46000E+02     9.99360E-01     5.00500E-03
   1.74600E+01     1.01056E+00     4.00700E-03            7.34000E+02     1.00099E+00     5.00400E-03
   1.74600E+01     1.00895E+00     4.00700E-03            9.89000E+02     9.99880E-01     5.00400E-03
   1.74600E+01     1.00810E+00     3.60800E-03            1.09800E+03     9.95258E-01     4.00400E-03
   1.74600E+01     1.02279E+00     3.10800E-03            5.26000E+02     9.91910E-01     5.00600E-03
   1.74600E+01     1.01940E+00     3.10900E-03            6.45000E+02     9.92980E-01     5.00600E-03
   1.74600E+01     1.01824E+00     3.10900E-03            7.34000E+02     9.94260E-01     5.00500E-03
   1.74600E+01     1.01839E+00     3.10900E-03            9.89000E+02     9.90540E-01     5.00400E-03
   2.76300E+01     1.00702E+00     2.81100E-03            1.00100E+03     9.93196E-01     3.70500E-03
   2.76300E+01     9.99631E-01     4.20700E-03            4.89000E+02     1.00094E+00     5.00600E-03
   2.76300E+01     9.99711E-01     2.11500E-03            4.89000E+02     1.00378E+00     5.00600E-03
   1.95630E+02     1.00029E+00     3.80800E-03            4.89000E+02     1.00489E+00     5.00600E-03
   2.94360E+02     1.00207E+00     3.90600E-03            4.89000E+02     9.96330E-01     5.00600E-03
   4.06750E+02     9.99730E-01     4.00600E-03            4.89000E+02     9.97500E-01     5.00600E-03
   4.96210E+02     9.98460E-01     3.90600E-03            4.89000E+02     9.97100E-01     5.00700E-03
   6.13270E+02     9.97660E-01     4.10400E-03            4.88000E+02     1.00173E+00     5.00700E-03
   9.72770E+02     9.92110E-01     5.10300E-03

  chi =  6.0702 (upper bound = 9.49). The data tests normal.


        Output from statistical treatment

  Homogeneous Experiments x=H/X

   Number of data points (n)                          57
   Linear regression, k(X)                          1.0088 + (-1.6339E-05)*X
   Confidence on fit (1-gamma) [input]                95.0%
   Confidence on proportion (alpha) [input]           95.0%
   Proportion of population falling above
   lower tolerance interval (rho) [input]             99.5%
   Minimum value of X                                 1.7460E+01
   Maximum value of X                                 1.0980E+03
   Average value of X                                 3.3372E+02
   Average value of k                                  1.00339
   Minimum value of k                                  0.98794
```

```
    Variance of fit, s(k,X)^2                          4.0172E-05
    Within variance, s(w)^2                            1.6151E-05
    Pooled variance, s(p)^2                            5.6324E-05
    Pooled std. deviation, s(p)                        7.5049E-03
    C(alpha,rho)*s(p)                                  2.8804E-02
    student-t @ (n-2,1-gamma)                          1.67425E+00
    Confidence band width, W                           1.3256E-02
    Minimum margin of subcriticality, C*s(p)-W         1.5548E-02

    Upper subcritical limits: (  17.460     <= X <=   1098.0    )
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)            USL1 = 0.9456 + (-1.6339E-05)*X (X >    541.33   )
                                           = 0.9367                   (X <=   541.33   )

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)   USL2 = 0.9800 + (-1.6339E-05)*X (X >  5.41327E+02)
                                           = 0.9712                   (X <= 5.41327E+02)


    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  1.75E+1  1.72E+2  3.26E+2  4.81E+2  6.35E+2  7.89E+2  9.44E+2  1.10E+3
       -----------------------------------------------------------------------
USL-1:    0.9367   0.9367   0.9367   0.9367   0.9352   0.9327   0.9302   0.9276
USL-2:    0.9712   0.9712   0.9712   0.9712   0.9697   0.9671   0.9646   0.9621
       -----------------------------------------------------------------------

  ****************************************************************************
                             Thus spake USLSTATS
                                    Finis.
         Plot file written to: Hom-hx.plt
```

USLSTATS output, Homogeneous systems Set 2, k_{eff} vs. H/X

```
            uslstats: a utility to calculate upper subcritical
                      limits for criticality safety applications

 ***************************************************************************
                        Version 1.4, April 23, 2003
                       Oak Ridge National Laboratory
 ***************************************************************************


  Input to statistical treatment from file:Hom-hx.in

Title: Homogeneous Experiments x=H/X

  Proportion of the population = .995
  Confidence of fit            = .950
  Confidence on proportion     = .950
  Number of observations       =   57
  Minimum value of closed band =    0.00
  Maximum value of closed band =    0.00
  Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
1.74600E+01	1.01201E+00	2.51200E-03	1.33270E+02	1.01173E+00	3.80800E-03
1.74600E+01	1.00734E+00	2.51200E-03	2.77210E+02	1.01218E+00	3.90800E-03
1.74600E+01	9.91535E-01	2.91000E-03	1.95630E+02	1.00096E+00	4.00700E-03
1.74600E+01	9.98754E-01	2.91000E-03	2.94360E+02	1.00370E+00	3.90700E-03
1.74600E+01	1.00409E+00	3.11000E-03	4.06750E+02	1.00114E+00	3.90700E-03
1.74600E+01	1.01080E+00	2.61200E-03	4.96210E+02	9.99540E-01	4.00600E-03
1.74600E+01	1.01537E+00	2.41100E-03	6.13270E+02	9.99000E-01	4.10500E-03
1.74600E+01	1.01448E+00	2.41100E-03	9.72770E+02	9.92160E-01	5.00300E-03
1.74600E+01	1.01170E+00	3.01000E-03	1.33270E+02	1.01520E+00	4.20800E-03
1.74600E+01	1.00684E+00	2.90900E-03	2.77210E+02	1.01704E+00	4.10900E-03
1.74600E+01	1.01827E+00	2.81100E-03	5.01000E+02	9.97380E-01	5.00500E-03
1.74600E+01	1.00637E+00	2.81100E-03	5.26000E+02	1.00434E+00	5.00500E-03
1.74600E+01	1.02106E+00	3.50800E-03	6.46000E+02	9.98350E-01	5.00500E-03
1.74600E+01	1.01395E+00	4.00700E-03	7.34000E+02	1.00008E+00	5.00400E-03
1.74600E+01	1.01183E+00	4.00700E-03	9.89000E+02	9.99220E-01	5.00400E-03
1.74600E+01	1.01101E+00	3.60700E-03	1.09800E+03	9.94660E-01	4.00400E-03
1.74600E+01	1.02413E+00	3.10900E-03	5.26000E+02	9.91680E-01	5.00700E-03
1.74600E+01	1.02177E+00	3.11000E-03	6.45000E+02	9.92210E-01	5.00600E-03
1.74600E+01	1.01988E+00	3.10900E-03	7.34000E+02	9.93860E-01	5.00500E-03
1.74600E+01	1.01958E+00	3.10900E-03	9.89000E+02	9.89490E-01	5.00400E-03
2.76300E+01	1.00996E+00	2.81100E-03	1.00100E+03	9.92438E-01	3.70500E-03
2.76300E+01	1.00279E+00	4.20700E-03	4.89000E+02	9.99830E-01	5.00600E-03
2.76300E+01	1.00317E+00	2.11500E-03	4.89000E+02	1.00237E+00	5.00600E-03
1.95630E+02	1.00180E+00	3.80900E-03	4.89000E+02	1.00393E+00	5.00600E-03
2.94360E+02	1.00390E+00	3.90600E-03	4.89000E+02	9.95480E-01	5.00600E-03
4.06750E+02	1.00107E+00	4.00600E-03	4.89000E+02	9.96670E-01	5.00600E-03
4.96210E+02	9.99030E-01	3.90600E-03	4.89000E+02	9.96670E-01	5.00600E-03
6.13270E+02	9.98500E-01	4.10400E-03	4.88000E+02	1.00148E+00	5.00700E-03
9.72770E+02	9.92240E-01	5.10300E-03			

```
  chi =  2.2105 (upper bound = 9.49). The data tests normal.


        Output from statistical treatment

  Homogeneous Experiments x=H/X

   Number of data points (n)                     57
   Linear regression, k(X)                     1.0112 + (-2.0121E-05)*X
   Confidence on fit (1-gamma) [input]           95.0%
   Confidence on proportion (alpha) [input]      95.0%
   Proportion of population falling above
   lower tolerance interval (rho) [input]        99.5%
   Minimum value of X                            1.7460E+01
   Maximum value of X                            1.0980E+03
   Average value of X                            3.3372E+02
   Average value of k                             1.00449
   Minimum value of k                             0.98949
```

```
    Variance of fit, s(k,X)^2                          3.7449E-05
    Within variance, s(w)^2                            1.6152E-05
    Pooled variance, s(p)^2                            5.3601E-05
    Pooled std. deviation, s(p)                        7.3213E-03
    C(alpha,rho)*s(p)                                  2.8100E-02
    student-t @ (n-2,1-gamma)                          1.67425E+00
    Confidence band width, W                           1.2932E-02
    Minimum margin of subcriticality, C*s(p)-W         1.5168E-02

    Upper subcritical limits: (  17.460     <= X <=   1098.0    )
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)            USL1 = 0.9483 + (-2.0121E-05)*X (X >    556.92   )
                                           = 0.9371                   (X <=   556.92   )

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9831 + (-2.0121E-05)*X (X >  5.56924E+02)
                                           = 0.9719                   (X <= 5.56924E+02)


    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  1.75E+1  1.72E+2  3.26E+2  4.81E+2  6.35E+2  7.89E+2  9.44E+2  1.10E+3
       ------------------------------------------------------------------------
USL-1:    0.9371   0.9371   0.9371   0.9371   0.9355   0.9324   0.9293   0.9262
USL-2:    0.9719   0.9719   0.9719   0.9719   0.9703   0.9672   0.9641   0.9610
       ------------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
         Plot file written to: Hom-hx.plt
```

