Development of a Noise-Based Method for the Determination of the Moderator Temperature Coefficient of Reactivity (MTC) in Pressurized Water Reactors (PWRs)

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THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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Cover: Radial space-dependence of the Fourier transform of the moderator temperature noise
(at a frequency of 0.5 Hz); figure derived from the noise measurement performed in Ringhals-2,
cycle 26 reported in Section 4.3.

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ABSTRACT

The Moderator Temperature Coefficient of reactivity (MTC) is an important safety parameter of Pressurized Water Reactors (PWRs). In most countries, the so-called at-power MTC has to be measured a few months before the reactor outage, in order to determine if the MTC will not become too negative. Usually, the at-power MTC is determined by inducing a change in the moderator temperature, which has to be compensated for by other means, such as a change in the boron concentration. An MTC measurement using the boron dilution method is analysed in this thesis. It is demonstrated that the uncertainty of such a measurement technique is so large, that the measured MTC could become more negative than what the Technical Specifications allow. Furthermore, this technique incurs a disturbance of the plant operation. For this reason, another technique relying on noise analysis was proposed a few years ago. In this technique, the MTC is inferred from the neutron noise measured inside the core and the moderator temperature noise measured at the core-exit, in the same or in a neighbouring fuel assembly. This technique does not require any perturbation of the reactor operation, but was nevertheless proven to underestimate the MTC by a factor of 2 to 5.

In this thesis, it is shown, both theoretically and experimentally, that the reason of the MTC underestimation by noise analysis is the radially loosely coupled character of the moderator temperature noise throughout the core. A new MTC noise estimator, accounting for this radially non-homogeneous moderator temperature noise is proposed and demonstrated to give the correct MTC value. This new MTC noise estimator relies on the neutron noise measured in a single point of the reactor and the radially averaged moderator temperature noise measured inside the core. In the case of the Ringhals-2 PWR in Sweden, Gamma-Thermometers (GTs) offer such a possibility since in dynamic mode they measure the moderator temperature noise, whereas in static mode they measure the spatial distribution of the neutron flux. Both of these are required to estimate the core average moderator temperature noise. There are 12 radial positions where GTs are installed, which makes it possible to approximate averages over the horizontal cross-section of the core quite well.

Keywords: Moderator Temperature Coefficient (MTC), noise analysis, temperature noise (structure of the), correlation length, point-kinetics, boron dilution method, core calculations, dynamic reactor transfer function, Decay Ratio (DR), Gamma-Thermometer (GT).

LIST OF PUBLICATIONS

This thesis is based on the work contained in the following papers, referred to by Roman numerals in the text:

I.   C. DEMAZIÈRE, I. PÁZSIT and G. PÓR, "Evaluation of the Boron Dilution Method for Moderator Temperature Coefficient Measurements," *Nucl. Tech.*, **140**, 147 (2002).

II.   T. ANDERSSON, C. DEMAZIÈRE, A. NAGY, U. SANDBERG, N. S. GARIS and I. PÁZSIT, "Development and Application of Core Diagnostics and Monitoring for the Ringhals PWRs," to appear in *Prog. Nucl. Energy.*

III.   C. DEMAZIÈRE and I. PÁZSIT, "Theoretical Investigation of the MTC Noise Estimate in 1-D Homogeneous Systems," *Ann. Nucl. Energy*, **29**, 75 (2002).

IV.   C. DEMAZIÈRE and I. PÁZSIT, "2-D 2-Group Neutron Noise Simulator and its Application to Anomaly Localisation," *Proc. Int. Mtg. Mathematical Methods for Nuclear Applications (M&C2001)*, Salt Lake City, Utah, USA, September 9-13, 2001, American Nuclear Society (2001).

V.   C. DEMAZIÈRE and I. PÁZSIT, "A Phenomenological Model for the Explanation of a Strongly Space-Dependent Decay Ratio," submitted to *Int. Mtg. Nuclear Mathematical and Computational Sciences (M&C2003)*, Gatlinburg, Tennessee, April 6-11, 2003, American Nuclear Society (2003).

VI.   C. DEMAZIÈRE and I. PÁZSIT, "Study of the MTC Estimation by Noise Analysis in 2-D Heterogeneous Systems," to appear in *Prog. Nucl. Energy.*

VII.   C. DEMAZIÈRE and I. PÁZSIT, "On-Line Determination of the MTC (Moderator Temperature Coefficient) by Neutron Noise and Gamma-Thermometer Signals," chapter in the book *Power Plant Surveillance and Diagnostics - Modern Approaches and Advanced Applications*, by D. RUAN and P. F. FANTONI (Eds.), ISBN 3-540-43247-7, Physica Verlag, Springler, 135 (2002).

VIII.   C. DEMAZIÈRE, I. PÁZSIT, T. ANDERSSON, B. SEVERINSSON and T. RANMAN, "Analysis of an MTC Noise Measurement Performed in Ringhals-2 Using Gamma-Thermometers and In-Core Neutron Detectors," to appear in *Prog. Nucl. Energy.*

LIST OF PUBLICATIONS NOT INCLUDED IN THIS THESIS

I.   C. DEMAZIÈRE, V. ARZHANOV, J. K-H. KARLSSON and I. PÁZSIT, "Final Report on the Research Project Ringhals Diagnostics and Monitoring, Stage 4," CTH-RF-145/RR6, Chalmers University of Technology (1999).

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III.   C. DEMAZIÈRE, "Analysis of the reactivity coefficients and the stability of a BWR loaded with MOX fuel," *Proc. Int. Topl. Mtg. Advances in Reactor Physics and Mathematics and Computation into the Next Millennium (PHYSOR2000)*, Pittsburgh, Pennsylvania, USA, May 7-12, 2000, American Nuclear Society (2000).

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V.   I. PÁZSIT, C. DEMAZIÈRE, S. AVDIC and B. DAHL, "Research and Development Program in Reactor Diagnostics and Monitoring with Neutron Noise Methods, Stage 6, Final Report," SKI Report 00:28, Statens Kärnkraftinspektion (2000).

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VIII. C. DEMAZIÈRE, I. PÁZSIT and G. PÓR, "Estimation of the moderator temperature coefficient (Analysis of an MTC measurement using boron dilution method)," *Proc. Int. Topl. Meet. Nuclear Plant Instrumentation, Controls, and Human-Machine Interface Technologies (NPIC&HMIT 2000)*, Washington, DC, USA, November 13-16, 2000 (2000).

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XIII.   C. DEMAZIÈRE and I. PÁZSIT, "Development of a Method for Measuring the MTC by Noise Analysis and its Experimental Verification in Ringhals-2," *Proc. Int. Mtg. New Frontiers of Nuclear Technology: Reactor Physics, Safety and High-Performance Computing (PHYSOR2002)*, Seoul, South-Korea, October 10-13, 2002, American Nuclear Society (2002).

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Contents

Chapter 1

Introduction

The Moderator Temperature Coefficient of reactivity (MTC) is an important safety parameter in Pressurized Water Reactors (PWRs). In such a type of reactors, the coolant, i.e. water at a pressure of typically 155.10^5 Pa, serves as a neutron moderator. The MTC is defined as the change of reactivity induced by a temperature change of the core coolant, divided by the core average coolant temperature change (see Ref. 1). Physically, the reactivity effects due to moderator temperature changes can be divided into two main categories: the direct effects and the indirect effects (see Ref. 2). The direct effects can themselves be separated into a temperature-only or spectral component at a microscopic level, and a density component at a macroscopic level. The spectral component is due to the modification of the thermal equilibrium temperature of the neutrons resulting from the modification of the thermal scattering of neutrons by water when the temperature of the moderator changes. The density component is simply the result of the change of the moderator density when the temperature of the moderator varies. The indirect effects are related to the redistribution of the axial flux. The axial redistribution is due to the fact that even if the coolant temperature change is relatively homogeneous in the core, the moderator density change is not axially homogeneous since the axial distribution of the moderator temperature is not homogeneous (increase from bottom to top because of nuclear heating). The MTC must account for both the direct and indirect effects, and only these. Therefore, if the MTC measurement technique significantly perturbs the axial power shape/moderator temperature distribution (such an example occurs when the MTC is measured by the so-called control rod swap method using measured rod worth), the resulting axial flux redistribution cannot be considered solely as an indirect MTC effect (see Refs. 1 and 2).

Furthermore, the MTC is part of the feedback mechanism of PWRs, since a reactivity perturbation of the core will affect the power, which in turn will modify the coolant temperature, via a change of the fuel temperature. The reactivity effect induced by the MTC and the Doppler effects are the main elements of the inherent stability in a PWR. Consequently, the MTC should be negative in most circumstances in order to give a stable reactor. Nevertheless, in some countries, a positive MTC can be allowed at Beginning Of Cycle (BOC) if transient analyses have proven that there is no safety issue in accidental situations, because the total temperature coefficient of reactivity (including fuel) is negative.

Currently in Sweden, only a negative MTC is permitted by the Swedish Nuclear Power Inspectorate (SKI, Statens Kärnkraftinspektion). They require two MTC measurements during each fuel cycle: at BOC and Hot Zero Power (HZP), and near End Of Cycle (EOC) and Hot Full Power (HFP). All the actual MTC measurement techniques are based on a perturbation of the reactor in order to induce a change of the moderator temperature. Because of the decrease of the boron content during the cycle to compensate for the fuel depletion, the magnitude of the negative MTC increases from BOC to EOC. Therefore, the objective of the measurement early in the cycle is to demonstrate that the MTC is negative (preventing the consequences of a positive power feedback), while the objective near the EOC is to show that it remains less negative than some prescribed limit (preventing the consequences of a reactivity increase following a cooldown event). The

near EOC measurement is actually performed when the boron content reaches 300 ppm in the core (approximately three months before the expected EOC), and indicates whether one may operate the reactor up to the expected EOC (the extrapolation of the MTC to the EOC is performed via static core calculations). During the BOC measurement, the Isothermal Temperature Coefficient (ITC), which comprises both moderator and fuel reactivity effect, is actually determined. The MTC is latter calculated by removing the Doppler effect estimated from static core calculations. At the Ringhals nuclear power plant in Sweden, this measurement is carried out by using a digital reactivity meter and the core-exit thermocouples. The digital reactivity meter uses the neutron flux signal as input and evaluates the corresponding reactivity by adopting the one-point reactor kinetics model. This ITC measurement, of which the precision is about 0.5 pcm/°C (see Ref. 4), can be considered as reliable for two main reasons. First, the temperature change is uniform both in the fuel and the moderator and can therefore be measured accurately. Second, today's reactivity meters can measure the reactivity with a high level of accuracy. In contrast, the near EOC measurement has to be carried out while the reactor is at full power, and the MTC cannot be determined with the same ease and level of accuracy. Namely, because the EOC MTC measurement requires several hours, the variation of parameters other than the moderator temperature and the counteracting parameter (such as the boron concentration, or a power change, or the control rods insertion) occurs. These variations cannot be measured in practice, only determined by core calculations. Finally, and most importantly, the traditional measurement techniques of the at-power MTC induce a plant transient that needs to be monitored during several hours.

The trend nowadays in many countries (Sweden being an exception) is to measure the MTC at BOC, and then to completely rely on core calculations for the variation of the MTC throughout the cycle. Nevertheless, the at-power MTC calculations have never been benchmarked (only the zero-power ITC calculations have been benchmarked against measurements - see Ref. 5). Since all the at-power MTC measurement techniques make use of calculated parameters, the at-power MTC cannot be benchmarked since the same calculation tools are used in both the measurement and the calculation, i.e. comparing the results of the measurements to the MTC calculation might hide some inaccuracy in the calculation scheme.

The at-power MTC measurement was given new attention some years ago with the development of a new measuring technique, namely the MTC estimation by noise analysis (see Refs. 6 and 7). In this technique, an in-core neutron detector and a core-exit thermocouple located above the same fuel assembly or one of the neighbouring fuel assemblies are used. The noise signals provided by these two detectors contain some information about the MTC, which can be extracted by using an appropriate noise estimator. The main advantage of this technique is that the reactor need not to be perturbed for estimating the MTC. Nevertheless, several attempts to monitor the MTC by noise analysis revealed that the MTC was systematically underestimated by a factor of 2 to 5 (see Refs. 6-25).

Although the noise-based method failed to provide a correct MTC estimation, such a measurement technique could be of particular interest since power utilities are willing to use new types of fuel assemblies that might have a positive contribution to the MTC. They argue that a positive MTC at BOC could be allowed since the Doppler effect will still ensure a negative feedback. These new types of fuel assemblies are for instance high burnup fuel assemblies (improved fuel economy, i.e. longer cycle length), or Mixed-Oxide (MOX)

fuel assemblies. For the former, the corresponding required amount of boron will be larger at BOC due to the high excess of reactivity of such fresh fuel assemblies. The boron content only affects the density component of the MTC, not the spectral one since ^{10}B has a 1/v absorption cross-section behaviour. Consequently, if high burnup fuel is used, the MTC could become positive at BOC, since the change of the thermal utilization factor with the moderator temperature, which is normally positive, is so large that it cannot be compensated any longer by the decrease of the resonance escape probability. This is equivalent to saying that increasing the boron concentration too much will reduce the probability of neutrons being scattered/captured by water, therefore minimizing the effect of decreasing reactivity when the moderator temperature is raised, which could lead to a positive MTC. In case of MOX fuel, the ^{239}Pu resonance might also render the MTC positive, due to the spectral effect only. The presence of the 0.3 eV resonance of ^{239}Pu implies that a coolant temperature increase will increase both the thermal utilization factor and the thermal fission factor. Consequently, monitoring of the MTC might become of prime importance in a near future, considering also the fact that at-power MTC core calculations were never benchmarked.

Therefore the goal of this thesis is to understand why the MTC is systematically underestimated by using the noise analysis technique and to propose a possible remedy, so that such a technique could provide an alternative to the actual trend, i.e. not measuring the at-power MTC and relying on core calculations only. The main advantage of the noise-based method would be that this technique would not disturb the reactor operation and would be suited to on-line MTC monitoring, if such a need arises. Several reasons could explain the underestimation of the MTC by noise analysis. Some of them were investigated in the past (see Refs. 14, 15, 24, 26-29). Corrections to the noise technique were proposed accordingly, but these corrections are usually small, cannot be easily estimated in practice, and cannot either explain solely the strong deviation of the MTC noise estimate from the true value. Another hypothesis that was not investigated in the past is considered in this thesis, namely that the temperature noise is not radially spatially homogeneous in the core, whereas the noise-based method relies on the measurement of both the neutron noise and the moderator temperature noise in one single radial point of the reactor.

In this thesis, the at-power MTC measurement techniques, both the traditional ones based on a perturbation of the reactor and the noise-based one, are first presented. The reason why the noise technique is not the only technique touched upon in the thesis is that there is no formal proof that the traditional measurement techniques give a correct MTC (since benchmarking of such techniques is usually impossible). The second part of this thesis is devoted to the theoretical investigation of the MTC noise estimate. Namely the hypothesis mentioned previously, i.e. a non-homogeneous radial moderator temperature noise in the core, is investigated theoretically, and its implications on the MTC noise-based method are given. The associated tools necessary to perform such calculations are also briefly explained. Finally, the last part of the thesis presents possible enhancements of the noise analysis method, which are based on the theoretical work. These improvements were also tested via an MTC noise measurement performed in January 2002 in the Ringhals-2 PWR in Sweden. Such a measurement is reported in detail in this thesis. The measurement confirmed the correctness of the new noise estimator.

A nomenclature explaining all the abbreviations used in this thesis can be found at the end of the introductory part to the Papers.

Chapter 2

The MTC and the different ways of measuring it

In this section, the definition of the MTC given by the newest American Standard is recalled (see Ref. 1). Emphasis is put on how the core average temperature should be defined in order to fulfil the MTC definition. An at-power MTC measurement performed in the Ringhals-4 PWR is then presented. This measurement was carried out according to the boron dilution method. Such a method is called in the rest of this thesis a *traditional* MTC measurement technique, since the reactor has to be perturbed (as opposed to the noise-based method that does not disturb the reactor operation). The MTC noise estimate is then derived, and a measurement performed in the Ringhals-4 PWR is briefly explained. Finally, the possible reasons of discrepancy between the two techniques investigated in the past are highlighted. The new hypothesis regarding the non-homogeneous radial structure of the moderator temperature noise throughout the core is touched upon separately, and a proof of such a hypothesis via a measurement performed in the Ringhals-2 PWR is presented.

2.1. Definition of the MTC

According to the newest American Standard and as mentioned briefly in the introduction, the MTC is defined as the partial derivative of the reactivity ρ with respect to the core average moderator temperature T_m^{ave} (see Ref. 1):

$$MTC = \frac{\partial \rho}{\partial T_m^{ave}} \tag{1}$$

For a small change in the average moderator temperature, the reactivity change would be

$$\delta\rho(t) = MTC \times \delta T_m^{ave}(t). \tag{2}$$

The Standard says that the way of calculating the core temperature average does not play a significant role as long as the same methodology is used in both the measurement and the calculations. The Standard thus implicitly proposes to simply use a volume average of the temperature change:

$$\delta T_m^{ave}(t) = \frac{\int \delta T_m(\boldsymbol{r}, t) d\boldsymbol{r}}{\int d\boldsymbol{r}} \tag{3}$$

As long as the temperature change is relatively homogeneous, the Standard suggests using the following definition for the temperature average:

$$\delta T_m^{ave}(t) = \frac{\delta T_{in}(t) + \delta T_{out}(t)}{2} \tag{4}$$

where *in* and *out* stand for the core inlet and the core outlet respectively.

If the temperature change is not homogeneous throughout the core, Eq. (4) will not accurately reflect the change in the core average moderator temperature, on which the true MTC is dependent according to the Standard. Although the Standard thus recommends using Eq. (3), it will be shown in the following (see Chapter 3) that the weighting function that needs to be used to estimate the core average moderator temperature change, i.e. $w(\boldsymbol{r})$ in the following Equation

$$\delta T_m^{ave}(t) = \frac{\int \delta T_m(\boldsymbol{r}, t)w(\boldsymbol{r})d\boldsymbol{r}}{\int w(\boldsymbol{r})d\boldsymbol{r}}, \tag{5}$$

cannot be equal simply to unity (volume average). As will be discussed later, the radial heterogeneous character of the moderator temperature noise is much more important than the axial one, so that emphasis is on the radial average in Eq. (5).

2.2. Traditional measurement techniques

Several measurement techniques exist for the measurement of the MTC that are based on the perturbation of the reactivity in the reactor and then compensating for it by a change of the coolant temperature of the core. The main traditional methods are (see Refs. 3, 30-32):

- The power change or xenon transient method, in which the power level of the reactor is changed (and so is the xenon concentration). The corresponding reactivity effect is then compensated by a modification of the inlet temperature of the core in order to keep the reactor critical.

- The depletion or stretch-out method, in which the boron concentration of the core is maintained constant during about 1.5 days. As a result of the fuel depletion, the coolant average temperature needs to be decreased.

- The control rod swap method, in which the inlet temperature of the core is modified. The corresponding reactivity effect is compensated by a modification of the insertion of some of the control rods (the control rod worth is either calculated or measured previously).

- The boron dilution method, in which the inlet temperature of the core is increased. The reactor is thus kept critical by diluting the boron content.

The boron dilution method is the most commonly used method worldwide and is the one briefly discussed in the following. The rest of this Section presents in more detail the boron dilution method reported in Paper I. The main advantages of the boron dilution method are (see Refs. 3 and 33):

- The high level of accuracy of the calculated differential boron worth, which is required for both the predetermination of the boron dilution and the estimation of the MTC itself. The boron reactivity effect is given by the product of the differential boron worth and the change in the boron concentration. This latter is a measured parameter, whereas the former is a calculated one. It is generally accepted that today's reactor codes predict this reactivity parameter very accurately [accuracy of 1 to 2% (see Ref. 3)].

- The small modification in the axial power shape. In a western-type PWR, only the core-inlet and core-outlet temperatures are actually measured. There is no thermocouple

installed inside the core. Consequently, the only average temperature that is measurable is the one defined in Eq. (4). This definition of the average temperature assumes that the coolant temperature change is relatively homogeneous in the core. Fortunately in the boron dilution method, core calculations show that the axial power shape of the reactor is only slightly modified by the modification of the coolant properties. Therefore, using Eq. (4) gives a rather accurate value of the core average temperature change, if the control rods are not used during the measurement for any axial offset compensation. Otherwise, a relatively simple adjustment is necessary to account for the change of the axial power shape.

The boron dilution method has also major drawbacks such as (see Ref. 3 and 34):

- The large uncertainty in the measurement of the boron concentration. At Ringhals, to cope with this problem, three different samples of the primary coolant are used to measure the boron content and the boron concentration is simply taken as the mean value of these three measurements. The standard deviation associated with these three measurements is then used to estimate the uncertainty of the measured MTC.

- The relatively long time required to perform the measurement (up to 12 h). During this time, other parameters can change, and their effect needs to be compensated for. The contribution of these can only be evaluated by core calculations, not measured directly.

- Loss of production. In some cases, although the test is performed at near full power, it is considered inappropriate to perform the measurement at exactly 100% power because of the proximity of the high flux trip setpoints and the effects of changing moderator density on ex-core detector response (see Ref. 3). Performing the test at reduced power during several hours represents a considerable loss of production for power utilities.

- The plant transient induced by the test. Because of the change of the reactor status in a relatively short time (typically 12 h), a plant transient is initiated and the operators must monitor it for about 24 h until steady-state conditions are again achieved.

- The relatively high number of parameters that need to be estimated by core calculations and cannot be measured in practice in order to estimate the MTC. This compromises the purity of the boron dilution method since a good measurement technique should rely on as few core-calculated parameters as possible.

In the following, an MTC measurement performed at Ringhals-4 using the boron dilution method is analysed in more detail. This measurement was carried out during the fuel cycle 16, at near EOC [the core average burnup was estimated to be 8,767 GWd/tHM (where HM stands for heavy metal), and the boron concentration was measured to be 295 ppm at the beginning of the measurement]. The purpose of this measurement was to check, according to the Technical Specifications of Ringhals-4, that the MTC at full power was larger (less negative) than -72 pcm/$°$C seven days equivalent full power after the boron concentration reached 300 ppm [about 2 to 3 months before the End Of Full Power (EOFP)]. Although this MTC estimation is called in the following the EOC MTC, this MTC measurement does not correspond exactly to the EOC MTCs (and not even the EOFP MTCs) since the boron concentration is about 300 ppm in the Reactor Coolant System (RCS). The EOFP is defined as the moment in the fuel cycle when the boron concentration in the RCS approaches 0 ppm. At that point, it is still possible to operate the reactor by decreasing the power level. The positive reactivity gained by the decrease of the power level

allows compensating the reactivity lost by the fuel depletion for about 2 more months. When it is not possible to operate the reactor any longer, the EOC is reached. Consequently, the MTC measurement is actually performed 4 to 5 months before the actual EOC, i.e. when the reactor has to be shut down. Since the purpose of the MTC measurement is to verify that the MTC will not become lower than some prescribed value during the remaining part of the cycle, the measured MTC needs to be extrapolated. In Ringhals, this extrapolation is carried out by using the static core simulator SIMULATE-3, in order to estimate the MTC change during that part of the cycle (see Ref. 35). The MTC measurement is therefore used as a calibration or a checking of the SIMULATE-3 ability to correctly predict the MTC at 300 ppm.