USLSTATS output, Homogeneous systems Set 1, k_{eff} vs. Enrichment

```
             uslstats: a utility to calculate upper subcritical
                       limits for criticality safety applications

  ****************************************************************************
                         Version 1.4, April 23, 2003
                        Oak Ridge National Laboratory
  ****************************************************************************


   Input to statistical treatment from file:Hom-hx.in

Title: Homogeneous Experiments x=H/X

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   57
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05

   independent     dependent       deviation              independent     dependent       deviation
   variable - x    variable - y      in y                 variable - x    variable - y      in y

    1.74600E+01     1.01201E+00     2.51200E-03            1.33270E+02     1.01173E+00     3.80800E-03
    1.74600E+01     1.00734E+00     2.51200E-03            2.77210E+02     1.01218E+00     3.90800E-03
    1.74600E+01     9.91535E-01     2.91000E-03            1.95630E+02     1.00096E+00     4.00700E-03
    1.74600E+01     9.98754E-01     2.91000E-03            2.94360E+02     1.00370E+00     3.90700E-03
    1.74600E+01     1.00409E+00     3.11000E-03            4.06750E+02     1.00114E+00     3.90700E-03
    1.74600E+01     1.01080E+00     2.61200E-03            4.96210E+02     9.99540E-01     4.00600E-03
    1.74600E+01     1.01537E+00     2.41100E-03            6.13270E+02     9.99000E-01     4.10500E-03
    1.74600E+01     1.01448E+00     2.41100E-03            9.72770E+02     9.92160E-01     5.00300E-03
    1.74600E+01     1.01170E+00     3.01000E-03            1.33270E+02     1.01520E+00     4.20800E-03
    1.74600E+01     1.00684E+00     2.90900E-03            2.77210E+02     1.01704E+00     4.10900E-03
    1.74600E+01     1.01827E+00     2.81100E-03            5.01000E+02     9.97380E-01     5.00500E-03
    1.74600E+01     1.00637E+00     2.81100E-03            5.26000E+02     1.00434E+00     5.00500E-03
    1.74600E+01     1.02106E+00     3.50800E-03            6.46000E+02     9.98350E-01     5.00500E-03
    1.74600E+01     1.01395E+00     4.00700E-03            7.34000E+02     1.00008E+00     5.00400E-03
    1.74600E+01     1.01183E+00     4.00700E-03            9.89000E+02     9.99220E-01     5.00400E-03
    1.74600E+01     1.01101E+00     3.60700E-03            1.09800E+03     9.94660E-01     4.00400E-03
    1.74600E+01     1.02413E+00     3.10900E-03            5.26000E+02     9.91680E-01     5.00700E-03
    1.74600E+01     1.02177E+00     3.11000E-03            6.45000E+02     9.92210E-01     5.00600E-03
    1.74600E+01     1.01988E+00     3.10900E-03            7.34000E+02     9.93860E-01     5.00500E-03
    1.74600E+01     1.01958E+00     3.10900E-03            9.89000E+02     9.89490E-01     5.00400E-03
    2.76300E+01     1.00996E+00     2.81100E-03            1.00100E+03     9.92438E-01     3.70500E-03
    2.76300E+01     1.00279E+00     4.20700E-03            4.89000E+02     9.99830E-01     5.00600E-03
    2.76300E+01     1.00317E+00     2.11500E-03            4.89000E+02     1.00237E+00     5.00600E-03
    1.95630E+02     1.00180E+00     3.80900E-03            4.89000E+02     1.00393E+00     5.00600E-03
    2.94360E+02     1.00390E+00     3.90600E-03            4.89000E+02     9.95480E-01     5.00600E-03
    4.06750E+02     1.00107E+00     4.00600E-03            4.89000E+02     9.96670E-01     5.00600E-03
    4.96210E+02     9.99030E-01     3.90600E-03            4.89000E+02     9.96670E-01     5.00600E-03
    6.13270E+02     9.98500E-01     4.10400E-03            4.88000E+02     1.00148E+00     5.00700E-03
    9.72770E+02     9.92240E-01     5.10300E-03

   chi =  2.2105 (upper bound = 9.49). The data tests normal.


         Output from statistical treatment

   Homogeneous Experiments x=H/X

    Number of data points (n)                         57
    Linear regression, k(X)                          1.0112 + (-2.0121E-05)*X
    Confidence on fit (1-gamma) [input]               95.0%
    Confidence on proportion (alpha) [input]          95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]            99.5%
    Minimum value of X                                1.7460E+01
    Maximum value of X                                1.0980E+03
    Average value of X                                3.3372E+02
    Average value of k                                 1.00449
    Minimum value of k                                 0.98949
```

```
    Variance of fit, s(k,X)^2                          3.7449E-05
    Within variance, s(w)^2                            1.6152E-05
    Pooled variance, s(p)^2                            5.3601E-05
    Pooled std. deviation, s(p)                        7.3213E-03
    C(alpha,rho)*s(p)                                  2.8100E-02
    student-t @ (n-2,1-gamma)                          1.67425E+00
    Confidence band width, W                           1.2932E-02
    Minimum margin of subcriticality, C*s(p)-W         1.5168E-02

    Upper subcritical limits: (  17.460     <= X <=   1098.0    )
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)            USL1 = 0.9483 + (-2.0121E-05)*X (X >    556.92   )
                                           = 0.9371                   (X <=   556.92   )

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9831 + (-2.0121E-05)*X (X >  5.56924E+02)
                                           = 0.9719                   (X <= 5.56924E+02)


    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  1.75E+1  1.72E+2  3.26E+2  4.81E+2  6.35E+2  7.89E+2  9.44E+2  1.10E+3
       -----------------------------------------------------------------------
USL-1:    0.9371   0.9371   0.9371   0.9371   0.9355   0.9324   0.9293   0.9262
USL-2:    0.9719   0.9719   0.9719   0.9719   0.9703   0.9672   0.9641   0.9610
       -----------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
         Plot file written to: Hom-hx.plt
```

USLSTATS output, Homogeneous systems Set 2, k_{eff} vs. Enrichment

```
            uslstats: a utility to calculate upper subcritical
                      limits for criticality safety applications

  ***************************************************************************
                        Version 1.4, April 23, 2003
                       Oak Ridge National Laboratory
  ***************************************************************************


   Input to statistical treatment from file:Hom-enr.in

Title: Homogeneous Experiments x=Enrichment

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   57
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
4.46000E-02	1.01201E+00	2.51000E-03	3.00000E-02	1.01173E+00	3.81000E-03
4.46000E-02	1.00734E+00	2.51000E-03	3.00000E-02	1.01218E+00	3.91000E-03
4.46000E-02	9.91530E-01	2.91000E-03	2.00000E-02	1.00096E+00	4.01000E-03
4.46000E-02	9.98750E-01	2.91000E-03	2.00000E-02	1.00370E+00	3.91000E-03
4.46000E-02	1.00409E+00	3.11000E-03	2.00000E-02	1.00114E+00	3.91000E-03
4.46000E-02	1.01080E+00	2.61000E-03	2.00000E-02	9.99540E-01	4.01000E-03
4.46000E-02	1.01537E+00	2.41000E-03	2.00000E-02	9.99000E-01	4.11000E-03
4.46000E-02	1.01448E+00	2.41000E-03	2.00000E-02	9.92160E-01	5.00000E-03
4.46000E-02	1.01170E+00	3.01000E-03	3.00000E-02	1.01520E+00	4.21000E-03
4.46000E-02	1.00684E+00	2.91000E-03	3.00000E-02	1.01704E+00	4.11000E-03
4.46000E-02	1.01827E+00	2.81000E-03	5.00000E-02	9.97380E-01	5.01000E-03
4.46000E-02	1.00637E+00	2.81000E-03	4.89000E-02	1.00434E+00	5.01000E-03
4.46000E-02	1.02106E+00	3.51000E-03	4.89000E-02	9.98350E-01	5.00000E-03
4.46000E-02	1.01395E+00	4.01000E-03	4.89000E-02	1.00008E+00	5.00000E-03
4.46000E-02	1.01183E+00	4.01000E-03	4.89000E-02	9.99220E-01	5.00000E-03
4.46000E-02	1.01101E+00	3.61000E-03	4.89000E-02	9.94660E-01	4.00000E-03
4.46000E-02	1.02413E+00	3.11000E-03	4.89000E-02	9.91680E-01	5.01000E-03
4.46000E-02	1.02177E+00	3.11000E-03	4.89000E-02	9.92210E-01	5.01000E-03
4.46000E-02	1.01988E+00	3.11000E-03	4.89000E-02	9.93860E-01	5.01000E-03
4.46000E-02	1.01958E+00	3.11000E-03	4.89000E-02	9.89490E-01	5.00000E-03
4.48000E-02	1.00996E+00	2.81000E-03	4.89000E-02	9.92440E-01	3.71000E-03
4.48000E-02	1.00279E+00	4.21000E-03	5.00000E-02	9.99830E-01	5.01000E-03
4.48000E-02	1.00317E+00	2.11000E-03	5.00000E-02	1.00237E+00	5.01000E-03
2.00000E-02	1.00180E+00	3.81000E-03	5.00000E-02	1.00393E+00	5.01000E-03
2.00000E-02	1.00390E+00	3.91000E-03	5.00000E-02	9.95480E-01	5.01000E-03
2.00000E-02	1.00107E+00	4.01000E-03	5.00000E-02	9.96670E-01	5.01000E-03
2.00000E-02	9.99030E-01	3.91000E-03	5.00000E-02	9.96670E-01	5.01000E-03
2.00000E-02	9.98500E-01	4.10000E-03	5.00000E-02	1.00148E+00	5.01000E-03
2.00000E-02	9.92240E-01	5.10000E-03			

```
   chi =  2.2105 (upper bound = 9.49). The data tests normal.


         Output from statistical treatment

   Homogeneous Experiments x=Enrichment

    Number of data points (n)                          57
    Linear regression, k(X)                          1.0027 + ( 4.5539E-02)*X
    Confidence on fit (1-gamma) [input]                95.0%
    Confidence on proportion (alpha) [input]           95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]             99.5%
    Minimum value of X                                 2.0000E-02
    Maximum value of X                                 5.0000E-02
    Average value of X                                 3.9919E-02
    Average value of k                                  1.00449
    Minimum value of k                                  0.98949
```

```
    Variance of fit, s(k,X)^2                          8.2227E-05
    Within variance, s(w)^2                            1.6162E-05
    Pooled variance, s(p)^2                            9.8389E-05
    Pooled std. deviation, s(p)                        9.9191E-03
    C(alpha,rho)*s(p)                                  3.6777E-02
    student-t @ (n-2,1-gamma)                          1.67425E+00
    Confidence band width, W                           1.7195E-02
    Minimum margin of subcriticality, C*s(p)-W         1.9583E-02

    Upper subcritical limits: ( 2.00000E-02 <= X <=  5.00000E-02)
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)             USL1 = 0.9328 ( 2.00000E-2< X < 5.00000E-2)

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9632 (2.00000E-02< X <5.00000E-02)


    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  2.00E-2  2.43E-2  2.86E-2  3.29E-2  3.71E-2  4.14E-2  4.57E-2  5.00E-2
       ------------------------------------------------------------------------
USL-1:    0.9328   0.9328   0.9328   0.9328   0.9328   0.9328   0.9328   0.9328
USL-2:    0.9632   0.9632   0.9632   0.9632   0.9632   0.9632   0.9632   0.9632
       ------------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
         Plot file written to: Hom-enr.plt
```

USLSTATS output, Homogeneous systems Set 1, k_{eff} vs. EALF (Lethargy)

```
              uslstats: a utility to calculate upper subcritical
                        limits for criticality safety applications

  ****************************************************************************
                         Version 1.4, April 23, 2003
                        Oak Ridge National Laboratory
  ****************************************************************************