The measured parameters during the Ringhals-4 measurement were the coolant inlet and outlet temperatures in each of the 3 loops [from which the core average temperature of the coolant can be estimated according to Eq. (4)], the power level, and the boron concentration. During the measurement procedure, the axial offset became more negative but remained within the operating limits. Consequently, the control rods were not used to adjust the axial offset of the core. Therefore, assuming that the reactivity does not change during the whole measurement (perfect reactivity compensation), the Moderator Temperature Coefficient can be estimated as (see Ref. 36)

$$MTC = \frac{\partial \rho}{\partial T_m} = \frac{1}{k_{eff,1} \cdot k_{eff,2}} \frac{\partial k_{eff}}{\partial T_m} \approx -\frac{1}{\Delta T_m^{ave}} \left(\frac{\partial \rho}{\partial T_f} \Delta T_f + \frac{\partial \rho}{\partial C_B} \Delta C_B + \Delta \rho^* \right) \qquad (6)$$

since the reactivity effect induced by the change of the moderator temperature between the beginning and the end of the measurement is fully compensated by the boron dilution, the Doppler effect, and the remaining effects mainly due to the fuel depletion during this time. The power level is the same at the end of the measurement as at the beginning. Thus no reactivity effect due to change in power appears in the preceding Eq. (6). The different terms in Eq. (6) have the following meaning:

- k_{eff} is the effective multiplication factor (the subscripts 1 and 2 refer to the initial and the final states respectively);

- ΔT_m^{ave} is the change in the core average moderator temperature [as defined in Eq. (4)] and is a measured parameter;

- $\partial \rho / \partial T_f \times \Delta T_f$ is the Doppler effect; $\partial \rho / \partial T_f$ and ΔT_f, the Doppler coefficient and the average fuel temperature change, respectively, are both estimated by core calculations;

- $\partial \rho / \partial C_B \times \Delta C_B$ is the boron effect; $\partial \rho / \partial C_B$ is the differential boron worth and is a calculated parameter; ΔC_B is the average change of the boron concentration and is a measured parameter;

- $\Delta \rho^*$ represents effects other than the boron effect, the Doppler effect, and MTC effect. Namely, $\Delta \rho^*$ can be associated to the fuel depletion, the xenon redistribution, the variation in the neutron leakage, and the change in the axial flux profile (only the change

due to the measurement technique itself, not due to the change in the moderator properties, which must be accounted for in the MTC according to the standard).

The MTC was then evaluated according to Eq. (6) from the data provided by Ringhals, and with the use of SIMULATE-3 for the calculation of the different terms that cannot be measured. The uncertainty of all the parameters was also estimated in a conservative way. For comparison purposes, it is also interesting to calculate the MTC directly via SIMULATE-3. One finally obtains:

- calculated MTC using SIMULATE-3: -45.7 pcm/°C;

- measured MTC using the boron dilution method: -58.1 pcm/°C with an uncertainty of ± 14.5 pcm/°C.

One notices that the MTC calculated by SIMULATE-3 lies in the confidence interval associated with the measured MTC. This confidence interval is nevertheless so large that the MTC might even be close to -72 pcm/°C. Consequently, the large confidence interval of the boron dilution method makes it impossible to determine if the MTC is always larger (less negative) than this threshold. One might thus question the usefulness of the boron dilution method.

Fig. 1 shows the results obtained in the present study of the MTC measurement, which is reported in detail in Paper I. In this Figure, the contribution of each term in Eq. (6) is plotted. The uncertainties associated with the MTC and with each of its contributing terms in Eq. (6) are also represented. The MTC calculated by SIMULATE-3 is also given on the right-hand side of the Figure as an indicative parameter. One notices that the Doppler contribution is negligibly small, even if its relative error is rather high. The main reactivity effect, which counteracts the moderator temperature change, is due to the change in the boron concentration. Nevertheless, the term $\Delta\rho^*$ (referenced to as "Remaining effects" in the Figure) contributes to the MTC by introducing a positive contribution of + 15.70 pcm/°C. As explained in Paper I, the estimation of this parameter is a very difficult task, and it is thus not granted that the term $-\Delta\rho^* / \Delta T_m^{ave}$ is estimated precisely. Since its contribution to the final MTC is relatively significant, the misestimation of this parameter might have major consequences on the final MTC value given by the boron dilution method.

Although the actual MTC value could differ from the measured one much more than previously expected, when the boron dilution method is used, it is very unlikely that the MTC is misestimated by a factor of 2 to 5 of magnitude. Furthermore, the MTC calculated by codes such as SIMULATE-3, even if no benchmark exists for the at-power MTC, seems to indicate that a "reasonable" MTC value should not differ so much from the measured one. Therefore, the noise analysis technique and the corresponding underlying assumptions need to be studied in more detail in order to find out why this technique underestimates systematically the MTC by a factor of 2 to 5.
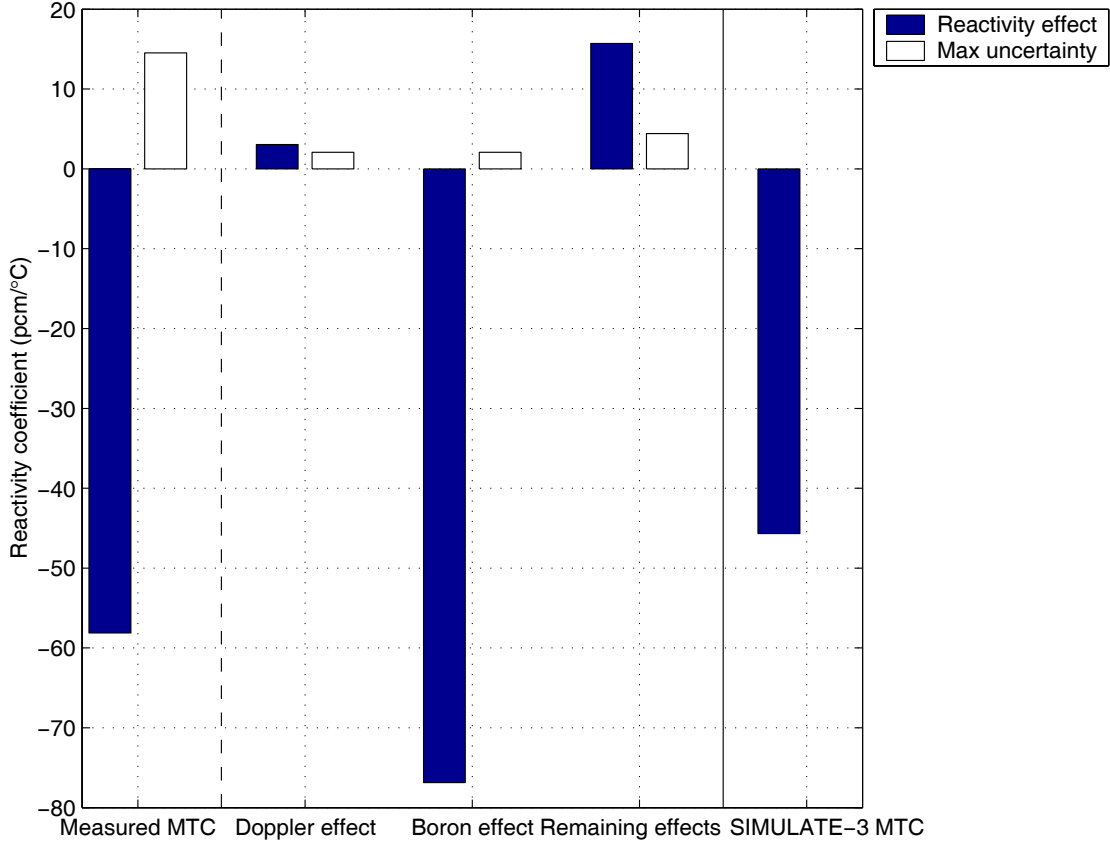
Fig. 1. Measured EOC MTC at Ringhals-4, cycle 16, with the contribution of the different reactivity components and their respective uncertainties (the MTC directly calculated by SIMULATE-3 is also given as an indicative parameter).

2.3. Derivation of the MTC noise estimate

If the reactivity depends on the stationary[1] ergodic[2] processes $s_i(t)$, $i = 1, ..., N$, expanding the reactivity variation around the stationary value and assuming linear theory will lead to:

$$\delta\rho(t) \approx MTC \times \delta T_m^{ave}(t) + \sum_{i=1, s_i \neq T_m^{ave}}^{N} \frac{\partial\rho}{\partial s_i} \times \delta s_i(t) \tag{7}$$

in which the MTC effect is separated from the other effects. This equation defines the so-called reactivity noise, i.e. the variation of the reactivity with time from which the static component is removed. Since in all the cases considered in the following, steady-state conditions are assumed to be fulfilled, the static reactivity is equal to zero. Likewise, all the $\delta s_i(t)$, $i = 1, ..., N$ terms in Eq. (7) are defined as the differences between the time-dependent parameters and their mean values. These noise sources can be manifold (see Ref. 37): random fluctuations due to the fission process itself (this effect is overwhelming at zero power, but negligible at full power), reactivity-induced global changes in the power or

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1. Random events are called *stationary* when their stochastic properties do not change with time.
2. A process is called *ergodic* when the time average equals the ensemble average.

local changes in power distribution, relative motion of the detector and the flux distribution close to it, fluctuation of the moderator mass distribution around the in-core detectors, variation of the field-of-view of the ex-core detectors due to the modification of the gap between the core and the detectors, vibration of control rods and of the core-barrel. The level of noise is usually low since the ratio between the standard deviation and the mean value of the signals is typically lower than 1%. Nevertheless, signal processing such as hardware elimination of the DC component and analog-to-digital conversion of the AC component provides a very accurate noise signal. Furthermore, only the normalised (to the mean value) neutron flux noise is needed. Since neutron detectors usually deliver a current directly proportional to the neutron flux, any error in the calibration of these detectors cancels out in noise analysis. For other noise signals such as the temperature noise necessary for the MTC estimation, the absolute value of the noise is required. This means that the uncertainty of the measured signal has to be negligible compared to the noise level. For thermocouples for instance, the standard deviation of the temperature signal is typically less than $0.1\,°C$, whereas the uncertainty of the measured signal is lower than $0.005\,°C$. Consequently, the level of noise is overwhelmingly large compared to the thermocouple uncertainty and the accuracy of the MTC noise estimation is expected to be relatively good.

The idea of using noise analysis to monitor the MTC in PWRs was probably firstly introduced by Thie in 1977 who suggested to use the root-mean-square values of the temperature noise and of the reactivity noise, which can be determined from the relative neutron noise under some assumptions (see Section 2.4), to evaluate the MTC (see Ref. 6):

$$MTC = \frac{\sigma_{\delta\rho}}{\sigma_{\delta T_m^{ave}}} \tag{8}$$

From Eq. (7), one can clearly see that the MTC evaluated by Eq. (8) is biased due to the contamination of the neutron noise from noise sources other than the moderator temperature noise. This bias was experimentally noticed by Türkcan at the Borssele PWR in the Netherlands (see Ref. 7).

Later in 1985, Pór et al. proved that the contribution of the other noise sources can be removed if spectral analysis of the signals is used, as explained in the following (see Ref. 18). Multiplying Eq. (7) by $\delta T_m^{ave}(t + \tau)$ and taking the average gives:

$$CCF_{\delta\rho,\,\delta T_m^{ave}}(\tau) = MTC \times ACF_{\delta T_m^{ave}}(\tau) + \sum_{i\,=\,1,\,s_i\,\neq\,T_m^{ave}}^{N} \frac{\partial\rho}{\partial s_i} \times CCF_{\delta s_s,\,\delta T_m^{ave}}(\tau) \tag{9}$$

where ACF and CCF stand for the Auto-Correlation Function and the Cross-Correlation Function, respectively:

$$CCF_{\delta\rho,\,\delta T_m^{ave}}(\tau) = \langle \delta\rho(t)\delta T_m^{ave}(t + \tau) \rangle \tag{10}$$

$$CCF_{\delta s_s,\,\delta T_m^{ave}}(\tau) = \langle \delta s_i(t)\delta T_m^{ave}(t + \tau) \rangle \tag{11}$$

$$ACF_{\delta T_m^{ave}}(\tau) \ = \ \langle \delta T_m^{ave}(t) \delta T_m^{ave}(t+\tau) \rangle \tag{12}$$

If it can be assumed that the fluctuations of the different s_i parameters ($s_i \neq T_m^{ave}$) are statistically independent of moderator temperature fluctuations, then their cross-correlation vanishes:

$$CCF_{\delta s_s, \delta T_m^{ave}}(\tau) \ = \ 0 \tag{13}$$

It can be shown that this assumption is valid within a certain frequency region (see Section 2.4). Then one has:

$$MTC \ = \ \frac{CCF_{\delta \rho, \delta T_m^{ave}}(\tau)}{ACF_{\delta T_m^{ave}}(\tau)} \tag{14}$$

Alternatively, the MTC can also be derived by multiplying Eq. (7) by $\delta \rho(t+\tau)$ and taking the average:

$$MTC \ = \ \frac{ACF_{\delta \rho}(\tau)}{CCF_{\delta T_m^{ave}, \delta \rho}(\tau)} \tag{15}$$

Eq. (14) or Eq. (15) represents the MTC noise estimator that should be used in noise analysis in order to get the correct value of the MTC.

2.4. Measurement by noise analysis technique

Although Eqs. (14) and (15) are written in the time domain, it is much more common (and practical) to perform the MTC estimation in the frequency domain. This reads as:

$$MTC \ = \ \frac{CPSD_{\delta \rho, \delta T_m^{ave}}(\omega)}{APSD_{\delta T_m^{ave}}(\omega)} \tag{16}$$

or

$$MTC \ = \ \frac{APSD_{\delta \rho}(\omega)}{CPSD_{\delta T_m^{ave}, \delta \rho}(\omega)} \tag{17}$$

where the APSD and CPSD stand for the Auto-Power Spectral Density and Cross-Power Spectral Density respectively. They are the Fourier transforms of the ACF and CCF defined previously.

Nevertheless, neither the at-power reactivity noise nor the average coolant temperature noise can be measured in practice in PWRs. Only the neutron flux (either via the ex-core neutron detectors or via the in-core neutron detectors, when they are present in the core) and the core-inlet/outlet temperatures are measurable quantities in PWRs. Simply

speaking, the MTC should be estimated by using global parameters, such as the reactivity and the average moderator temperature, whereas only local quantities (flux and temperature) can be measured.

The reactivity noise can, under some assumptions, be inferred from the flux noise, as explained in the following. The flux is first factorised into an amplitude function $P(t)$ and a shape function $\psi(r, t)$ as follows

$$\phi(r, t) = P(t)\psi(r, t) \tag{18}$$

where

$$\frac{\partial}{\partial t}\int \phi_0(r)\psi(r, t)dr = 0 \tag{19}$$

and

$$\phi_0(r) = \psi(r, t = 0) \tag{20}$$

From Eqs. (19) and (20), one can see that the fluctuation of the shape function and the static flux are orthogonal:

$$\int \phi_0(r)\delta\psi(r, t)dr = 0 \tag{21}$$

The flux noise in the frequency domain can thus be approximated by (second-order terms neglected):

$$\delta\phi(r, \omega) = \delta\phi^{pk}(r, \omega) + \delta\psi(r, \omega), \tag{22}$$

where

$$\delta\phi^{pk}(r, \omega) = \phi_0(r)\delta P(\omega) \tag{23}$$

is the point-kinetic component of the flux noise. The amplitude function itself satisfies the so-called point-kinetic equations that can be written in the frequency domain:

$$\delta P(\omega) = \delta\rho(\omega)G(\omega) \tag{24}$$

In this equation, $G(\omega)$ is the so-called closed-loop or at-power reactor transfer function which is related to the open-loop or zero-power reactor transfer function $G_0(\omega)$ as follows:

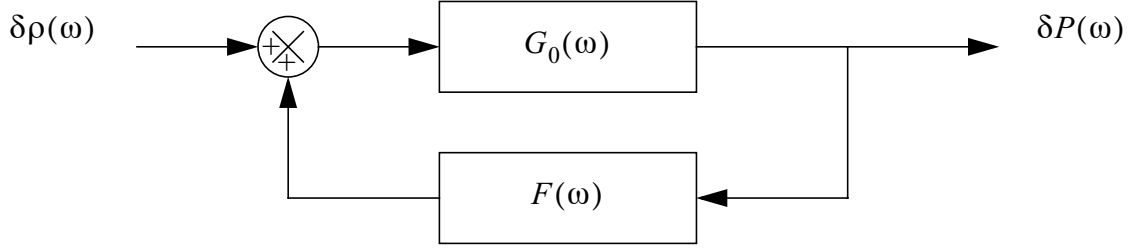$$G(\omega) = \frac{G_0(\omega)}{1 - F(\omega)G_0(\omega)} \tag{25}$$

where

Fig. 2. Point-kinetic model of a reactor with feedback.

$$G_0(\omega) = \frac{1}{i\omega\left(\Lambda + \dfrac{\beta}{i\omega + \lambda}\right)} \tag{26}$$

and all the other terms have their usual meaning. In Eq. (25) and Fig. 2, $F(\omega)$ represents the power to reactivity transfer function or simply the feedback mechanism. As a matter of fact, the MTC and the Doppler effects are hidden in this transfer function, which can only be evaluated if the Doppler and the Moderator Temperature Coefficients of reactivity are known. Since the MTC is the parameter one is looking for, the closed-loop transfer function cannot be used for the MTC noise determination. Spectral or frequency analysis allows coping with this problem. Due to the relatively large time constant of the heat transfer dynamics from fuel to coolant, feedback effects will only occur at low frequencies, typically below 0.1 Hz. By filtering the low frequencies of the signals, the coolant temperature fluctuation can be measured before the feedback begins to alter the coolant temperature. Consequently, $F(\omega) \approx 0$ for frequencies larger than 0.1 Hz and then replacing $G(\omega)$ by $G_0(\omega)$ in Eq. (24) is a valid approximation, so that:

$$\delta\rho(\omega) = \frac{1}{G_0(\omega)}\frac{\delta\phi^{pk}(r, \omega)}{\phi_0(r)} \tag{27}$$

Nevertheless, in-core neutron detectors do not measure solely the point-kinetic component of the neutron noise, but the total flux noise, as defined by Eq. (22). Consequently, the reactivity noise can be calculated from the flux noise only if $\delta\phi^{pk}(r, \omega)$ is overwhelmingly large compared to $\delta\psi(r, \omega)$, i.e. the reactor behaves in a point-kinetic way. It is well known that large power reactors do not necessarily behave in a point-kinetic way. Consequently, the reactivity noise can only be approximated by the following expression:

$$\delta\rho^{approx}(r, \omega) = \frac{1}{G_0(\omega)}\frac{\delta\phi(r, \omega)}{\phi_0(r)} \neq \delta\rho(\omega) \tag{28}$$

Regarding the temperature noise, only the local temperature noise can be measured in a PWR, usually at the top of only a few fuel assemblies. In all the experimental work based on the noise analysis method so far, only a single core-exit thermocouple was used, whereas there is no reason to believe that the moderator temperature noise is homogeneous in the core:

$$\delta T_m(\boldsymbol{r}, \omega) \neq \delta T_m^{ave}(\omega) = \frac{\int \delta T_m(\boldsymbol{r}, \omega) w(\boldsymbol{r}) d\boldsymbol{r}}{\int w(\boldsymbol{r}) d\boldsymbol{r}} \tag{29}$$

This is equivalent to saying that in all the experimental investigations carried out so far, the moderator temperature noise was assumed to be spatially homogeneous in the core. The validity of this hypothesis is discussed in more details in Section 2.6.

Assuming the validity of this approximation, another problem surfaces with the separation distance between the in-core neutron detector and the core-exit thermocouple. The MTC can only be accurately determined if no temperature noise is generated between the neutron and the temperature detectors. There is some experimental evidence (see Section 4.3) that this fact is actually verified. Furthermore, due to the time constant of the detectors, the overwhelmingly large background noise at high frequencies, and the separation distance between the two detectors, the moderator temperature noise recorded by the core-exit thermocouple is damped, compared with the one recorded by the in-core neutron detector. As will be seen in Section 4.2, there is a large enough coherence for frequencies lower than 1 Hz between two in-core thermocouples located within the same fuel assembly to prove that the axial damping is not crucial for such frequencies.

Consequently, the MTC has to be evaluated using frequencies smaller than 1 Hz (because of the temperature noise), and frequencies larger than 0.1 Hz (because of the unknown closed-loop reactor transfer function that has to be replaced by the known open-loop reactor transfer function). Another advantage of using the frequency band 0.1 - 1.0 Hz is that the open-loop transfer function, as can be seen in Fig. 3, further simplifies into

$$G_0(\omega) \approx G_0^{plateau} = \frac{1}{\beta}, \tag{30}$$

which is called the plateau approximation, and where β is the effective fraction of delayed neutrons (see Ref. 38).

Consequently, the usual MTC noise estimator that can be used in practice in the frequency range 0.1 to 1.0 Hz is defined as the H_1^{biased} estimator:

$$H_1^{biased}(\boldsymbol{r}, \omega) = \frac{1}{G_0(\omega)} \frac{CPSD_{\delta\phi/\phi_0, \delta T_m}(\boldsymbol{r}, \omega)}{APSD_{\delta T_m}(\boldsymbol{r}, \omega)} \tag{31}$$

or the H_2^{biased} estimator:

$$H_2^{biased}(\boldsymbol{r}, \omega) = \frac{1}{G_0(\omega)} \frac{APSD_{\delta\phi/\phi_0}(\boldsymbol{r}, \omega)}{CPSD_{\delta T_m, \delta\phi/\phi_0}(\boldsymbol{r}, \omega)} \tag{32}$$

depending on whether Eq. (16) or Eq. (17) is used. These estimators are biased since it is obvious from what was explained previously that they lead to the correct MTC only when very specific conditions are fulfilled. In case of a perfect correlation between the neutron noise and the local temperature noise, these two estimators give the same result since the ratio between H_1^{biased} and H_2^{biased} is simply the coherence $\gamma_{\delta\phi/\phi_0, \delta T_m}$ between the two signals:
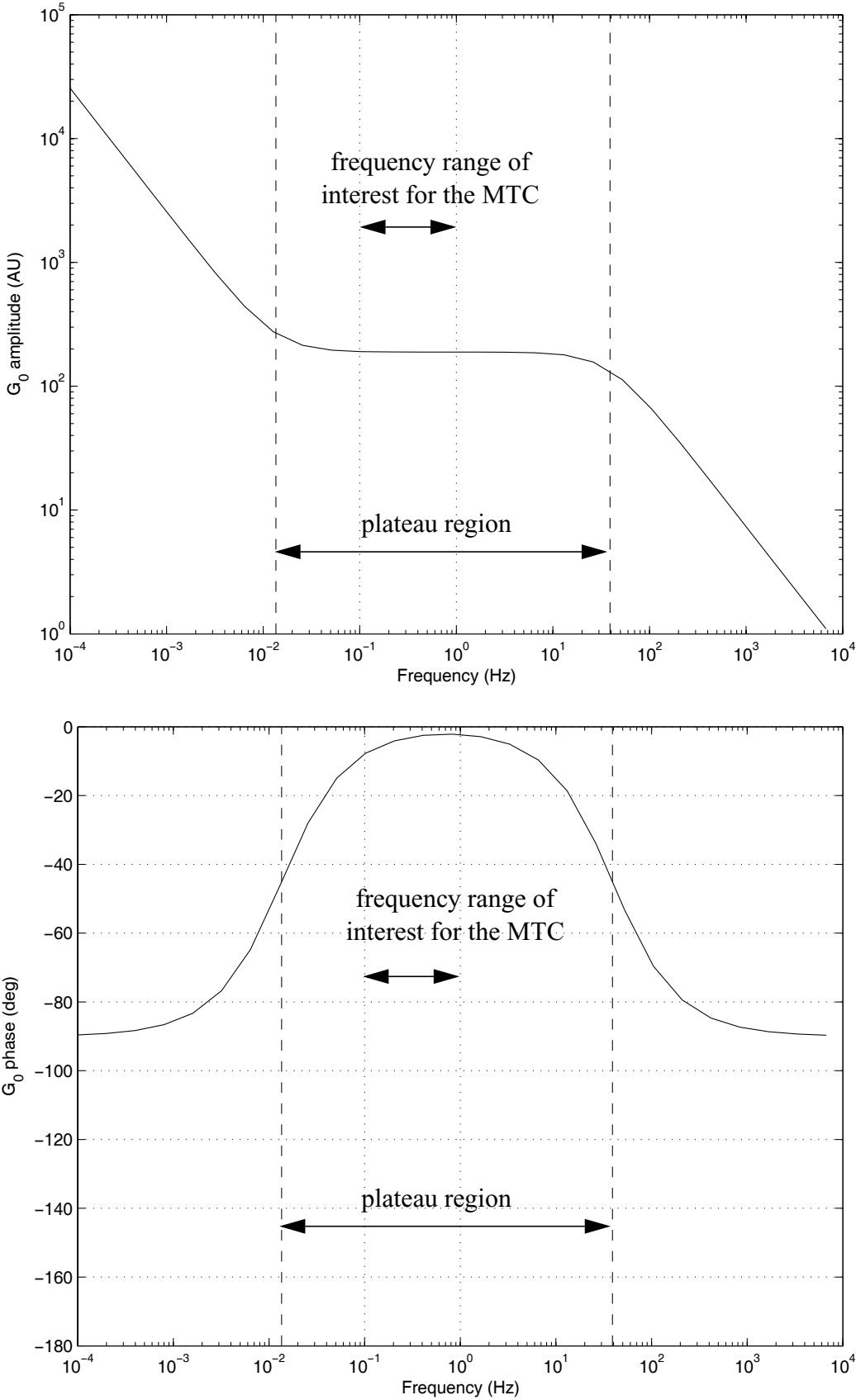
Fig. 3. The zero-power or open-loop reactor transfer function G_0 as a function of frequency for a typical PWR.