   Input to statistical treatment from file:Hom-leth.in

Title: Homogeneous Experiments x=EALF MeV

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   57
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y
4.37300E-07	1.00969E+00	2.51000E-03
4.40600E-07	1.00442E+00	2.51000E-03
1.51500E-06	9.87940E-01	2.91000E-03
1.36300E-06	9.95450E-01	2.91000E-03
9.02700E-07	1.00117E+00	3.11000E-03
4.68400E-07	1.00885E+00	2.61000E-03
4.78800E-07	1.01279E+00	2.41000E-03
4.78500E-07	1.01276E+00	2.41000E-03
4.62300E-07	1.00903E+00	3.01000E-03
1.09700E-06	1.00392E+00	2.91000E-03
4.20600E-07	1.01581E+00	2.81000E-03
4.26900E-07	1.00369E+00	2.81000E-03
4.76500E-07	1.01917E+00	3.51000E-03
1.97900E-06	1.01056E+00	4.01000E-03
2.00100E-06	1.00895E+00	4.01000E-03
1.71300E-06	1.00810E+00	3.61000E-03
5.05900E-07	1.02279E+00	3.11000E-03
5.07100E-07	1.01940E+00	3.11000E-03
5.22900E-07	1.01824E+00	3.11000E-03
5.23200E-07	1.01839E+00	3.11000E-03
3.70100E-07	1.00702E+00	2.81000E-03
1.11100E-06	9.99630E-01	4.21000E-03
1.11100E-06	9.99710E-01	2.11000E-03
2.13400E-07	1.00029E+00	3.81000E-03
1.25800E-07	1.00207E+00	3.91000E-03
9.06200E-08	9.99730E-01	4.01000E-03
7.70200E-08	9.98460E-01	3.91000E-03
6.60800E-08	9.97660E-01	4.10000E-03
5.10400E-08	9.92110E-01	5.10000E-03
2.54600E-07	1.00964E+00	3.81000E-03
1.00300E-07	1.01075E+00	3.91000E-03
2.55000E-07	9.99770E-01	4.01000E-03
1.43600E-07	1.00293E+00	3.91000E-03
9.97600E-08	1.00026E+00	3.91000E-03
8.28000E-08	9.98470E-01	4.01000E-03
6.95500E-08	9.98450E-01	4.10000E-03
5.20200E-08	9.91660E-01	5.00000E-03
3.42400E-07	1.01354E+00	4.21000E-03
1.17700E-07	1.01616E+00	4.11000E-03
5.17400E-08	9.98540E-01	5.01000E-03
5.14500E-08	1.00474E+00	5.01000E-03
4.67900E-08	9.99360E-01	5.01000E-03
4.41400E-08	1.00099E+00	5.00000E-03
3.96800E-08	9.99880E-01	5.00000E-03
3.83400E-08	9.95260E-01	4.00000E-03
5.43100E-08	9.91910E-01	5.01000E-03
4.84600E-08	9.92980E-01	5.01000E-03
4.55500E-08	9.94260E-01	5.01000E-03
4.04300E-08	9.90540E-01	5.00000E-03
4.02000E-08	9.93200E-01	3.71000E-03
5.31000E-08	1.00094E+00	5.01000E-03
5.61600E-08	1.00378E+00	5.01000E-03
5.65100E-08	1.00489E+00	5.01000E-03
5.38000E-08	9.96330E-01	5.01000E-03
5.63300E-08	9.97500E-01	5.01000E-03
5.65600E-08	9.97100E-01	5.01000E-03
5.63200E-08	1.00173E+00	5.01000E-03

```
   chi =  6.0702 (upper bound = 9.49). The data tests normal.


         Output from statistical treatment

   Homogeneous Experiments x=EALF MeV

    Number of data points (n)                         57
    Linear regression, k(X)                         1.0020 + ( 3.6589E+03)*X
    Confidence on fit (1-gamma) [input]               95.0%
    Confidence on proportion (alpha) [input]          95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]            99.5%
    Minimum value of X                                3.8340E-08
    Maximum value of X                                2.0010E-06
    Average value of X                                3.9199E-07
    Average value of k                                 1.00339
    Minimum value of k                                 0.98794
```

```
    Variance of fit, s(k,X)^2                          6.6419E-05
    Within variance, s(w)^2                            1.6162E-05
    Pooled variance, s(p)^2                            8.2581E-05
    Pooled std. deviation, s(p)                        9.0874E-03
    C(alpha,rho)*s(p)                                  3.6858E-02
    student-t @ (n-2,1-gamma)                          1.67425E+00
    Confidence band width, W                           1.6662E-02
    Minimum margin of subcriticality, C*s(p)-W         2.0196E-02

    Upper subcritical limits: ( 3.83400E-08 <= X <=  2.00100E-06)
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)            USL1 = 0.9333 ( 3.83400E-8< X < 2.00100E-6)

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9631 (3.83400E-08< X <2.00100E-06)



    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  3.83E-8  3.19E-7  5.99E-7  8.79E-7  1.16E-6  1.44E-6  1.72E-6  2.00E-6
       -----------------------------------------------------------------------
USL-1:     0.9333   0.9333   0.9333   0.9333   0.9333   0.9333   0.9333   0.9333
USL-2:     0.9631   0.9631   0.9631   0.9631   0.9631   0.9631   0.9631   0.9631
       -----------------------------------------------------------------------

  ****************************************************************************
                             Thus spake USLSTATS
                                    Finis.
         Plot file written to: Hom-leth.plt
```

USLSTATS output, Homogeneous systems Set 2, k_{eff} vs. EALF (Lethargy)

```
              uslstats: a utility to calculate upper subcritical
                        limits for criticality safety applications

  ****************************************************************************
                         Version 1.4, April 23, 2003
                        Oak Ridge National Laboratory
  ****************************************************************************


   Input to statistical treatment from file:Hom-leth.in

Title: Homogeneous Experiments x=EALF MeV

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   57
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05

   independent     dependent       deviation              independent     dependent       deviation
   variable - x    variable - y      in y                 variable - x    variable - y      in y

    4.40000E-07     1.01201E+00     2.51000E-03             2.56200E-07     1.01173E+00     3.81000E-03
    4.41800E-07     1.00734E+00     2.51000E-03             1.01000E-07     1.01218E+00     3.91000E-03
    1.50100E-06     9.91530E-01     2.91000E-03             2.56900E-07     1.00096E+00     4.01000E-03
    1.35300E-06     9.98750E-01     2.91000E-03             1.44400E-07     1.00370E+00     3.91000E-03
    9.00700E-07     1.00409E+00     3.11000E-03             1.00600E-07     1.00114E+00     3.91000E-03
    4.71100E-07     1.01080E+00     2.61000E-03             8.34200E-08     9.99540E-01     4.01000E-03
    4.79800E-07     1.01537E+00     2.41000E-03             7.01100E-08     9.99000E-01     4.11000E-03
    4.82200E-07     1.01448E+00     2.41000E-03             5.23200E-08     9.92160E-01     5.00000E-03
    4.64700E-07     1.01170E+00     3.01000E-03             3.43000E-07     1.01520E+00     4.21000E-03
    1.09500E-06     1.00684E+00     2.91000E-03             1.18400E-07     1.01704E+00     4.11000E-03
    4.21000E-07     1.01827E+00     2.81000E-03             5.20400E-08     9.97380E-01     5.01000E-03
    4.28900E-07     1.00637E+00     2.81000E-03             5.16500E-08     1.00434E+00     5.01000E-03
    4.79000E-07     1.02106E+00     3.51000E-03             4.70400E-08     9.98350E-01     5.00000E-03
    1.96300E-06     1.01395E+00     4.01000E-03             4.43500E-08     1.00008E+00     5.00000E-03
    1.99200E-06     1.01183E+00     4.01000E-03             3.98200E-08     9.99220E-01     5.00000E-03
    1.70500E-06     1.01101E+00     3.61000E-03             3.84400E-08     9.94660E-01     4.00000E-03
    5.08700E-07     1.02413E+00     3.11000E-03             5.46100E-08     9.91680E-01     5.01000E-03
    5.07900E-07     1.02177E+00     3.11000E-03             4.86900E-08     9.92210E-01     5.01000E-03
    5.25300E-07     1.01988E+00     3.11000E-03             4.57700E-08     9.93860E-01     5.01000E-03
    5.26800E-07     1.01958E+00     3.11000E-03             4.06000E-08     9.89490E-01     5.00000E-03
    3.71600E-07     1.00996E+00     2.81000E-03             4.03600E-08     9.92440E-01     3.71000E-03
    1.10800E-06     1.00279E+00     4.21000E-03             5.34100E-08     9.99830E-01     5.01000E-03
    1.11000E-06     1.00317E+00     2.11000E-03             5.64700E-08     1.00237E+00     5.01000E-03
    2.14600E-07     1.00180E+00     3.81000E-03             5.67000E-08     1.00393E+00     5.01000E-03
    1.26900E-07     1.00390E+00     3.91000E-03             5.41200E-08     9.95480E-01     5.01000E-03
    9.15700E-08     1.00107E+00     4.01000E-03             5.66500E-08     9.96670E-01     5.01000E-03
    7.76300E-08     9.99030E-01     3.91000E-03             5.68400E-08     9.96670E-01     5.01000E-03
    6.65200E-08     9.98500E-01     4.10000E-03             5.67000E-08     1.00148E+00     5.01000E-03
    5.13300E-08     9.92240E-01     5.10000E-03

   chi =  2.2105 (upper bound = 9.49). The data tests normal.


         Output from statistical treatment

   Homogeneous Experiments x=EALF MeV

    Number of data points (n)                          57
    Linear regression, k(X)                          1.0021 + ( 6.0060E+03)*X
    Confidence on fit (1-gamma) [input]                95.0%
    Confidence on proportion (alpha) [input]           95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]             99.5%
    Minimum value of X                                 3.8440E-08
    Maximum value of X                                 1.9920E-06
    Average value of X                                 3.9168E-07
    Average value of k                                  1.00449
    Minimum value of k                                  0.98949
```

```
Variance of fit, s(k,X)^2                            7.3272E-05
Within variance, s(w)^2                              1.6162E-05
Pooled variance, s(p)^2                              8.9434E-05
Pooled std. deviation, s(p)                          9.4569E-03
C(alpha,rho)*s(p)                                    3.8361E-02
student-t @ (n-2,1-gamma)                            1.67425E+00
Confidence band width, W                             1.7341E-02
Minimum margin of subcriticality, C*s(p)-W           2.1020E-02

Upper subcritical limits: ( 3.84400E-08 <= X <=  1.99200E-06)
***** *********** *******

USL Method 1 (Confidence Band with
Administrative Margin)            USL1 = 0.9327 ( 3.84400E-8< X < 1.99200E-6)

USL Method 2 (Single-Sided Uniform
Width Closed Interval Approach)    USL2 = 0.9616 (3.84400E-08< X <1.99200E-06)


USLs Evaluated Over Range of Parameter X:
**** ********* **** ***** ** ********* **

    X:  3.84E-8  3.18E-7  5.97E-7  8.76E-7  1.15E-6  1.43E-6  1.71E-6  1.99E-6
       ------------------------------------------------------------------------
USL-1:    0.9327   0.9327   0.9327   0.9327   0.9327   0.9327   0.9327   0.9327
USL-2:    0.9616   0.9616   0.9616   0.9616   0.9616   0.9616   0.9616   0.9616
       ------------------------------------------------------------------------

  ****************************************************************************
                            Thus spake USLSTATS
                                   Finis.
         Plot file written to: Hom-leth.plt
```

USLSTATS output, Heterogeneous systems Set 1, k_{eff} vs. H/U

```
             uslstats: a utility to calculate upper subcritical
                       limits for criticality safety applications

  ***************************************************************************
                         Version 1.4, April 23, 2003
                        Oak Ridge National Laboratory
  ***************************************************************************


   Input to statistical treatment from file:Het-hu.in

Title: Heterogeneous Experiments x=H/U

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   90
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
5.40000E+00	9.91650E-01	1.82000E-03	4.60000E+00	9.97570E-01	4.02000E-03
5.40000E+00	9.94210E-01	3.20000E-03	4.60000E+00	9.92220E-01	1.85000E-03
6.32000E+00	9.93760E-01	3.20000E-03	4.60000E+00	9.98620E-01	4.02000E-03
6.16000E+00	9.90590E-01	1.70000E-03	5.21000E+00	9.83540E-01	3.42000E-03
7.16000E+00	9.89880E-01	1.70000E-03	4.60000E+00	9.95820E-01	4.02000E-03
7.16000E+00	9.90150E-01	1.70000E-03	5.21000E+00	9.93250E-01	1.64000E-03
8.22000E+00	9.88890E-01	1.70000E-03	4.60000E+00	9.94170E-01	1.84000E-03
8.22000E+00	9.89070E-01	1.70000E-03	5.21000E+00	9.83950E-01	3.42000E-03
9.33000E+00	9.88430E-01	1.80000E-03	4.60000E+00	9.97910E-01	4.02000E-03
8.46000E+00	9.90130E-01	2.00000E-03	9.51000E+00	9.94090E-01	3.12000E-03
6.18000E+00	9.95740E-01	2.40000E-03	1.11900E+01	9.95410E-01	2.14000E-03
7.15000E+00	9.93390E-01	1.90000E-03	4.60000E+00	9.97730E-01	4.02000E-03
6.08000E+00	9.95220E-01	1.90000E-03	4.60000E+00	9.97930E-01	4.02000E-03
6.08000E+00	9.91380E-01	1.90000E-03	4.60000E+00	9.96790E-01	4.02000E-03
6.08000E+00	9.86820E-01	2.20000E-03	4.60000E+00	9.98280E-01	4.02000E-03
7.04000E+00	9.86060E-01	1.90000E-03	5.21000E+00	9.82190E-01	3.42000E-03
6.08000E+00	9.90200E-01	2.40000E-03	4.60000E+00	9.97050E-01	4.02000E-03
7.04000E+00	9.89020E-01	2.00000E-03	5.21000E+00	9.92550E-01	1.64000E-03
6.08000E+00	9.91400E-01	2.70000E-03	4.60000E+00	9.94250E-01	1.85000E-03
7.04000E+00	9.89760E-01	2.10000E-03	3.40000E+00	9.95210E-01	5.01000E-03
8.05000E+00	9.88420E-01	1.90000E-03	3.37000E+00	9.94510E-01	5.01000E-03
2.28500E+01	9.98060E-01	6.11000E-03	3.54000E+00	9.97160E-01	5.01000E-03
2.28500E+01	1.00224E+00	6.11000E-03	3.54000E+00	9.95070E-01	5.01000E-03
2.28500E+01	1.00364E+00	6.11000E-03	3.47000E+00	9.94450E-01	5.01000E-03
2.28500E+01	1.00317E+00	6.11000E-03	3.66000E+00	9.95250E-01	5.01000E-03
2.28500E+01	1.00296E+00	6.11000E-03	3.48000E+00	9.95610E-01	5.01000E-03
2.28500E+01	1.00272E+00	6.12000E-03	6.70000E+00	9.96020E-01	5.01000E-03
5.26000E+00	9.91250E-01	1.66000E-03	6.70000E+00	9.92230E-01	5.01000E-03
5.26000E+00	9.89050E-01	1.47000E-03	6.72000E+00	9.89760E-01	5.01000E-03
5.26000E+00	9.89020E-01	1.46000E-03	7.07000E+00	1.00629E+00	5.01000E-03
5.21000E+00	9.92390E-01	2.83000E-03	7.41000E+00	1.00140E+00	5.01000E-03
5.21000E+00	9.93000E-01	2.82000E-03	7.70000E+00	1.01663E+00	5.01000E-03
5.21000E+00	9.93970E-01	2.83000E-03	7.84000E+00	9.93690E-01	5.01000E-03
5.21000E+00	9.91860E-01	2.82000E-03	6.70000E+00	9.91770E-01	5.01000E-03
9.51000E+00	9.94290E-01	3.12000E-03	6.74000E+00	9.93910E-01	5.01000E-03
9.51000E+00	9.90650E-01	3.12000E-03	7.64000E+00	1.02156E+00	5.01000E-03
4.60000E+00	9.94830E-01	2.83000E-03	8.33000E+00	1.01211E+00	5.01000E-03
4.60000E+00	9.96120E-01	2.83000E-03	8.88000E+00	1.02123E+00	5.01000E-03
4.60000E+00	9.96250E-01	2.83000E-03	9.04000E+00	9.93690E-01	5.01000E-03
4.60000E+00	9.95940E-01	2.83000E-03	6.70000E+00	9.94560E-01	5.01000E-03
4.60000E+00	9.95820E-01	2.83000E-03	6.79000E+00	9.96840E-01	5.01000E-03
4.60000E+00	9.94280E-01	2.83000E-03	8.66000E+00	1.00438E+00	5.01000E-03
1.11900E+01	9.96770E-01	2.14000E-03	1.01500E+01	1.01752E+00	5.01000E-03
1.11900E+01	9.97110E-01	2.14000E-03	1.14300E+01	9.99820E-01	5.01000E-03
1.11900E+01	9.95950E-01	2.14000E-03	1.16100E+01	9.95040E-01	5.01000E-03

```
   chi = 24.1111 (upper bound = 9.49). The data tests NOT normal
```

```
      Output from statistical treatment

   Heterogeneous Experiments x=H/U

    Number of data points (n)                        90
    Linear regression, k(X)                         0.9919 + ( 4.8379E-04)*X
    Confidence on fit (1-gamma) [input]              95.0%
    Confidence on proportion (alpha) [input]         95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]           99.5%
    Minimum value of X                               3.3700E+00
    Maximum value of X                               2.2850E+01
    Average value of X                               7.5318E+00
    Average value of k                                0.99549
    Minimum value of k                                0.98219
    Variance of fit, s(k,X)^2                        4.5805E-05
    Within variance, s(w)^2                          1.4478E-05
    Pooled variance, s(p)^2                          6.0282E-05
    Pooled std. deviation, s(p)                      7.7642E-03
    C(alpha,rho)*s(p)                                2.9128E-02
    student-t @ (n-2,1-gamma)                        1.66493E+00
    Confidence band width, W                         1.3775E-02
    Minimum margin of subcriticality, C*s(p)-W       1.5353E-02

    Upper subcritical limits: (  3.3700     <= X <=   22.850    )
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)           USL1 = 0.9281 + ( 4.8379E-04)*X (X <    16.844   )
                                          = 0.9362                    (X >=   16.844   )

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)   USL2 = 0.9627 + ( 4.8379E-04)*X (X <  1.68439E+01)
                                          = 0.9709                    (X >= 1.68439E+01)


    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  3.37E+0  6.15E+0  8.94E+0  1.17E+1  1.45E+1  1.73E+1  2.01E+1  2.29E+1
       ---------------------------------------------------------------------
USL-1:    0.9297   0.9311   0.9324   0.9337   0.9351   0.9362   0.9362   0.9362
USL-2:    0.9644   0.9657   0.9670   0.9684   0.9697   0.9709   0.9709   0.9709
       ---------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
      Plot file written to: Het-hu.plt
```

USLSTATS output, Heterogeneous systems Set 2, k_{eff} vs. H/U

```
              uslstats: a utility to calculate upper subcritical
                        limits for criticality safety applications

  ****************************************************************************
                          Version 1.4, April 23, 2003
                         Oak Ridge National Laboratory
  ****************************************************************************


   Input to statistical treatment from file:Het-hu.in

Title: Heterogeneous Experiments x=H/U

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   90
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
5.40000E+00	9.93480E-01	1.81000E-03	4.60000E+00	9.99920E-01	4.02000E-03
5.40000E+00	9.95610E-01	3.20000E-03	4.60000E+00	9.94210E-01	1.84000E-03
6.32000E+00	9.95150E-01	3.20000E-03	4.60000E+00	1.00009E+00	4.02000E-03
6.16000E+00	9.91930E-01	1.70000E-03	5.21000E+00	9.85380E-01	3.42000E-03
7.16000E+00	9.91240E-01	1.70000E-03	4.60000E+00	9.98130E-01	4.03000E-03
7.16000E+00	9.91480E-01	1.70000E-03	5.21000E+00	9.94280E-01	1.64000E-03
8.22000E+00	9.90590E-01	1.70000E-03	4.60000E+00	9.96740E-01	1.85000E-03
8.22000E+00	9.90490E-01	1.70000E-03	5.21000E+00	9.85160E-01	3.42000E-03
9.33000E+00	9.90090E-01	1.80000E-03	4.60000E+00	1.00024E+00	4.02000E-03
8.46000E+00	9.91740E-01	2.00000E-03	9.51000E+00	9.94530E-01	3.12000E-03
6.18000E+00	9.97200E-01	2.40000E-03	1.11900E+01	9.95790E-01	2.13000E-03
7.15000E+00	9.94550E-01	1.90000E-03	4.60000E+00	1.00059E+00	4.02000E-03
6.08000E+00	9.96440E-01	1.90000E-03	4.60000E+00	1.00098E+00	4.02000E-03
6.08000E+00	9.92860E-01	1.90000E-03	4.60000E+00	9.97640E-01	4.02000E-03
6.08000E+00	9.88100E-01	2.20000E-03	4.60000E+00	9.99580E-01	4.02000E-03
7.04000E+00	9.87540E-01	1.90000E-03	5.21000E+00	9.84510E-01	3.42000E-03
6.08000E+00	9.91580E-01	2.40000E-03	4.60000E+00	9.99550E-01	4.02000E-03
7.04000E+00	9.90290E-01	2.00000E-03	5.21000E+00	9.93740E-01	1.64000E-03
6.08000E+00	9.92740E-01	2.70000E-03	4.60000E+00	9.96410E-01	1.85000E-03
7.04000E+00	9.91410E-01	2.10000E-03	3.40000E+00	9.96360E-01	5.01000E-03
8.05000E+00	9.89880E-01	1.90000E-03	3.37000E+00	9.96020E-01	5.01000E-03
2.28500E+01	9.97920E-01	6.11000E-03	3.54000E+00	9.98540E-01	5.01000E-03
2.28500E+01	1.00156E+00	6.11000E-03	3.54000E+00	9.95560E-01	5.01000E-03
2.28500E+01	1.00298E+00	6.11000E-03	3.47000E+00	9.94670E-01	5.01000E-03
2.28500E+01	1.00241E+00	6.11000E-03	3.66000E+00	9.97030E-01	5.01000E-03
2.28500E+01	1.00161E+00	6.11000E-03	3.48000E+00	9.96740E-01	5.01000E-03
2.28500E+01	1.00267E+00	6.11000E-03	6.70000E+00	9.96620E-01	5.01000E-03
5.26000E+00	9.91660E-01	1.66000E-03	6.70000E+00	9.93710E-01	5.01000E-03
5.26000E+00	9.91740E-01	1.46000E-03	6.72000E+00	9.91450E-01	5.01000E-03
5.26000E+00	9.90930E-01	1.47000E-03	7.07000E+00	1.00737E+00	5.01000E-03
5.21000E+00	9.93360E-01	2.82000E-03	7.41000E+00	1.00312E+00	5.01000E-03
5.21000E+00	9.94640E-01	2.82000E-03	7.70000E+00	1.01807E+00	5.01000E-03
5.21000E+00	9.94770E-01	2.82000E-03	7.84000E+00	9.95350E-01	5.01000E-03
5.21000E+00	9.94310E-01	2.83000E-03	6.70000E+00	9.93940E-01	5.01000E-03
9.51000E+00	9.95280E-01	3.12000E-03	6.74000E+00	9.95090E-01	5.01000E-03
9.51000E+00	9.92300E-01	3.12000E-03	7.64000E+00	1.02303E+00	5.01000E-03
4.60000E+00	9.98040E-01	2.83000E-03	8.33000E+00	1.01326E+00	5.01000E-03
4.60000E+00	9.97690E-01	2.83000E-03	8.88000E+00	1.02224E+00	5.01000E-03
4.60000E+00	9.98720E-01	2.83000E-03	9.04000E+00	9.95350E-01	5.01000E-03
4.60000E+00	9.98050E-01	2.83000E-03	6.70000E+00	9.96480E-01	5.01000E-03
4.60000E+00	9.97480E-01	2.83000E-03	6.79000E+00	9.98710E-01	5.01000E-03
4.60000E+00	9.95900E-01	2.83000E-03	8.66000E+00	1.00620E+00	5.01000E-03
1.11900E+01	9.97030E-01	2.14000E-03	1.01500E+01	1.01893E+00	5.01000E-03
1.11900E+01	9.97140E-01	2.13000E-03	1.14300E+01	1.00144E+00	5.01000E-03
1.11900E+01	9.95040E-01	2.13000E-03	1.16100E+01	9.95960E-01	5.01000E-03

```
   chi = 19.3333 (upper bound = 9.49). The data tests NOT normal
```

```
      Output from statistical treatment

   Heterogeneous Experiments x=H/U

    Number of data points (n)                         90
    Linear regression, k(X)                          0.9942 + ( 3.5507E-04)*X
    Confidence on fit (1-gamma) [input]               95.0%
    Confidence on proportion (alpha) [input]          95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]            99.5%
    Minimum value of X                                3.3700E+00
    Maximum value of X                                2.2850E+01
    Average value of X                                7.5318E+00
    Average value of k                                 0.99685
    Minimum value of k                                 0.98451
    Variance of fit, s(k,X)^2                         4.6125E-05
    Within variance, s(w)^2                           1.4475E-05
    Pooled variance, s(p)^2                           6.0600E-05
    Pooled std. deviation, s(p)                       7.7846E-03
    C(alpha,rho)*s(p)                                 2.9204E-02
    student-t @ (n-2,1-gamma)                         1.66493E+00
    Confidence band width, W                          1.3811E-02
    Minimum margin of subcriticality, C*s(p)-W        1.5393E-02

    Upper subcritical limits: (  3.3700     <= X <=   22.850    )
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)           USL1 = 0.9304 + ( 3.5507E-04)*X (X <    16.409   )
                                          = 0.9362                   (X >=   16.409   )

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9650 + ( 3.5507E-04)*X (X <  1.64086E+01)
                                          = 0.9708                   (X >= 1.64086E+01)


    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  3.37E+0  6.15E+0  8.94E+0  1.17E+1  1.45E+1  1.73E+1  2.01E+1  2.29E+1
       ------------------------------------------------------------------------
USL-1:    0.9316   0.9325   0.9335   0.9345   0.9355   0.9362   0.9362   0.9362
USL-2:    0.9662   0.9672   0.9681   0.9691   0.9701   0.9708   0.9708   0.9708
       ------------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
      Plot file written to: Het-hu.plt
```