$$\frac{H_1^{biased}(\boldsymbol{r}, \omega)}{H_2^{biased}(\boldsymbol{r}, \omega)} = \gamma_{\delta\phi/\phi_0, \delta T_m}(\boldsymbol{r}, \omega) = \frac{\left|CPSD_{\delta\phi/\phi_0, \delta T_m}(\boldsymbol{r}, \omega)\right|^2}{APSD_{\delta\phi/\phi_0}(\boldsymbol{r}, \omega) \times APSD_{\delta T_m}(\boldsymbol{r}, \omega)} \tag{33}$$

When the coherence is not unity, i.e. when there is extraneous noise in the signals, H_1^{biased} is biased low and gives the lower bound of the estimate, whereas H_2^{biased} is biased high and corresponds to the upper bound of the estimate. It was considered that the H_2^{biased} estimator could lead to a better MTC estimation since the APSD of the neutron flux, measured directly within the core, is less biased than the APSD of the temperature, measured outside the core, i.e. at the core-exit (see Ref. 23). Nevertheless, in practice, the H_1^{biased} estimator will give a better estimate of the transfer function than H_2^{biased} since the relative measurement noise, i.e. extraneous noise, in the neutron signal is usually much larger than that in the temperature signal (see Ref. 39). Consequently, only the H_1^{biased} MTC noise estimator will be discussed and used in the following. It has to be emphasized that this estimator will lead to the correct MTC estimate only if the reactor response is point-kinetic, and the temperature noise is (radially) space-independent and generated outside the core.

Although it is generally assumed that the ex-core neutron detectors follow more closely the so-called point-kinetic component of the flux noise (which is directly proportional to the reactivity noise as explained previously) than the in-core neutron detectors, using ex-core neutron detectors would only give a correct MTC estimate if the average moderator temperature noise could be determined. Most importantly, the coherence between ex-core neutron detectors and core-exit thermocouples is usually very low. Using local measurements for both the temperature and the flux gives a higher coherence and a more reliable MTC value if both detectors are positioned close to each other. Therefore, the MTC is inferred in practice from the signals provided by an in-core neutron detector and a core-exit thermocouple located in the same fuel channel or in two neighbouring channels.

An MTC noise estimation, carried out by using the H_1^{biased} MTC noise estimator and measurement data from Ringhals-4 during the fuel cycle 15 at near EOC (core average burnup of 9.378 GWd/tHM), is presented in the following (see Ref. 40). The purpose of the presentation of such a measurement in this thesis is to give general characteristics of the MTC noise estimation performed according to the methodology used in all the experimental work so far. The available detectors and their location during this measurement are depicted in the following Fig. 4.

The MTC was estimated to be equal to -51 pcm/°C at the corresponding core average burnup by using SIMULATE-3 and the real core history and layout. The MTC was also estimated by noise analysis for the following detector pairs: C-T3, B-T7, D-T48, E-43, and A-T32 assuming that only detectors located in the same channel or in two neighbouring channels could lead to a correct MTC value, i.e. the coherence should be high enough between the two detectors. The noise estimations, as a function of frequency, are represented in Fig. 5. It can be noticed that only the detector pairs E-T43 and A-T32 seem to give a meaningful MTC estimate. A more thorough analysis of the coherence function of all the detector pairs revealed that E-T43 and A-T32 had a coherence much higher than any other pair. In the frequency range of interest for the MTC investigation (between 0.1 and 0.8 Hz in this case), the maximum of the magnitude of the MTC noise estimator gives approximately a value equal to 30-35 pcm/°C, i.e. roughly half of the value predicted by core calculations. Experimental investigations carried out by other research teams all
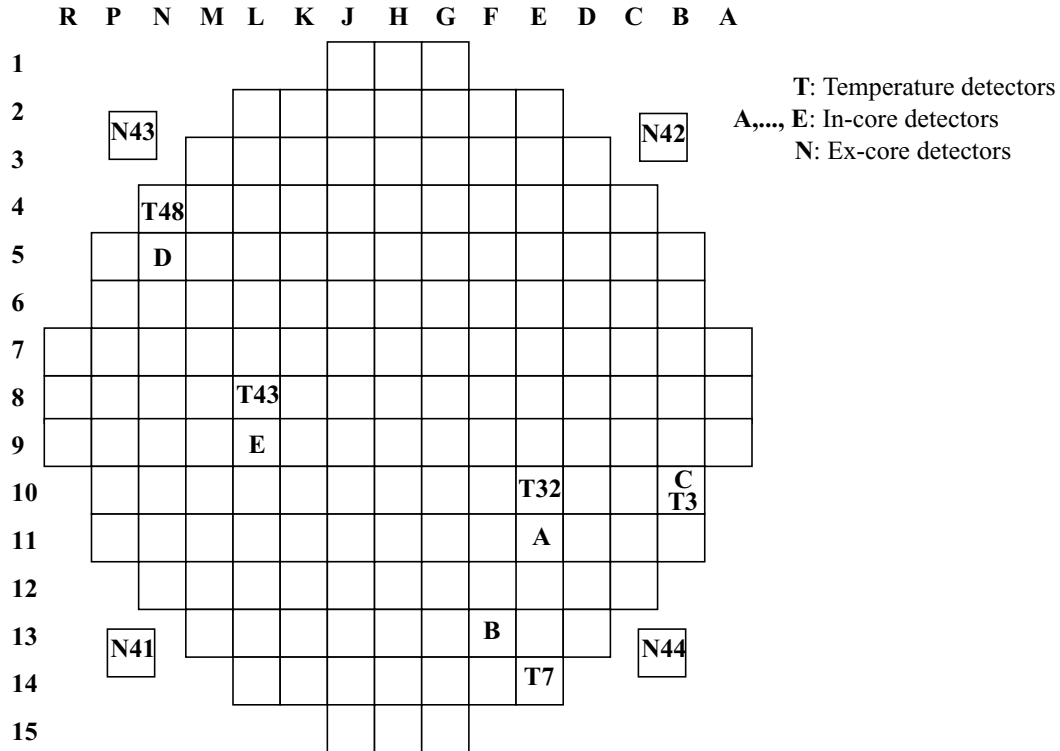
Fig. 4. Radial location of the available detectors for the EOC MTC noise measurement at Ringhals-4, cycle 15.

showed the same tendency: the MTC noise estimate is systematically underestimated by a factor of 2 to 5 (see Refs. 8, 10, 12-14, 16, 17, 19). This factor seems nevertheless to be independent of burnup, so that calibrating the MTC noise estimate to a known value at a given burnup allows determining successfully the MTC for the remaining part of the cycle (see Refs. 16 and 17). It was even noticed that the same calibration factor can be used from cycle to cycle with different fuel loading as long as one uses the same pairs of detectors for the temperature and the neutron noise. The idea of a proportionality relationship between the noise estimate and the true MTC value was also confirmed in Ref. 13, since the true MTC value was obtained from measurements by using the back propagation neural network technique with in-core neutron signals and core-exit thermocouple signals (the calibration is in fact realized during the training of the network).

2.5. Possible reasons of the inaccuracy of the MTC noise estimation investigated in the past

Many theoretical investigations have already been carried out by different research groups in order to find out and to explain the reasons why the MTC noise estimate is systematically biased low compared with its actual value. An extensive literature survey can be found in Ref. 40 regarding this matter.

The main source of concern is the fact that the neutron noise and the temperature noise are not measured at the same location, and might therefore measure different phenomena. Using spectral analysis or more specifically the CPSD between the neutron and the temperature noise prevents from including in the MTC effects that are not related to moderator temperature fluctuations. But depending on how these fluctuations are generated,
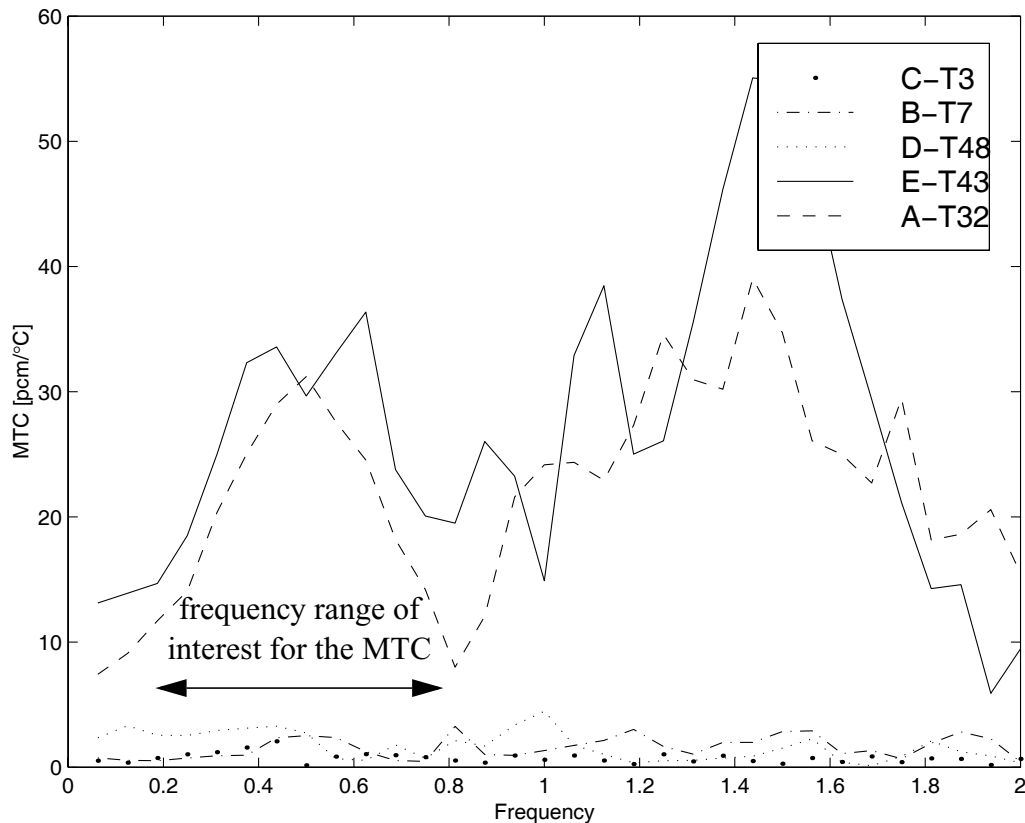
Fig. 5. EOC MTC noise measurement at Ringhals-4, cycle 15 (from Ref. 40).

the moderator temperature noise sources might be axially and non-homogeneously distributed (see Refs. 14, 15, 26-28).

Another source of concern comes from one of the basic assumptions underlying the derivation of the MTC noise estimate, namely the deviation of the reactor response from point-kinetics. It is well known that large power reactors deviate appreciably from point-kinetics compared to small research reactors for localised noise sources. But it was noticed that only a large deviation from point-kinetics could be responsible for the underestimation of the MTC (see Ref. 27).

Recently, the use of the open-loop reactor transfer function was questioned (see Ref. 29). More specifically, the effect of the Doppler coefficient and of the fuel time constant, which characterize the feedback chain in Fig. 2, was investigated. It was found that in the most usual cases, the feedback loop does not play any significant role. In any case, if it was not possible to neglect the feedback chain, the MTC estimation by noise analysis would be simply impossible since the feedback chain contains also the MTC, which is the parameter one is looking for.