USLSTATS output, Heterogeneous systems Set 1, k_{eff} vs. H/X

```
              uslstats: a utility to calculate upper subcritical
                        limits for criticality safety applications

 ****************************************************************************
                          Version 1.4, April 23, 2003
                         Oak Ridge National Laboratory
 ****************************************************************************


  Input to statistical treatment from file:Het-hx.in

Title: Heterogeneous Experiments x=H/X

  Proportion of the population = .995
  Confidence of fit            = .950
  Confidence on proportion     = .950
  Number of observations       =   90
  Minimum value of closed band =    0.00
  Maximum value of closed band =    0.00
  Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
2.16800E+02	9.91650E-01	1.82000E-03	1.05510E+02	9.97570E-01	4.02000E-03
2.16800E+02	9.94210E-01	3.20000E-03	1.05490E+02	9.92220E-01	1.85000E-03
2.53680E+02	9.93760E-01	3.20000E-03	1.05380E+02	9.98620E-01	4.02000E-03
2.47130E+02	9.90590E-01	1.70000E-03	2.19080E+02	9.83540E-01	3.42000E-03
2.87280E+02	9.89880E-01	1.70000E-03	1.05380E+02	9.95820E-01	4.02000E-03
2.87280E+02	9.90150E-01	1.70000E-03	2.19080E+02	9.93250E-01	1.64000E-03
3.29930E+02	9.88890E-01	1.70000E-03	1.05490E+02	9.94170E-01	1.84000E-03
3.29930E+02	9.89070E-01	1.70000E-03	2.19080E+02	9.83950E-01	3.42000E-03
3.74600E+02	9.88430E-01	1.80000E-03	1.05510E+02	9.97910E-01	4.02000E-03
3.39510E+02	9.90130E-01	2.00000E-03	3.99640E+02	9.94090E-01	3.12000E-03
2.48150E+02	9.95740E-01	2.40000E-03	2.56440E+02	9.95410E-01	2.14000E-03
2.87060E+02	9.93390E-01	1.90000E-03	1.05380E+02	9.97730E-01	4.02000E-03
2.43910E+02	9.95220E-01	1.90000E-03	1.05380E+02	9.97930E-01	4.02000E-03
2.43910E+02	9.91380E-01	1.90000E-03	1.05380E+02	9.96790E-01	4.02000E-03
2.43910E+02	9.86820E-01	2.20000E-03	1.05380E+02	9.98280E-01	4.02000E-03
2.82620E+02	9.86060E-01	1.90000E-03	2.19080E+02	9.82190E-01	3.42000E-03
2.43910E+02	9.90200E-01	2.40000E-03	1.05380E+02	9.97050E-01	4.02000E-03
2.82620E+02	9.89020E-01	2.00000E-03	2.19080E+02	9.92550E-01	1.64000E-03
2.43910E+02	9.91400E-01	2.70000E-03	1.05490E+02	9.94250E-01	1.85000E-03
2.82620E+02	9.89760E-01	2.10000E-03	1.17660E+02	9.95210E-01	5.01000E-03
3.23080E+02	9.88420E-01	1.90000E-03	1.16650E+02	9.94510E-01	5.01000E-03
4.46200E+02	9.98060E-01	6.11000E-03	1.22990E+02	9.97160E-01	5.01000E-03
4.46200E+02	1.00224E+00	6.11000E-03	1.23390E+02	9.95070E-01	5.01000E-03
4.46200E+02	1.00364E+00	6.11000E-03	1.21070E+02	9.94450E-01	5.01000E-03
4.46200E+02	1.00317E+00	6.11000E-03	1.27670E+02	9.95250E-01	5.01000E-03
4.46200E+02	1.00296E+00	6.11000E-03	1.21590E+02	9.95610E-01	5.01000E-03
4.46200E+02	1.00272E+00	6.12000E-03	1.39630E+02	9.96020E-01	5.01000E-03
1.09700E+02	9.91250E-01	1.66000E-03	1.39630E+02	9.92230E-01	5.01000E-03
1.09700E+02	9.89050E-01	1.47000E-03	1.40100E+02	9.89760E-01	5.01000E-03
1.09700E+02	9.89020E-01	1.46000E-03	1.47310E+02	1.00629E+00	5.01000E-03
2.19080E+02	9.92390E-01	2.83000E-03	1.54470E+02	1.00140E+00	5.01000E-03
2.19080E+02	9.93000E-01	2.82000E-03	1.60390E+02	1.01663E+00	5.01000E-03
2.19080E+02	9.93970E-01	2.83000E-03	1.63430E+02	9.93690E-01	5.01000E-03
2.19080E+02	9.91860E-01	2.82000E-03	1.39630E+02	9.91770E-01	5.01000E-03
3.99640E+02	9.94290E-01	3.12000E-03	1.40450E+02	9.93910E-01	5.01000E-03
3.99640E+02	9.90650E-01	3.12000E-03	1.59130E+02	1.02156E+00	5.01000E-03
1.05490E+02	9.94830E-01	2.83000E-03	1.73480E+02	1.01211E+00	5.01000E-03
1.05490E+02	9.96120E-01	2.83000E-03	1.85100E+02	1.02123E+00	5.01000E-03
1.05490E+02	9.96250E-01	2.83000E-03	1.88420E+02	9.93690E-01	5.01000E-03
1.05490E+02	9.95940E-01	2.83000E-03	1.39630E+02	9.94560E-01	5.01000E-03
1.05490E+02	9.95820E-01	2.83000E-03	1.41570E+02	9.96840E-01	5.01000E-03
1.05490E+02	9.94280E-01	2.83000E-03	1.80460E+02	1.00438E+00	5.01000E-03
2.56440E+02	9.96770E-01	2.14000E-03	2.11420E+02	1.01752E+00	5.01000E-03
2.56440E+02	9.97110E-01	2.14000E-03	2.38080E+02	9.99820E-01	5.01000E-03
2.56440E+02	9.95950E-01	2.14000E-03	2.41980E+02	9.95040E-01	5.01000E-03

chi = 24.1111 (upper bound = 9.49). The data tests NOT normal

```
      Output from statistical treatment

   Heterogeneous Experiments x=H/X

    Number of data points (n)                         90
    Linear regression, k(X)                          0.9963 + (-3.6655E-06)*X
    Confidence on fit (1-gamma) [input]               95.0%
    Confidence on proportion (alpha) [input]          95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]            99.5%
    Minimum value of X                                1.0538E+02
    Maximum value of X                                4.4620E+02
    Average value of X                                2.1050E+02
    Average value of k                                 0.99549
    Minimum value of k                                 0.98219
    Variance of fit, s(k,X)^2                         5.0683E-05
    Within variance, s(w)^2                           1.4478E-05
    Pooled variance, s(p)^2                           6.5161E-05
    Pooled std. deviation, s(p)                       8.0722E-03
    C(alpha,rho)*s(p)                                 2.8694E-02
    student-t @ (n-2,1-gamma)                         1.66493E+00
    Confidence band width, W                          1.3913E-02
    Minimum margin of subcriticality, C*s(p)-W        1.4780E-02

    Upper subcritical limits: (  105.38     <= X <=   446.20    )
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)            USL1 = 0.9324 + (-3.6655E-06)*X

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9676 + (-3.6655E-06)*X



    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  1.05E+2  1.54E+2  2.03E+2  2.51E+2  3.00E+2  3.49E+2  3.98E+2  4.46E+2
       ------------------------------------------------------------------------
USL-1:    0.9320   0.9318   0.9316   0.9314   0.9313   0.9311   0.9309   0.9307
USL-2:    0.9672   0.9670   0.9668   0.9667   0.9665   0.9663   0.9661   0.9659
       ------------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
      Plot file written to: Het-hx.plt
```

USLSTATS output, Heterogeneous systems Set 2, k_{eff} vs. H/X

```
             uslstats: a utility to calculate upper subcritical
                       limits for criticality safety applications

 ***************************************************************************
                         Version 1.4, April 23, 2003
                        Oak Ridge National Laboratory
 ***************************************************************************


  Input to statistical treatment from file:Het-hx.in

Title: Heterogeneous Experiments x=H/X

  Proportion of the population = .995
  Confidence of fit            = .950
  Confidence on proportion     = .950
  Number of observations       =   90
  Minimum value of closed band =    0.00
  Maximum value of closed band =    0.00
  Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
2.16800E+02	9.93480E-01	1.81000E-03	1.05510E+02	9.99920E-01	4.02000E-03
2.16800E+02	9.95610E-01	3.20000E-03	1.05490E+02	9.94210E-01	1.84000E-03
2.53680E+02	9.95150E-01	3.20000E-03	1.05380E+02	1.00009E+00	4.02000E-03
2.47130E+02	9.91930E-01	1.70000E-03	2.19080E+02	9.85380E-01	3.42000E-03
2.87280E+02	9.91240E-01	1.70000E-03	1.05380E+02	9.98130E-01	4.03000E-03
2.87280E+02	9.91480E-01	1.70000E-03	2.19080E+02	9.94280E-01	1.64000E-03
3.29930E+02	9.90590E-01	1.70000E-03	1.05490E+02	9.96740E-01	1.85000E-03
3.29930E+02	9.90490E-01	1.70000E-03	2.19080E+02	9.85160E-01	3.42000E-03
3.74600E+02	9.90090E-01	1.80000E-03	1.05510E+02	1.00024E+00	4.02000E-03
3.39510E+02	9.91740E-01	2.00000E-03	3.99640E+02	9.94530E-01	3.12000E-03
2.48150E+02	9.97200E-01	2.40000E-03	2.56440E+02	9.95790E-01	2.13000E-03
2.87060E+02	9.94550E-01	1.90000E-03	1.05380E+02	1.00059E+00	4.02000E-03
2.43910E+02	9.96440E-01	1.90000E-03	1.05380E+02	1.00098E+00	4.02000E-03
2.43910E+02	9.92860E-01	1.90000E-03	1.05380E+02	9.97640E-01	4.02000E-03
2.43910E+02	9.88100E-01	2.20000E-03	1.05380E+02	9.99580E-01	4.02000E-03
2.82620E+02	9.87540E-01	1.90000E-03	2.19080E+02	9.84510E-01	3.42000E-03
2.43910E+02	9.91580E-01	2.40000E-03	1.05380E+02	9.99550E-01	4.02000E-03
2.82620E+02	9.90290E-01	2.00000E-03	2.19080E+02	9.93740E-01	1.64000E-03
2.43910E+02	9.92740E-01	2.70000E-03	1.05490E+02	9.96410E-01	1.85000E-03
2.82620E+02	9.91410E-01	2.10000E-03	1.17660E+02	9.96360E-01	5.01000E-03
3.23080E+02	9.89880E-01	1.90000E-03	1.16650E+02	9.96020E-01	5.01000E-03
4.46200E+02	9.97920E-01	6.11000E-03	1.22990E+02	9.98540E-01	5.01000E-03
4.46200E+02	1.00156E+00	6.11000E-03	1.23390E+02	9.95560E-01	5.01000E-03
4.46200E+02	1.00298E+00	6.11000E-03	1.21070E+02	9.94670E-01	5.01000E-03
4.46200E+02	1.00241E+00	6.11000E-03	1.27670E+02	9.97030E-01	5.01000E-03
4.46200E+02	1.00161E+00	6.11000E-03	1.21590E+02	9.96740E-01	5.01000E-03
4.46200E+02	1.00267E+00	6.11000E-03	1.39630E+02	9.96620E-01	5.01000E-03
1.09700E+02	9.91660E-01	1.66000E-03	1.39630E+02	9.93710E-01	5.01000E-03
1.09700E+02	9.91740E-01	1.46000E-03	1.40100E+02	9.91450E-01	5.01000E-03
1.09700E+02	9.90930E-01	1.47000E-03	1.47310E+02	1.00737E+00	5.01000E-03
2.19080E+02	9.93360E-01	2.82000E-03	1.54470E+02	1.00312E+00	5.01000E-03
2.19080E+02	9.94640E-01	2.82000E-03	1.60390E+02	1.01807E+00	5.01000E-03
2.19080E+02	9.94770E-01	2.82000E-03	1.63430E+02	9.95350E-01	5.01000E-03
2.19080E+02	9.94310E-01	2.83000E-03	1.39630E+02	9.93940E-01	5.01000E-03
3.99640E+02	9.95280E-01	3.12000E-03	1.40450E+02	9.95090E-01	5.01000E-03
3.99640E+02	9.92300E-01	3.12000E-03	1.59130E+02	1.02303E+00	5.01000E-03
1.05490E+02	9.98040E-01	2.83000E-03	1.73480E+02	1.01326E+00	5.01000E-03
1.05490E+02	9.97690E-01	2.83000E-03	1.85100E+02	1.02224E+00	5.01000E-03
1.05490E+02	9.98720E-01	2.83000E-03	1.88420E+02	9.95350E-01	5.01000E-03
1.05490E+02	9.98050E-01	2.83000E-03	1.39630E+02	9.96480E-01	5.01000E-03
1.05490E+02	9.97480E-01	2.83000E-03	1.41570E+02	9.98710E-01	5.01000E-03
1.05490E+02	9.95900E-01	2.83000E-03	1.80460E+02	1.00620E+00	5.01000E-03
2.56440E+02	9.97030E-01	2.14000E-03	2.11420E+02	1.01893E+00	5.01000E-03
2.56440E+02	9.97140E-01	2.13000E-03	2.38080E+02	1.00144E+00	5.01000E-03
2.56440E+02	9.95040E-01	2.13000E-03	2.41980E+02	9.95960E-01	5.01000E-03

chi = 19.3333 (upper bound = 9.49). The data tests NOT normal

```
        Output from statistical treatment

   Heterogeneous Experiments x=H/X

    Number of data points (n)                      90
    Linear regression, k(X)                       0.9987 + (-8.6729E-06)*X
    Confidence on fit (1-gamma) [input]            95.0%
    Confidence on proportion (alpha) [input]       95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]         99.5%
    Minimum value of X                             1.0538E+02
    Maximum value of X                             4.4620E+02
    Average value of X                             2.1050E+02
    Average value of k                              0.99685
    Minimum value of k                              0.98451
    Variance of fit, s(k,X)^2                      4.8044E-05
    Within variance, s(w)^2                        1.4475E-05
    Pooled variance, s(p)^2                        6.2519E-05
    Pooled std. deviation, s(p)                    7.9069E-03
    C(alpha,rho)*s(p)                              2.8106E-02
    student-t @ (n-2,1-gamma)                      1.66493E+00
    Confidence band width, W                       1.3628E-02
    Minimum margin of subcriticality, C*s(p)-W     1.4478E-02

    Upper subcritical limits: (  105.38     <= X <=   446.20    )
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)            USL1 = 0.9350 + (-8.6729E-06)*X

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9706 + (-8.6729E-06)*X



    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  1.05E+2  1.54E+2  2.03E+2  2.51E+2  3.00E+2  3.49E+2  3.98E+2  4.46E+2
       ------------------------------------------------------------------------
USL-1:    0.9341   0.9337   0.9333   0.9329   0.9324   0.9320   0.9316   0.9312
USL-2:    0.9697   0.9692   0.9688   0.9684   0.9680   0.9675   0.9671   0.9667
       ------------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
        Plot file written to: Het-hx.plt
```

USLSTATS output, Heterogeneous systems Set 1, k_{eff} vs. Enrichment

```
              uslstats: a utility to calculate upper subcritical
                        limits for criticality safety applications

 ***************************************************************************
                          Version 1.4, April 23, 2003
                         Oak Ridge National Laboratory
 ***************************************************************************


   Input to statistical treatment from file:Het-enr.in

Title: Heterogeneous Experiments x=Enrichment

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   90
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
2.45900E-02	9.91650E-01	1.82000E-03	4.31000E-02	9.97570E-01	4.02000E-03
2.45900E-02	9.94210E-01	3.20000E-03	4.31000E-02	9.92220E-01	1.85000E-03
2.45900E-02	9.93760E-01	3.20000E-03	4.31000E-02	9.98620E-01	4.02000E-03
2.45900E-02	9.90590E-01	1.70000E-03	2.35000E-02	9.83540E-01	3.42000E-03
2.45900E-02	9.89880E-01	1.70000E-03	4.31000E-02	9.95820E-01	4.02000E-03
2.45900E-02	9.90150E-01	1.70000E-03	2.35000E-02	9.93250E-01	1.64000E-03
2.45900E-02	9.88890E-01	1.70000E-03	4.31000E-02	9.94170E-01	1.84000E-03
2.45900E-02	9.89070E-01	1.70000E-03	2.35000E-02	9.83950E-01	3.42000E-03
2.45900E-02	9.88430E-01	1.80000E-03	4.31000E-02	9.97910E-01	4.02000E-03
2.45900E-02	9.90130E-01	2.00000E-03	2.35000E-02	9.94090E-01	3.12000E-03
2.45900E-02	9.95740E-01	2.40000E-03	4.31000E-02	9.95410E-01	2.14000E-03
2.45900E-02	9.93390E-01	1.90000E-03	4.31000E-02	9.97730E-01	4.02000E-03
2.45900E-02	9.95220E-01	1.90000E-03	4.31000E-02	9.97930E-01	4.02000E-03
2.45900E-02	9.91380E-01	1.90000E-03	4.31000E-02	9.96790E-01	4.02000E-03
2.45900E-02	9.86820E-01	2.20000E-03	4.31000E-02	9.98280E-01	4.02000E-03
2.45900E-02	9.86060E-01	1.90000E-03	2.35000E-02	9.82190E-01	3.42000E-03
2.45900E-02	9.90200E-01	2.40000E-03	4.31000E-02	9.97050E-01	4.02000E-03
2.45900E-02	9.89020E-01	2.00000E-03	2.35000E-02	9.92550E-01	1.64000E-03
2.45900E-02	9.91400E-01	2.70000E-03	4.31000E-02	9.94250E-01	1.85000E-03
2.45900E-02	9.89760E-01	2.10000E-03	2.64000E-02	9.95210E-01	5.01000E-03
2.45900E-02	9.88420E-01	1.90000E-03	2.64400E-02	9.94510E-01	5.01000E-03
5.06000E-02	9.98060E-01	6.11000E-03	2.62200E-02	9.97160E-01	5.01000E-03
5.06000E-02	1.00224E+00	6.11000E-03	2.60500E-02	9.95070E-01	5.01000E-03
5.06000E-02	1.00364E+00	6.11000E-03	2.60500E-02	9.94450E-01	5.01000E-03
5.06000E-02	1.00317E+00	6.11000E-03	2.60400E-02	9.95250E-01	5.01000E-03
5.06000E-02	1.00296E+00	6.11000E-03	2.60400E-02	9.95610E-01	5.01000E-03
5.06000E-02	1.00272E+00	6.12000E-03	4.74200E-02	9.96020E-01	5.01000E-03
4.74000E-02	9.91250E-01	1.66000E-03	4.74200E-02	9.92230E-01	5.01000E-03
4.74000E-02	9.89050E-01	1.47000E-03	4.74200E-02	9.89760E-01	5.01000E-03
4.74000E-02	9.89020E-01	1.46000E-03	4.74200E-02	1.00629E+00	5.01000E-03
2.35000E-02	9.92390E-01	2.83000E-03	4.74200E-02	1.00140E+00	5.01000E-03
2.35000E-02	9.93000E-01	2.82000E-03	4.74200E-02	1.01663E+00	5.01000E-03
2.35000E-02	9.93970E-01	2.83000E-03	4.74200E-02	9.93690E-01	5.01000E-03
2.35000E-02	9.91860E-01	2.82000E-03	4.74200E-02	9.91770E-01	5.01000E-03
2.35000E-02	9.94290E-01	3.12000E-03	4.74200E-02	9.93910E-01	5.01000E-03
2.35000E-02	9.90650E-01	3.12000E-03	4.74200E-02	1.02156E+00	5.01000E-03
4.31000E-02	9.94830E-01	2.83000E-03	4.74200E-02	1.01211E+00	5.01000E-03
4.31000E-02	9.96120E-01	2.83000E-03	4.74200E-02	1.02123E+00	5.01000E-03
4.31000E-02	9.96250E-01	2.83000E-03	4.74200E-02	9.93690E-01	5.01000E-03
4.31000E-02	9.95940E-01	2.83000E-03	4.74200E-02	9.94560E-01	5.01000E-03
4.31000E-02	9.95820E-01	2.83000E-03	4.74200E-02	9.96840E-01	5.01000E-03
4.31000E-02	9.94280E-01	2.83000E-03	4.74200E-02	1.00438E+00	5.01000E-03
4.31000E-02	9.96770E-01	2.14000E-03	4.74200E-02	1.01752E+00	5.01000E-03
4.31000E-02	9.97110E-01	2.14000E-03	4.74200E-02	9.99820E-01	5.01000E-03
4.31000E-02	9.95950E-01	2.14000E-03	4.74200E-02	9.95040E-01	5.01000E-03

chi = 24.1111 (upper bound = 9.49). The data tests NOT normal

```
      Output from statistical treatment

   Heterogeneous Experiments x=Enrichment

    Number of data points (n)                         90
    Linear regression, k(X)                          0.9822 + ( 3.6433E-01)*X
    Confidence on fit (1-gamma) [input]               95.0%
    Confidence on proportion (alpha) [input]          95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]            99.5%
    Minimum value of X                                2.3500E-02
    Maximum value of X                                5.0600E-02
    Average value of X                                3.6407E-02
    Average value of k                                 0.99549
    Minimum value of k                                 0.98219
    Variance of fit, s(k,X)^2                         3.4947E-05
    Within variance, s(w)^2                           1.4478E-05
    Pooled variance, s(p)^2                           4.9424E-05
    Pooled std. deviation, s(p)                       7.0302E-03
    C(alpha,rho)*s(p)                                 2.3940E-02
    student-t @ (n-2,1-gamma)                         1.66493E+00
    Confidence band width, W                          1.1881E-02
    Minimum margin of subcriticality, C*s(p)-W        1.2060E-02

    Upper subcritical limits: ( 2.35000E-02 <= X <=  5.06000E-02)
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)           USL1 = 0.9204 + ( 3.6433E-01)*X (X <   4.87721E-2)
                                          = 0.9381                   (X >=  4.87721E-2)

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)   USL2 = 0.9583 + ( 3.6433E-01)*X (X <  4.87721E-02)
                                          = 0.9761                   (X >= 4.87721E-02)


    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  2.35E-2  2.74E-2  3.12E-2  3.51E-2  3.90E-2  4.29E-2  4.67E-2  5.06E-2
       ------------------------------------------------------------------------
USL-1:    0.9289   0.9303   0.9317   0.9331   0.9346   0.9360   0.9374   0.9381
USL-2:    0.9669   0.9683   0.9697   0.9711   0.9725   0.9739   0.9753   0.9761
       ------------------------------------------------------------------------