In any of the previous investigations, correction factors were proposed. These are either negligible or cannot be easily estimated in practice, so that calibrating the MTC noise estimate to a known value still remains.

Finally, another effect that has not been investigated so far is the difference between the neutron detector and the thermocouples with respect to their field of view. The

neutron detectors measure a spatial response of the neutron noise due to temperature noise, while the thermocouples only measure the local behaviour of the temperature noise.

2.6. New hypothesis of the reason of the inaccuracy of the MTC noise estimation

Another hypothesis, which surprisingly was never given any attention before, is the fact that the moderator temperature noise might be spatially non-homogeneous in the core. This Section presents the results of some experimental investigation in this respect, which is reported in Paper II. It is very unlikely that there is equality between the local moderator temperature noise and the core average moderator temperature noise [see Eq. (29)]. When there is no equality, it is expected that the MTC estimated by noise analysis will deviate from its actual value, since with a spatially randomly distributed moderator temperature noise, the total reactivity effect is much smaller than in the homogeneous case. This thus leads to an MTC underestimation by the noise analysis technique. This situation is equivalent to trying to calibrate the worth of a bank of control rods where the individual rods move up and down uncorrelated, and one measures the induced neutron noise and the axial displacement of one rod only. Clearly, the evaluation of the rod bank worth, based on the assumption that all rods move coherently, will underestimate the rod bank worth.

The radially spatially incoherent and loosely coupled character of the moderator temperature noise was actually noticed in a noise measurement performed in Ringhals-2, during the fuel cycle 24 (core average burnup of 8.5 GWd/tHM) as described in Paper II. As discussed in Section 2.4, for frequencies lower than 1 Hz, the axial damping of the moderator temperature noise, assumed to be created outside the core, is negligible (the validity of this approximation will be discussed in Section 4.3). This means that the axial coupling of the temperature noise should be much higher than the one in the radial case, therefore explaining why our concern was mainly focused on the radial structure of the moderator temperature noise in the core. In this measurement, the moderator temperature noise was recorded by 3 strings containing 9 Gamma-Thermometers (GTs) located at different axial elevations. GTs, in the frequency range of interest for the MTC noise investigation by noise analysis, i.e. between 0.1 and 1.0 Hz, are working as ordinary thermocouples (this fact will be explained later in Section 4.2). As can be seen in Fig. 6, the coherence between 3 detectors in these 3 different strings at the same axial level in the frequency band 0.1 - 0.5 Hz shows that the radial temperature correlations decay fast with increasing radial distance. If the moderator temperature noise was radially homogeneous, the coherence between the different strings would be roughly independent of the separation distance between the detectors and would have values much higher, too.

Consequently, one of the approximations used to derive the MTC noise estimator H_1^{biased}, i.e. the radial homogeneity of the moderator temperature noise, is not fulfilled. One further consequence of this is the deviation of the reactor response from point-kinetics, as explained in Paper III and briefly in the following. This means that the second approximation used to derive the MTC noise estimator H_1^{biased} is not valid either. In a one-group model relying on the diffusion approximation, the flux noise induced by a spatially randomly distributed noise source (here expressed as a macroscopic absorption cross-section noise source - see Section 3.1 for further detail) is expressed in a homogeneous reactor as:

$$\nabla^2 \delta\phi(\boldsymbol{r}, \omega) + B^2(\omega)\delta\phi(\boldsymbol{r}, \omega) = \frac{\delta\Sigma_a(\boldsymbol{r}, \omega)\phi_0(\boldsymbol{r})}{D_0} \tag{34}$$
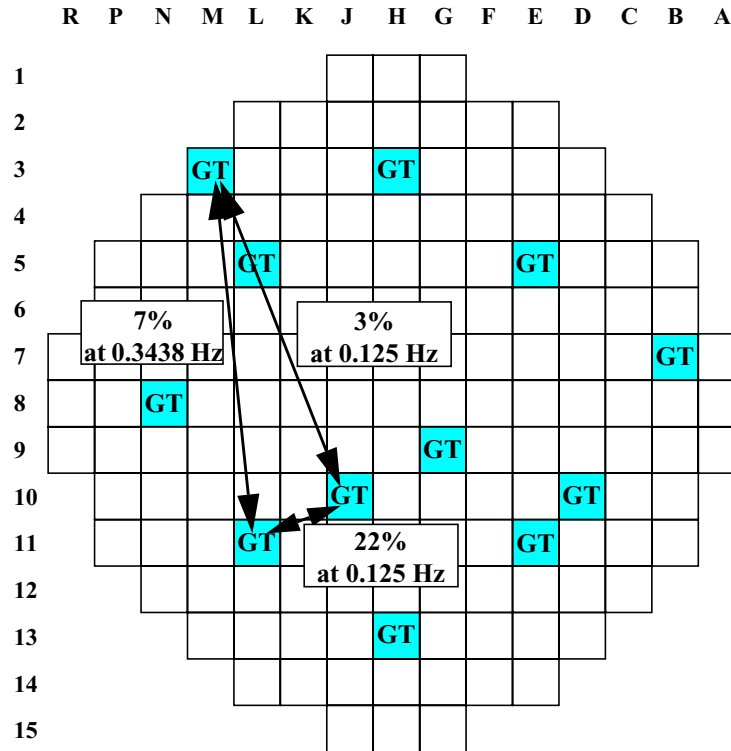
Fig. 6. Maximum of the coherence between GT signals measured in Ringhals-2, cycle 24 (detectors located in the lower part of the core).

with

$$B^2(\omega) = B_0^2\left(1 - \frac{1}{\rho_\infty G_0(\omega)}\right) \tag{35}$$

All the other terms have their usual meaning. If one expands the flux noise with respect to spatial static eigenfunctions $\phi_k(r)$, which are solutions of the generic following equation:

$$\nabla^2\phi_k(r) + B_k^2\phi_k(r) = 0, \tag{36}$$

one can show that:

$$\delta\phi(r, \omega) = \delta\phi^{pk}(r, \omega) + \sum_{k > 0} A_k(\omega)\phi_k(r) \tag{37}$$

where

$$A_k(\omega) = \frac{\int \delta\Sigma_a(r, \omega)\phi_0(r)\phi_k(r)dr}{D_0[B^2(\omega) - B_k^2]\int \phi_k^2(r)dr} \tag{38}$$

From Eqs. (37) and (38), it is easy to see that only the case of a homogeneous structure of the temperature noise, i.e. $\delta\Sigma_a(r, \omega) = \delta\Sigma_a(\omega), \forall r$, gives a point-kinetic response of the reactor, since the orthogonality of the eigenfunctions $\phi_k(r)$ implies that $A_k(\omega) = 0, k \geq 1$.

Therefore, the following Chapter tries to assess in a theoretical manner the implications of these two hypotheses on the MTC estimation by noise analysis. For that purpose, the moderator temperature noise sources will be assumed to be spatially randomly distributed. Using the local moderator temperature noise instead of the core average one and assuming a point-kinetic response of the reactor will be studied separately, in order to see if any of these approximations can be valid or not.

Chapter 3

Theoretical investigation of the MTC noise estimate

This Chapter investigates the effect of a non-homogeneous distribution of the moderator temperature noise on the MTC estimation by noise analysis. This will be done by calculating the neutron noise, induced by randomly distributed temperature fluctuations. Indeed the temperature noise is supposed to be randomly distributed both in time and in space. A white noise is considered for the time behaviour, whereas the spatial properties of the noise are not defined in a deterministic manner, rather through their statistical properties. Basically, the cross-correlation function of the temperature noise between two points is assumed to be described by a shape function representing the spatial distribution of the noise source strength throughout the core and an exponential decay function, which simply states that for increasing distances the correlation between two points decreases. From these driving noise sources, the neutron noise, and hence also the MTC derived according to the H_1^{biased} estimator can be calculated and compared to its actual value.

Two cases are investigated in the following: the case of a homogeneous bare 1-D reactor in the 1-group diffusion approximation, and the case of a heterogeneous reflected 2-D reactor in the 2-group diffusion approximation. In both cases, the spatial coordinate represents the radial position in the core. Neglecting the axial dependence is equivalent to assume that no noise source is created between the in-core neutron detector and the core-exit thermocouple. This means that the temperature noise is only travelling upwards from one detector to the other. This transport time only affects the phase of the CPSD between neutron and temperature noise, without changing its magnitude. Therefore the magnitude of the H_1^{biased} estimator, which should correspond to the MTC magnitude in the ideal case, does not depend on the separation distance between the two detectors, i.e between the neutron detector and the thermocouple. It has to be emphasized that this approximation probably does not hold in reality, but the effect of radially inhomogeneous temperature noise sources will be shown to give much stronger effects. The axial effect is actually investigated experimentally in Section 4.3. The moderator temperature noise is therefore referenced as inlet temperature noise, since the temperature noise is generated outside the core.

The 1-D homogeneous model relies on the so-called 1-group Green's function, which allows estimating the neutron noise induced by a localised noise source and which can be derived analytically. On the contrary, the 2-D heterogeneous model relies on the so-called 2-D 2-group discretised Green's function, which also estimates the neutron noise induced by a localised noise source and which cannot be derived analytically, only via a numerical model. The derivation of this numerical model and its benchmark are briefly presented in this thesis. Some other practical applications of this model are also touched upon, such as the possibility of locating a noise source from Local Power Range Monitors (LPRMs) signals in Boiling Water Reactors (BWRs), or the explanation of a spatially non-homogeneous Decay Ratio (DR) in BWRs too.

3.1. MTC noise estimation in 1-D homogeneous systems

This Section presents the theoretical investigation reported in Paper III. In this Paper, a 1-dimensional homogeneous bare reactor is considered. The space variable is thus assumed to describe the radial position in the core. The moderator temperature noise may have many different sources (inlet temperature noise, coolant velocity noise, heat generation noise, heat transfer noise, etc.). At any rate, the moderator temperature noise causes density fluctuations, which in turn cause removal macroscopic cross-section noise. Nevertheless, the change induced by the removal cross-section change can only be accounted for in a 2-group representation. On the other hand, a shift of the thermal spectrum of the moderator and thus that of the thermal neutrons will lead to increased absorption in the fuel. This phenomenon can be modelled in a one-group model and this is what we shall use here. Therefore, one can assume that in a 1-group model the space-dependent change of the macroscopic absorption cross-section of the homogeneous mixture fuel + moderator is directly proportional to the space-dependent temperature change via a space-independent coefficient K as follows:

$$\delta\Sigma_a(r, t) = \frac{1}{K} \times \delta T_m(r, t) \tag{39}$$

The approximation of proportionality between the change of the macroscopic absorption cross-section and the change of the moderator temperature has been verified via SIMU-LATE-3 calculations performed at different core-inlet temperatures on a 0-D system, i.e equivalent to a homogeneous reactor. Further, $\delta\Sigma_a(r, t)$ is supposed to be stationary and ergodic in time with a zero expected value:

$$\langle \delta\Sigma_a(r, t) \rangle = 0 \quad \forall \ r, t \tag{40}$$

As mentioned previously, the temperature noise, or more precisely, the corresponding fluctuation of the absorption cross-section is not known in a deterministic way. It rather can be defined in a statistical sense through its temporal and spatial cross-correlation function. For this correlation function, we shall assume the simplest non-trivial model which is described with a few parameters. This model was introduced for cross-section fluctuations by Williams (see Ref. 41) and then was developed further to describe spatial density correlations of two-phase flow (see Ref. 42), and later it was also applied to fusion plasma transport (see Ref. 43). It is assumed that the correlations can be factorised into a temporal component and a spatial component. The temporal part is given by $\delta(\tau)$ (a white noise in the frequency range of interest for the MTC investigations, i.e. typically from 0.1 to 1.0 Hz). The spatial part, given by $R(r, r')$, is further factorised into a fast decaying function of the distance of the two points, representing the decay of correlations, and a much slower varying shape (amplitude) function $\sigma^2(\hat{r})$ of the midpoint $\hat{r}$ of the two spatial coordinates, representing the space-dependent strength of the noise source [it can be related to the Root-Mean-Square (RMS) or the variance of the noise source]. This can be summarized by the following formula:

$$CCF_{\delta\Sigma_a}(r, r', \tau) = \langle \delta\Sigma_a(r, t)\delta\Sigma_a(r', t+\tau) \rangle = \delta(\tau)R(r, r') = \delta(\tau)\sigma^2(\hat{r})e^{-\frac{|r-r'|}{l}} \tag{41}$$

with:

$$\hat{r} \equiv \frac{r + r'}{2} \tag{42}$$

In this model, l is called the correlation length of the temperature fluctuations and is supposed to be space independent. The correlation length indicates roughly the maximum distance between two points that can be considered as having a coherent behaviour. For greater distances, their behaviour can be assumed to be uncorrelated. On the opposite, if the correlation length is infinite and the shape function $\sigma^2(\hat{r})$ space-independent, i.e. $R(r, r') = \text{constant}$, the noise sources are spatially homogeneous throughout the core.