  ***************************************************************************
                          Thus spake USLSTATS
                                 Finis.
      Plot file written to: Het-enr.plt
```

USLSTATS output, Heterogeneous systems Set 2, k_{eff} vs. Enrichment

```
             uslstats: a utility to calculate upper subcritical
                       limits for criticality safety applications

  ***************************************************************************
                         Version 1.4, April 23, 2003
                        Oak Ridge National Laboratory
  ***************************************************************************


   Input to statistical treatment from file:Het-enr.in

Title: Heterogeneous Experiments x=Enrichment

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   90
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
2.45900E-02	9.93480E-01	1.81000E-03	4.31000E-02	9.99920E-01	4.02000E-03
2.45900E-02	9.95610E-01	3.20000E-03	4.31000E-02	9.94210E-01	1.84000E-03
2.45900E-02	9.95150E-01	3.20000E-03	4.31000E-02	1.00009E+00	4.02000E-03
2.45900E-02	9.91930E-01	1.70000E-03	2.35000E-02	9.85380E-01	3.42000E-03
2.45900E-02	9.91240E-01	1.70000E-03	4.31000E-02	9.98130E-01	4.03000E-03
2.45900E-02	9.91480E-01	1.70000E-03	2.35000E-02	9.94280E-01	1.64000E-03
2.45900E-02	9.90590E-01	1.70000E-03	4.31000E-02	9.96740E-01	1.85000E-03
2.45900E-02	9.90490E-01	1.70000E-03	2.35000E-02	9.85160E-01	3.42000E-03
2.45900E-02	9.90090E-01	1.80000E-03	4.31000E-02	1.00024E+00	4.02000E-03
2.45900E-02	9.91740E-01	2.00000E-03	2.35000E-02	9.94530E-01	3.12000E-03
2.45900E-02	9.97200E-01	2.40000E-03	4.31000E-02	9.95790E-01	2.13000E-03
2.45900E-02	9.94550E-01	1.90000E-03	4.31000E-02	1.00059E+00	4.02000E-03
2.45900E-02	9.96440E-01	1.90000E-03	4.31000E-02	1.00098E+00	4.02000E-03
2.45900E-02	9.92860E-01	1.90000E-03	4.31000E-02	9.97640E-01	4.02000E-03
2.45900E-02	9.88100E-01	2.20000E-03	4.31000E-02	9.99580E-01	4.02000E-03
2.45900E-02	9.87540E-01	1.90000E-03	2.35000E-02	9.84510E-01	3.42000E-03
2.45900E-02	9.91580E-01	2.40000E-03	4.31000E-02	9.99550E-01	4.02000E-03
2.45900E-02	9.90290E-01	2.00000E-03	2.35000E-02	9.93740E-01	1.64000E-03
2.45900E-02	9.92740E-01	2.70000E-03	4.31000E-02	9.96410E-01	1.85000E-03
2.45900E-02	9.91410E-01	2.10000E-03	2.64000E-02	9.96360E-01	5.01000E-03
2.45900E-02	9.89880E-01	1.90000E-03	2.64400E-02	9.96020E-01	5.01000E-03
5.06000E-02	9.97920E-01	6.11000E-03	2.62200E-02	9.98540E-01	5.01000E-03
5.06000E-02	1.00156E+00	6.11000E-03	2.60500E-02	9.95560E-01	5.01000E-03
5.06000E-02	1.00298E+00	6.11000E-03	2.60500E-02	9.94670E-01	5.01000E-03
5.06000E-02	1.00241E+00	6.11000E-03	2.60400E-02	9.97030E-01	5.01000E-03
5.06000E-02	1.00161E+00	6.11000E-03	2.60400E-02	9.96740E-01	5.01000E-03
5.06000E-02	1.00267E+00	6.11000E-03	4.74200E-02	9.96620E-01	5.01000E-03
4.74000E-02	9.91660E-01	1.66000E-03	4.74200E-02	9.93710E-01	5.01000E-03
4.74000E-02	9.91740E-01	1.46000E-03	4.74200E-02	9.91450E-01	5.01000E-03
4.74000E-02	9.90930E-01	1.47000E-03	4.74200E-02	1.00737E+00	5.01000E-03
2.35000E-02	9.93360E-01	2.82000E-03	4.74200E-02	1.00312E+00	5.01000E-03
2.35000E-02	9.94640E-01	2.82000E-03	4.74200E-02	1.01807E+00	5.01000E-03
2.35000E-02	9.94770E-01	2.82000E-03	4.74200E-02	9.95350E-01	5.01000E-03
2.35000E-02	9.94310E-01	2.83000E-03	4.74200E-02	9.93940E-01	5.01000E-03
2.35000E-02	9.95280E-01	3.12000E-03	4.74200E-02	9.95090E-01	5.01000E-03
2.35000E-02	9.92300E-01	3.12000E-03	4.74200E-02	1.02303E+00	5.01000E-03
4.31000E-02	9.98040E-01	2.83000E-03	4.74200E-02	1.01326E+00	5.01000E-03
4.31000E-02	9.97690E-01	2.83000E-03	4.74200E-02	1.02224E+00	5.01000E-03
4.31000E-02	9.98720E-01	2.83000E-03	4.74200E-02	9.95350E-01	5.01000E-03
4.31000E-02	9.98050E-01	2.83000E-03	4.74200E-02	9.96480E-01	5.01000E-03
4.31000E-02	9.97480E-01	2.83000E-03	4.74200E-02	9.98710E-01	5.01000E-03
4.31000E-02	9.95900E-01	2.83000E-03	4.74200E-02	1.00620E+00	5.01000E-03
4.31000E-02	9.97030E-01	2.14000E-03	4.74200E-02	1.01893E+00	5.01000E-03
4.31000E-02	9.97140E-01	2.13000E-03	4.74200E-02	1.00144E+00	5.01000E-03
4.31000E-02	9.95040E-01	2.13000E-03	4.74200E-02	9.95960E-01	5.01000E-03

```
   chi = 19.3333 (upper bound = 9.49). The data tests NOT normal
```

```
      Output from statistical treatment

   Heterogeneous Experiments x=Enrichment

    Number of data points (n)                        90
    Linear regression, k(X)                         0.9839 + ( 3.5645E-01)*X
    Confidence on fit (1-gamma) [input]              95.0%
    Confidence on proportion (alpha) [input]         95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]           99.5%
    Minimum value of X                               2.3500E-02
    Maximum value of X                               5.0600E-02
    Average value of X                               3.6407E-02
    Average value of k                                0.99685
    Minimum value of k                                0.98451
    Variance of fit, s(k,X)^2                        3.3631E-05
    Within variance, s(w)^2                          1.4475E-05
    Pooled variance, s(p)^2                          4.8106E-05
    Pooled std. deviation, s(p)                      6.9358E-03
    C(alpha,rho)*s(p)                                2.3619E-02
    student-t @ (n-2,1-gamma)                        1.66493E+00
    Confidence band width, W                         1.1721E-02
    Minimum margin of subcriticality, C*s(p)-W       1.1898E-02

    Upper subcritical limits: ( 2.35000E-02 <= X <=  5.06000E-02)
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)            USL1 = 0.9221 + ( 3.5645E-01)*X (X <   4.52491E-2)
                                           = 0.9383                   (X >=  4.52491E-2)

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)   USL2 = 0.9603 + ( 3.5645E-01)*X (X <  4.52491E-02)
                                           = 0.9764                   (X >= 4.52491E-02)


    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  2.35E-2  2.74E-2  3.12E-2  3.51E-2  3.90E-2  4.29E-2  4.67E-2  5.06E-2
       -----------------------------------------------------------------------
USL-1:    0.9305   0.9319   0.9333   0.9347   0.9361   0.9374   0.9383   0.9383
USL-2:    0.9686   0.9700   0.9714   0.9728   0.9742   0.9755   0.9764   0.9764
       -----------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
      Plot file written to: Het-enr.plt
```

USLSTATS output, Heterogeneous systems Set 1, k_{eff} vs. EALF (Lethargy)

```
              uslstats: a utility to calculate upper subcritical
                        limits for criticality safety applications

  ****************************************************************************
                          Version 1.4, April 23, 2003
                         Oak Ridge National Laboratory
  ****************************************************************************


   Input to statistical treatment from file:Het-leth.in

Title: Heterogeneous Experiments x=EALF MeV

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   90
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
1.74900E-07	9.91648E-01	1.81593E-03	3.24900E-07	9.97570E-01	4.02199E-03
2.54800E-07	9.94205E-01	3.20156E-03	3.07100E-07	9.92220E-01	1.84610E-03
2.01000E-07	9.93756E-01	3.20127E-03	3.18800E-07	9.98620E-01	4.01995E-03
1.93600E-07	9.90589E-01	1.70294E-03	1.81700E-07	9.83540E-01	3.42007E-03
1.69000E-07	9.89880E-01	1.70294E-03	2.95000E-07	9.95820E-01	4.01897E-03
1.73200E-07	9.90150E-01	1.70294E-03	1.81700E-07	9.93250E-01	1.64000E-03
1.53300E-07	9.88891E-01	1.70294E-03	3.03700E-07	9.94170E-01	1.84391E-03
1.56600E-07	9.89071E-01	1.70238E-03	1.82900E-07	9.83950E-01	3.42117E-03
1.44600E-07	9.88432E-01	1.80225E-03	2.96300E-07	9.97910E-01	4.02305E-03
1.52000E-07	9.90130E-01	2.00202E-03	9.86100E-08	9.94090E-01	3.12083E-03
2.03200E-07	9.95744E-01	2.40208E-03	1.17400E-07	9.95410E-01	2.13965E-03
1.73000E-07	9.93387E-01	1.90263E-03	2.38200E-07	9.97730E-01	4.01897E-03
2.00500E-07	9.95225E-01	1.90263E-03	2.56800E-07	9.97930E-01	4.02096E-03
2.03700E-07	9.91379E-01	1.90263E-03	2.45400E-07	9.96790E-01	4.02096E-03
2.09900E-07	9.86823E-01	2.20227E-03	2.57500E-07	9.98280E-01	4.02096E-03
1.77000E-07	9.86064E-01	1.90263E-03	1.74400E-07	9.82190E-01	3.41901E-03
2.09700E-07	9.90200E-01	2.40208E-03	2.83800E-07	9.97050E-01	4.01801E-03
1.76700E-07	9.89021E-01	2.00202E-03	1.75600E-07	9.92550E-01	1.63783E-03
2.10500E-07	9.91399E-01	2.70185E-03	2.96300E-07	9.94250E-01	1.84835E-03
1.77100E-07	9.89760E-01	2.10238E-03	1.77600E-07	9.95210E-01	5.01023E-03
1.57700E-07	9.88422E-01	1.90263E-03	1.73900E-07	9.94510E-01	5.01088E-03
8.12400E-08	9.98060E-01	6.11183E-03	1.68600E-07	9.97160E-01	5.00960E-03
7.00400E-08	1.00224E+00	6.11183E-03	1.91000E-07	9.95070E-01	5.00899E-03
6.78300E-08	1.00364E+00	6.11310E-03	1.76700E-07	9.94450E-01	5.00960E-03
6.65900E-08	1.00317E+00	6.11183E-03	1.87000E-07	9.95250E-01	5.01023E-03
6.56100E-08	1.00296E+00	6.11183E-03	1.36400E-07	9.95610E-01	5.00960E-03
6.46900E-08	1.00272E+00	6.11514E-03	2.82900E-07	9.96020E-01	5.00729E-03
2.52000E-07	9.91250E-01	1.65940E-03	2.64400E-07	9.92230E-01	5.00675E-03
2.32000E-07	9.89050E-01	1.47054E-03	2.65400E-07	9.89760E-01	5.00675E-03
2.00600E-07	9.89020E-01	1.46164E-03	2.45700E-07	1.00629E+00	5.00675E-03
1.84600E-07	9.92390E-01	2.82567E-03	2.55200E-07	1.00140E+00	5.00729E-03
1.78100E-07	9.93000E-01	2.82179E-03	2.33700E-07	1.01663E+00	5.00625E-03
1.72000E-07	9.93970E-01	2.82567E-03	1.96900E-07	9.93690E-01	5.00675E-03
1.65700E-07	9.91860E-01	2.82305E-03	2.51900E-07	9.91770E-01	5.00625E-03
9.84700E-08	9.94290E-01	3.11970E-03	2.49900E-07	9.93910E-01	5.00625E-03
9.83600E-08	9.90650E-01	3.11751E-03	2.06100E-07	1.02156E+00	5.00625E-03
3.20800E-07	9.94830E-01	2.83436E-03	2.25700E-07	1.01211E+00	5.00625E-03
3.07400E-07	9.96120E-01	2.83283E-03	1.94300E-07	1.02123E+00	5.00625E-03
2.97000E-07	9.96250E-01	2.83283E-03	1.96900E-07	9.93690E-01	5.00675E-03
2.91200E-07	9.95940E-01	2.83436E-03	2.32000E-07	9.94560E-01	5.00625E-03
2.85400E-07	9.95820E-01	2.83132E-03	2.29100E-07	9.96840E-01	5.00625E-03
2.79400E-07	9.94280E-01	2.83436E-03	1.80900E-07	1.00438E+00	5.00576E-03
1.25800E-07	9.96770E-01	2.13965E-03	1.91700E-07	1.01752E+00	5.00625E-03
1.19300E-07	9.97110E-01	2.13776E-03	1.76900E-07	9.99820E-01	5.00625E-03
1.15800E-07	9.95950E-01	2.13965E-03	1.77200E-07	9.95040E-01	5.00625E-03

```
   chi = 24.1111 (upper bound = 9.49). The data tests NOT normal
```

```
      Output from statistical treatment

   Heterogeneous Experiments x=EALF MeV

    Number of data points (n)                         90
    Linear regression, k(X)                          0.9956 + (-4.0613E+02)*X
    Confidence on fit (1-gamma) [input]               95.0%
    Confidence on proportion (alpha) [input]          95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]            99.5%
    Minimum value of X                                6.4690E-08
    Maximum value of X                                3.2490E-07
    Average value of X                                2.0018E-07
    Average value of k                                 0.99549
    Minimum value of k                                 0.98219
    Variance of fit, s(k,X)^2                         5.0821E-05
    Within variance, s(w)^2                           1.4474E-05
    Pooled variance, s(p)^2                           6.5295E-05
    Pooled std. deviation, s(p)                       8.0805E-03
    C(alpha,rho)*s(p)                                 2.8689E-02
    student-t @ (n-2,1-gamma)                         1.66493E+00
    Confidence band width, W                          1.3848E-02
    Minimum margin of subcriticality, C*s(p)-W        1.4840E-02

    Upper subcritical limits: ( 6.46900E-08 <= X <=  3.24900E-07)
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)            USL1 = 0.9317 + (-4.0613E+02)*X

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9669 + (-4.0613E+02)*X



    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  6.47E-8  1.02E-7  1.39E-7  1.76E-7  2.13E-7  2.51E-7  2.88E-7  3.25E-7
       -----------------------------------------------------------------------
USL-1:    0.9317   0.9317   0.9317   0.9317   0.9316   0.9316   0.9316   0.9316
USL-2:    0.9669   0.9668   0.9668   0.9668   0.9668   0.9668   0.9668   0.9668
       -----------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
      Plot file written to: Het-leth.plt
```

USLSTATS output, Heterogeneous systems Set 2, k_{eff} vs. EALF (Lethargy)

```
              uslstats: a utility to calculate upper subcritical
                        limits for criticality safety applications

  ***************************************************************************
                          Version 1.4, April 23, 2003
                         Oak Ridge National Laboratory
  ***************************************************************************


   Input to statistical treatment from file:Het-leth.in

Title: Heterogeneous Experiments x=EALF MeV

   Proportion of the population = .995
   Confidence of fit            = .950
   Confidence on proportion     = .950
   Number of observations       =   90
   Minimum value of closed band =    0.00
   Maximum value of closed band =    0.00
   Administrative margin        =    0.05
```

independent variable - x	dependent variable - y	deviation in y	independent variable - x	dependent variable - y	deviation in y
1.76200E-07	9.93477E-01	1.81221E-03	3.27900E-07	9.99920E-01	4.01995E-03
2.56200E-07	9.95614E-01	3.20156E-03	3.09500E-07	9.94210E-01	1.84391E-03
2.02600E-07	9.95154E-01	3.20127E-03	3.21000E-07	1.00009E+00	4.02305E-03
1.95000E-07	9.91928E-01	1.70294E-03	1.82400E-07	9.85380E-01	3.42230E-03
1.70400E-07	9.91239E-01	1.70238E-03	2.97100E-07	9.98130E-01	4.02523E-03
1.74400E-07	9.91479E-01	1.70294E-03	1.83000E-07	9.94280E-01	1.64222E-03
1.54500E-07	9.90589E-01	1.70238E-03	3.06300E-07	9.96740E-01	1.85065E-03
1.57800E-07	9.90490E-01	1.70294E-03	1.83700E-07	9.85160E-01	3.41901E-03
1.45700E-07	9.90090E-01	1.80225E-03	2.98500E-07	1.00024E+00	4.01995E-03
1.53300E-07	9.91738E-01	2.00202E-03	9.95600E-08	9.94530E-01	3.12083E-03
2.04600E-07	9.97203E-01	2.40208E-03	1.17900E-07	9.95790E-01	2.13410E-03
1.74300E-07	9.94545E-01	1.90213E-03	2.38900E-07	1.00059E+00	4.02199E-03
2.02000E-07	9.96444E-01	1.90263E-03	2.58700E-07	1.00098E+00	4.01801E-03
2.05200E-07	9.92857E-01	1.90263E-03	2.48100E-07	9.97640E-01	4.01897E-03
2.11300E-07	9.88102E-01	2.20227E-03	2.58900E-07	9.99580E-01	4.02305E-03
1.78300E-07	9.87542E-01	1.90263E-03	1.75600E-07	9.84510E-01	3.42117E-03
2.11200E-07	9.91578E-01	2.40208E-03	2.85600E-07	9.99550E-01	4.02096E-03
1.78000E-07	9.90290E-01	2.00250E-03	1.76900E-07	9.93740E-01	1.64000E-03
2.12200E-07	9.92737E-01	2.70185E-03	2.98300E-07	9.96410E-01	1.84835E-03
1.78500E-07	9.91409E-01	2.10238E-03	1.79200E-07	9.96360E-01	5.01088E-03
1.58900E-07	9.89880E-01	1.90213E-03	1.75200E-07	9.96020E-01	5.01088E-03
8.17400E-08	9.97920E-01	6.11245E-03	1.70200E-07	9.98540E-01	5.00960E-03
7.04300E-08	1.00156E+00	6.11444E-03	1.92600E-07	9.95560E-01	5.01023E-03
6.81600E-08	1.00298E+00	6.11183E-03	1.77600E-07	9.94670E-01	5.01023E-03
6.70800E-08	1.00241E+00	6.11444E-03	1.87900E-07	9.97030E-01	5.00899E-03
6.57300E-08	1.00161E+00	6.11121E-03	1.37600E-07	9.96740E-01	5.00960E-03
6.50600E-08	1.00267E+00	6.11121E-03	2.85500E-07	9.96620E-01	5.00675E-03
2.53900E-07	9.91660E-01	1.65677E-03	2.66800E-07	9.93710E-01	5.00675E-03
2.33900E-07	9.91740E-01	1.46455E-03	2.67000E-07	9.91450E-01	5.00625E-03
2.01800E-07	9.90930E-01	1.47054E-03	2.48300E-07	1.00737E+00	5.00729E-03
1.86100E-07	9.93360E-01	2.82305E-03	2.57400E-07	1.00312E+00	5.00625E-03
1.79200E-07	9.94640E-01	2.82434E-03	2.35400E-07	1.01807E+00	5.00625E-03
1.72400E-07	9.94770E-01	2.82434E-03	1.98500E-07	9.95350E-01	5.00675E-03
1.66400E-07	9.94310E-01	2.82567E-03	2.52700E-07	9.93940E-01	5.00625E-03
9.92400E-08	9.95280E-01	3.11859E-03	2.51700E-07	9.95090E-01	5.00625E-03
9.93800E-08	9.92300E-01	3.12083E-03	2.07900E-07	1.02303E+00	5.00625E-03
3.22300E-07	9.98040E-01	2.82986E-03	2.27900E-07	1.01326E+00	5.00625E-03
3.08400E-07	9.97690E-01	2.82986E-03	1.95900E-07	1.02224E+00	5.00576E-03
2.98300E-07	9.98720E-01	2.82986E-03	1.98500E-07	9.95350E-01	5.00675E-03
2.92700E-07	9.98050E-01	2.83132E-03	2.33700E-07	9.96480E-01	5.00625E-03
2.87300E-07	9.97480E-01	2.83132E-03	2.30600E-07	9.98710E-01	5.00576E-03
2.80200E-07	9.95900E-01	2.82843E-03	1.81900E-07	1.00620E+00	5.00576E-03
1.27000E-07	9.97030E-01	2.13776E-03	1.93800E-07	1.01893E+00	5.00576E-03
1.19900E-07	9.97140E-01	2.13410E-03	1.78200E-07	1.00144E+00	5.00576E-03
1.17000E-07	9.95040E-01	2.13410E-03	1.78900E-07	9.95960E-01	5.00576E-03

```
   chi = 19.3333 (upper bound = 9.49). The data tests NOT normal
```

```
      Output from statistical treatment

   Heterogeneous Experiments x=EALF MeV

    Number of data points (n)                         90
    Linear regression, k(X)                          0.9951 + ( 8.5263E+03)*X
    Confidence on fit (1-gamma) [input]               95.0%
    Confidence on proportion (alpha) [input]          95.0%
    Proportion of population falling above
    lower tolerance interval (rho) [input]            99.5%
    Minimum value of X                                6.5060E-08
    Maximum value of X                                3.2790E-07
    Average value of X                                2.0159E-07
    Average value of k                                 0.99685
    Minimum value of k                                 0.98451
    Variance of fit, s(k,X)^2                         4.8512E-05
    Within variance, s(w)^2                           1.4473E-05
    Pooled variance, s(p)^2                           6.2985E-05
    Pooled std. deviation, s(p)                       7.9363E-03
    C(alpha,rho)*s(p)                                 2.8181E-02
    student-t @ (n-2,1-gamma)                         1.66493E+00
    Confidence band width, W                          1.3602E-02
    Minimum margin of subcriticality, C*s(p)-W        1.4579E-02

    Upper subcritical limits: ( 6.50600E-08 <= X <=  3.27900E-07)
    ***** *********** *******

    USL Method 1 (Confidence Band with
    Administrative Margin)           USL1 = 0.9315 + ( 8.5263E+03)*X

    USL Method 2 (Single-Sided Uniform
    Width Closed Interval Approach)    USL2 = 0.9669 + ( 8.5263E+03)*X



    USLs Evaluated Over Range of Parameter X:
    **** ********* **** ***** ** ********* **

    X:  6.51E-8  1.03E-7  1.40E-7  1.78E-7  2.15E-7  2.53E-7  2.90E-7  3.28E-7
       ------------------------------------------------------------------------
USL-1:    0.9321   0.9324   0.9327   0.9330   0.9334   0.9337   0.9340   0.9343
USL-2:    0.9675   0.9678   0.9681   0.9685   0.9688   0.9691   0.9694   0.9697
       ------------------------------------------------------------------------

  ****************************************************************************
                           Thus spake USLSTATS
                                  Finis.
      Plot file written to: Het-leth.plt
```