In the frequency domain, only the spatial part of the CCF is retained due to the white noise characteristic of the temporal part, so that one obtains:

$$CPSD_{\delta\Sigma_a}(r, r', \omega) = R(r, r') = \sigma^2(\hat{r})e^{-\frac{|r - r'|}{l}} \tag{43}$$

If $r = r'$, the previous equation leads to the APSD:

$$APSD_{\delta\Sigma_a}(r, \omega) = \sigma^2(\hat{r}) \tag{44}$$

This is why the spatial part of the CPSD has been factorised into an amplitude function and a fast decaying exponential function: the amplitude function corresponds to the APSD, describing the strength of the temperature fluctuation throughout the core and can even be fitted to real plant measurements, whereas the fast decaying exponential function only represents how two points are correlated to each other and can be fitted to actual data simply by scaling the correlation length l. As will be seen later on, scaling this correlation length is completely identical to calibrating the MTC noise estimation to a known value of the MTC.

Further, it is easy to show that in case of a homogeneous distribution of the moderator temperature noise throughout the core, i.e. for an infinite correlation length, the MTC is simply given in the first-order 1-group perturbation theory by:

$$MTC = -\frac{1}{K\nu\Sigma_{f,0}} \tag{45}$$

By definition, this is the exact MTC of the system, irrespective of the space-dependence of the temperature fluctuations. The MTC given by Eq. (45) will thus be called in the following the "reference" MTC with which the noise estimate will be compared. So far, nothing was said about how the MTC is determined experimentally. If the temperature fluctuations are not homogeneous in space, one needs to define the average temperature variation, i.e. the weighting function $w(r)$ in Eq. (5) that leads to the same MTC. Starting with the reactivity noise in a 1-group homogeneous model relying on the first-order 1-group perturbation theory as

$$\delta\rho(\omega) = \frac{-\int \delta\Sigma_a(r, \omega)\phi_0^2(r)dr}{\nu\Sigma_{f,0}\int \phi_0^2(r)dr}, \tag{46}$$

Eqs. (39) and (2) allow writing the core average moderator temperature noise as follows:

$$\delta T_m^{ave}(\omega) = \frac{1}{MTC} \times \frac{-\int \delta T_m(r, \omega)\phi_0^2(r)dr}{K\nu\Sigma_{f,0}\int \phi_0^2(r)dr} \tag{47}$$

Using the reference MTC value given by Eq. (45), one gets

$$\delta T_m^{ave}(\omega) = \frac{\int \delta T_m(r, \omega)\phi_0^2(r)dr}{\int \phi_0^2(r)dr}, \tag{48}$$

so that the weighting function that has to be used to calculate the average moderator temperature noise is the square of the static flux, i.e.

$$w(r) = \phi_0^2(r) \tag{49}$$

Regarding the MTC estimation by noise analysis, it has first to be pointed out that the ideal MTC noise estimator, i.e. the noise estimator that provides the correct MTC given by Eq. (45), is known. Using the H_1 noise estimator will lead to the following ideal MTC noise estimator in the frequency band 0.1 - 1.0 Hz:

$$H_1^{ideal} = \frac{1}{G_0(\omega)} \frac{CPSD_{\delta\phi^{pk}/\phi_0, \delta T_m^{ave}}(\omega)}{APSD_{\delta T_m^{ave}}(\omega)} \tag{50}$$

This ideal MTC noise estimator has to be compared to the one used in all the experimental work so far, i.e. the one given by Eq. (31). As pointed out previously, if the moderator temperature noise is not radially spatially homogeneous in the core, the noise estimator in Eq. (31) will deviate from the reference MTC value given by either Eq. (45) or Eq. (50). This deviation is due to the fact that the local moderator temperature noise is different from the core average one, and due to the fact that the reactor does not consequently behave in a point-kinetic way, as explained in Section 2.6. Both effects can be studied separately in a theoretical manner by calculating the space-dependent neutron noise through the use of the Green's function, which is the solution of the following Equation:

$$\nabla_r^2 G(r, r', \omega) + B^2(\omega)G(r, r', \omega) = \delta(r - r') \tag{51}$$

with $B^2(\omega)$ defined by Eq. (35), so that one has:

$$\delta\phi(r, \omega) = \frac{1}{D_0}\int G(r, r', \omega)\delta\Sigma_a(r', \omega)\phi_0(r')dr' \tag{52}$$

If one first assumes that one could measure the core average moderator temperature noise, one can study the effect of the approximation of the reactivity noise by $\delta\rho^{approx}(\omega)$ defined by Eq. (28). Replacing the exact reactivity noise $\delta\rho(\omega)$ by its approximated expression $\delta\rho^{approx}(\omega)$ in the ideal MTC noise estimator given by Eq. (50)

leads to a biased MTC noise estimator that can be numerically estimated in the present case via the use of the Green's function technique as follows:
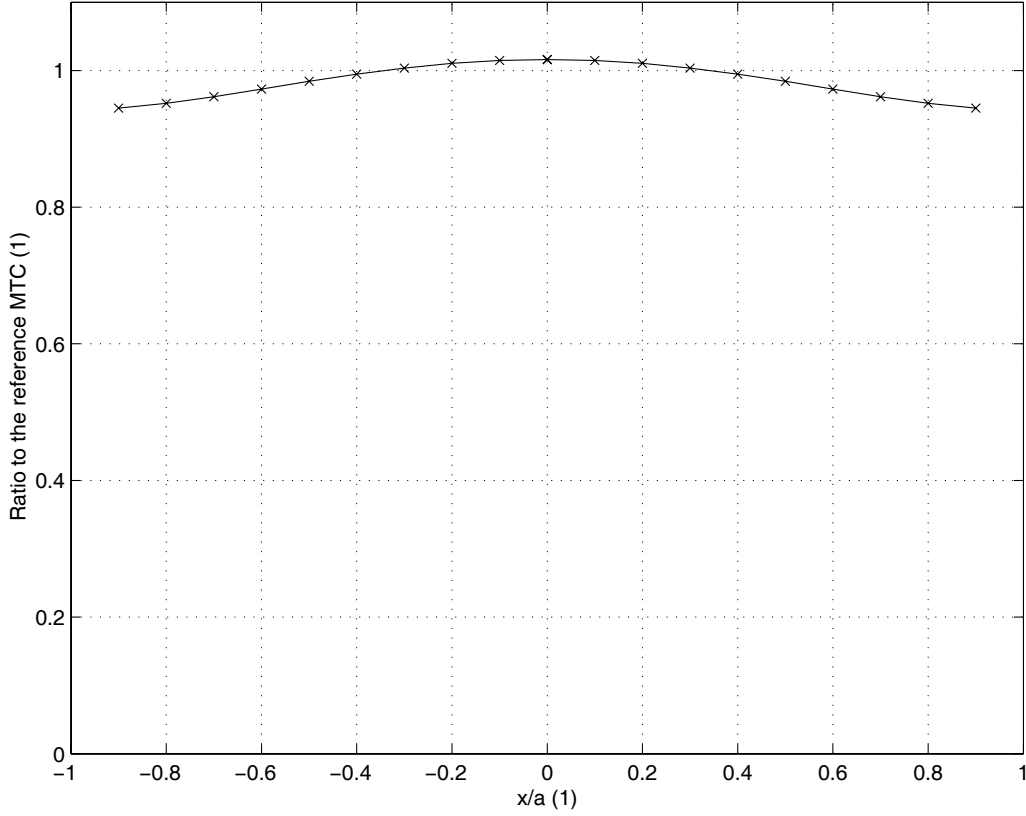
$$\tilde{H}_1^{biased}(\boldsymbol{r}, \omega) = \frac{1}{G_0(\omega)} \frac{CPSD_{\delta\phi/\phi_0, \delta T_m^{ave}}(\boldsymbol{r}, \omega)}{APSD_{\delta T_m^{ave}}(\omega)} \tag{53}$$

$$= \frac{\int w(\boldsymbol{r}')d\boldsymbol{r}'}{G_0(\omega)\phi_0(\boldsymbol{r})KD_0} \frac{\iint G(\boldsymbol{r}, \boldsymbol{r}', \omega)CPSD_{\delta\Sigma_a}(\boldsymbol{r}', \boldsymbol{r}'', \omega)\phi_0(\boldsymbol{r}')w(\boldsymbol{r}'')d\boldsymbol{r}'d\boldsymbol{r}''}{\iint CPSD_{\delta\Sigma_a}(\boldsymbol{r}', \boldsymbol{r}'', \omega)w(\boldsymbol{r}')w(\boldsymbol{r}'')d\boldsymbol{r}'d\boldsymbol{r}''}$$

Due to the above approximation, the $\tilde{H}_1^{biased}$ MTC noise estimator is both space- and frequency-dependent. This new $\tilde{H}_1^{biased}$ MTC noise estimator is compared to the actual MTC value in Fig. 7, where due to the symmetry of the 1-D system the origin of the abscissa represents the core centre, and the parameter a is the core radius. In this Figure and in Eq. (53), the space variable $\boldsymbol{r}$ represents the radial location of the neutron noise measurement only, since for the temperature noise measurement the core average is supposed to be taken. The calculations were performed with a set of cross-sections/point-kinetic parameters typical of a PWR core at near EOC conditions. The chosen frequency for the calculations was 1 Hz. Several correlation lengths and shape functions were studied, but only the case of a correlation length of 15 cm (to be compared to the core radius a equal to 150 cm) and the following shape function are reported in this thesis:

$$\sigma(\hat{x}) = \frac{1}{1 - \left(\dfrac{\hat{x}}{a + a/5}\right)} \tag{54}$$

This shape function is based on experimental evidence which shows that, contrary to the expectations, there are observed cases when the temperature noise is somewhat larger close to the core boundary than at the core centre (see Ref. 44). The results were nevertheless found to be completely independent of the choice of the shape function. As can be seen in Fig. 7, the $\tilde{H}_1^{biased}$ MTC noise estimator gives a relatively good estimation of the actual value of the MTC, wherever the neutron noise is measured in the core. Consequently, the deviation of the reactor response from point-kinetics does not play a significant role regarding the MTC noise estimation. This also means that the neutron noise induced by a spatially random noise source with short correlation length is nearly point-kinetic even in a large core. This is in contrast to localised perturbations such as a vibrating rod or a local thermalhydraulic instability (see Section 3.2). One can therefore conclude that approximating the reactivity noise $\delta\rho(\omega)$ by $\delta\rho^{approx}(\boldsymbol{r}, \omega)$ does not induce any appreciable discrepancy in the MTC noise estimation.

In the next step, one can study the effect of using the local moderator temperature noise instead of the core average moderator temperature noise. This is accomplished by using in Eq. (53) $\delta T_m(\boldsymbol{r}, \omega)$ instead of $\delta T_m^{ave}(\omega)$. This leads to the usual MTC noise estimator H_1^{biased} given by Eq. (31). As before, this biased MTC noise estimator can be numerically estimated in the present case from Eqs. (51) and (52) via the use of the Green's function technique as follows:
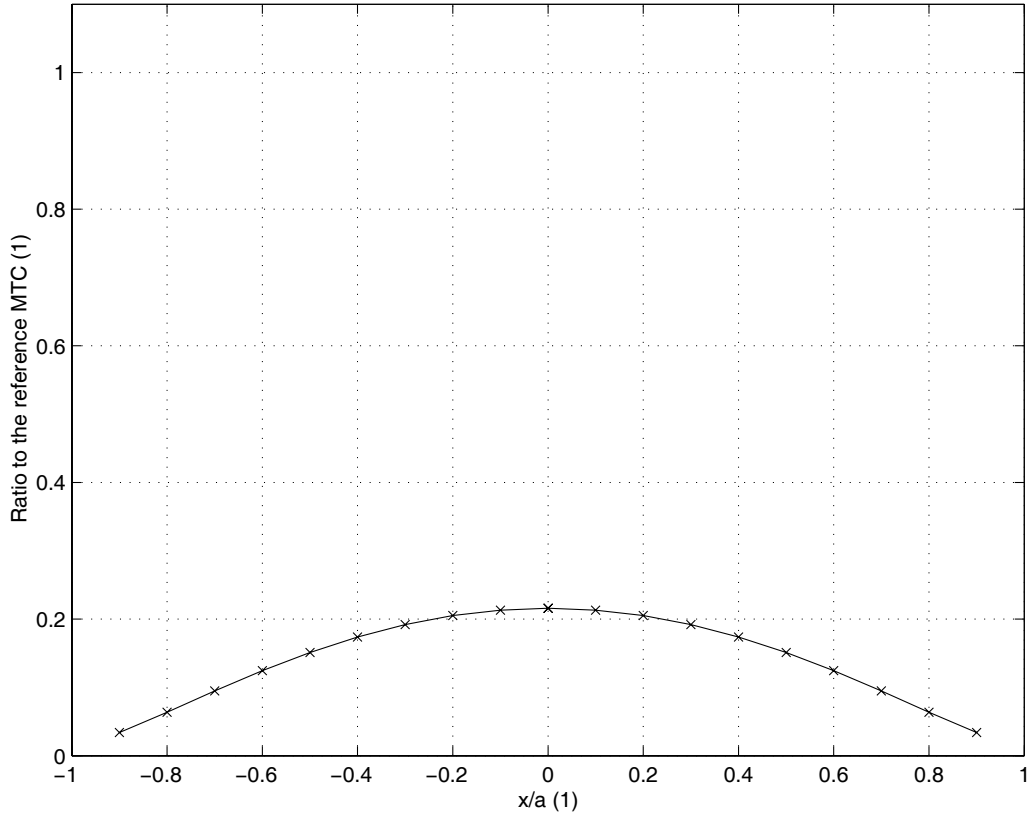
Fig. 7. Ratio between the MTC noise estimate $\tilde{H}_1^{biased}$ and its actual value (due to the deviation from point-kinetics of the reactor response solely) in a 1-D 1-group homogeneous model.

$$
\begin{aligned}
H_1^{biased}(\boldsymbol{r}, \omega) &= \frac{1}{G_0(\omega)} \frac{CPSD_{\delta\phi/\phi_0,\, \delta T_m}(\boldsymbol{r}, \omega)}{APSD_{\delta T_m}(\boldsymbol{r}, \omega)} \\[2mm]
&= \frac{1}{G_0(\omega)\phi_0(\boldsymbol{r})KD_0} \frac{\int G(\boldsymbol{r}, \boldsymbol{r}', \omega) CPSD_{\delta\Sigma_a}(\boldsymbol{r}', \boldsymbol{r}, \omega)\phi_0(\boldsymbol{r}')d\boldsymbol{r}'}{APSD_{\delta\Sigma_a}(\boldsymbol{r}, \omega)}
\end{aligned}
\tag{55}
$$

This MTC noise estimator is, as before, both space- and frequency-dependent. Using the same parameters as for the $\tilde{H}_1^{biased}$ MTC noise estimator, i.e. set of cross-sections, point-kinetic parameters, correlation length, and shape function, the H_1^{biased} MTC noise estimator can be compared to the actual MTC value. Such a comparison was done at a frequency of 1 Hz and is represented in Fig. 8. In this Figure and in Eq. (55), the space variable $\boldsymbol{r}$ represents the radial location of the measurement of both the neutron noise and the moderator temperature noise. As can be seen in this Figure, there is a very strong underestimation of the actual value of the MTC, and this underestimation is noticeably space-dependent. Close to the core centre, the MTC is roughly underestimated by a factor 5, which corresponds to what was noticed in the experimental investigations relying of the usual H_1^{biased} MTC noise estimator.

Our calculations show that using the usual H_1^{biased} estimator (where only the local temperature noise is measured) will lead to a significantly underestimated value of the MTC in the case of short correlation lengths. Furthermore, the deviation increases with the deviation of the perturbation from homogeneous, i.e. with decreasing correlation lengths.

Fig. 8. Ratio between the MTC noise estimate H_1^{biased} and its actual value (due to the deviation from point-kinetics of the reactor response *and* the approximation of the average temperature noise by the local one) in a 1-D 1-group homogeneous model.

The above result shows that the reason of the underestimation of the MTC by noise analysis in the traditional way lies with the overestimation of the reactivity effect in case of heterogeneous noise sources. With spatially inhomogeneous temperature fluctuations, the reactivity effect will be smaller than it is assumed in the traditional method, since the ratio between the true reactivity effect and the assumed one (infinite correlation, i.e. homogeneous temperature noise) will be smaller than unity, as shown in Paper III. This fact would lead to an underestimation of the MTC even if the reactor response was point-kinetic. Furthermore, with decreasing correlation length, the true reactivity effect becomes a smaller and smaller fraction of the one assumed in the traditional MTC formula.

Nevertheless, the above described reactivity underestimation is space-independent. Since, as explained above, the deviation from point-kinetics is space-dependent but weak, another strongly space-dependent effect takes place in the MTC underestimation when the H_1^{biased} MTC noise estimator is used. This last effect is due to the underestimation of the cross-correlation between the neutron noise and the temperature noise when the temperature fluctuations are non-homogeneous. As before, the ratio between the true CPSD and the assumed one (infinite correlation length, i.e homogeneous temperature noise), assuming a point-kinetic behaviour of the reactor, is partly smaller than unity, and partly it is space-dependent, as can be seen in Paper III.

To summarise, it is the strong space-dependence of the temperature fluctuations that is the reason for the significant underestimation of the MTC by the usual noise estimator H_1^{biased}. One way of alleviating this discrepancy is to use integral methods

regarding the temperature noise, i.e. to use the average temperature as defined by Eqs. (5) and (49). Integral methods have so far been suggested only for the extraction of the reactivity from the flux measurements (see Ref. 28). These latter play however a much less significant role, as was shown by the numerical analysis presented in the foregoing.

The above model used to estimate the different MTC noise estimators H_1^{biased} and $\tilde{H}_1^{biased}$ was based on a 1-group 1-D bare homogeneous reactor relying on the diffusion approximation. The extension of this model to a 2-group 2-D reflected heterogeneous reactor requires being able to calculate the 2-D 2-group discretised Green's function of the reactor. This can only be done via a fully numeric model, which is presented in the next Section.

3.2. Development of a 2-D 2-group neutron noise simulator

This Section deals with the development of a so-called 2-D 2-group neutron noise simulator, which is reported in detail in Paper IV. The neutron noise simulator is able to calculate the spatial distribution of the neutron noise induced by any given spatially distributed or localised noise sources. Several types of noise sources can be simultaneously investigated, i.e. a perturbation of the macroscopic absorption cross-section (fast and/or thermal), and/or a perturbation of the macroscopic removal cross-section, and/or a perturbation of the macroscopic fission cross-section (fast and/or thermal). The neutron noise simulator is able to model the noise sources of the "absorber of variable strength" type (the so-called reactor oscillator). The simulator cannot model noise sources of the "moving absorber" type. Furthermore, the calculations are directly performed in the frequency domain, which is equivalent to define complex cross-sections. The main advantage of using the frequency domain instead of the time domain is twofold. First, it is common practice to use the Fourier transform of the measured signals. Second, because of the Fourier transform, the time derivative in the equations is eliminated. There is consequently no need to properly choose a time discretisation which allows taking into account the phenomena one wants to study, and for which the neutron noise has to be evaluated at each time step. In the frequency domain instead, the calculation needs only to be performed once, assuming that the frequency of interest is known. If not, scanning a frequency range does not appear to be a big burden.

If the noise source is a point source of unit strength, the neutron noise simulator actually estimates the 2-D 2-group discretised Green's function $G_{\delta XS \to i}(r, r', \omega)$, the index $i = 1, 2$ representing the fast and thermal groups, respectively. More specifically, these transfer functions give the flux noise $\delta\phi_i$ in r and at a frequency $f = \omega/2\pi$ induced by a unit cross-section noise source $\delta XS = 1$ located at r' at the same frequency. Such a 2-D 2-group discretised Green's function will be required in Section 3.3 in order to calculate the different MTC noise estimators.

In the linear two-group diffusion theory, the neutron noise in the frequency domain can be expressed as a solution of the following matrix equation:

$$[\bar{\bar{D}}(r)\nabla^2 + \bar{\bar{\Sigma}}(r, \omega)] \times \begin{bmatrix} \delta\phi_1(r, \omega) \\ \delta\phi_2(r, \omega) \end{bmatrix}$$

$$= \bar{\phi}_{rem}(r)\delta\Sigma_{rem}(r, \omega) + \bar{\bar{\phi}}_a(r)\begin{bmatrix} \delta\Sigma_{a, 1}(r, \omega) \\ \delta\Sigma_{a, 2}(r, \omega) \end{bmatrix} + \bar{\bar{\phi}}_f(r, \omega)\begin{bmatrix} \delta\nu\Sigma_{f, 1}(r, \omega) \\ \delta\nu\Sigma_{f, 2}(r, \omega) \end{bmatrix} \quad (56)$$

where the different matrices/vectors are given as:

$$\bar{\bar{D}}(r) = \begin{bmatrix} D_1(r) & 0 \\ 0 & D_2(r) \end{bmatrix} \quad (57)$$

$$\bar{\bar{\Sigma}}(r, \omega) = \begin{bmatrix} -\Sigma_1(r, \omega) & \nu\Sigma_{f, 2}(r, \omega) \\ \Sigma_{rem}(r) & -\Sigma_{a, 2}(r, \omega) \end{bmatrix} \quad (58)$$

$$\bar{\phi}_{rem}(r) = \begin{bmatrix} \phi_1(r) \\ -\phi_1(r) \end{bmatrix} \quad (59)$$

$$\bar{\bar{\phi}}_a(r) = \begin{bmatrix} \phi_1(r) & 0 \\ 0 & \phi_2(r) \end{bmatrix} \quad (60)$$

$$\bar{\bar{\phi}}_f(r, \omega) = \begin{bmatrix} -\phi_1(r)\left(1 - \dfrac{i\omega\beta}{i\omega + \lambda}\right) & -\phi_2(r)\left(1 - \dfrac{i\omega\beta}{i\omega + \lambda}\right) \\ 0 & 0 \end{bmatrix} \quad (61)$$

and the different coefficients are defined as:

$$\Sigma_1(r, \omega) = \Sigma_{a, 1}(r) + \frac{i\omega}{v_1} + \Sigma_{rem}(r) - \nu\Sigma_{f, 1}(r)\left(1 - \frac{i\omega\beta}{i\omega + \lambda}\right) \quad (62)$$

$$\nu\Sigma_{f, 2}(r, \omega) = \nu\Sigma_{f, 2}(r)\left(1 - \frac{i\omega\beta}{i\omega + \lambda}\right) \quad (63)$$

$$\Sigma_{a, 2}(r, \omega) = \Sigma_{a, 2}(r) + \frac{i\omega}{v_2} \quad (64)$$

The spatial discretisation of Eq. (56) was carried out according to the finite differences scheme, and more precisely the so-called "box-scheme" (see Ref. 45). The equation giving the neutron noise, i.e. Eq. (56), is a non-homogeneous equation (source problem). Consequently, the discretised form of the matrix $[\bar{D}(r)\nabla^2 + \bar{\Sigma}(r, \omega)]$ can be directly inverted, and the flux noise calculated accordingly.

The only data required to use the 2-D 2-group neutron noise simulator are the 2-D 2-group material constants, the point-kinetic parameters of the reactor, and the 2-D 2-group static fluxes. All these data can be obtained by any commercial 3-D static core simulator. Then they have to be properly homogenized from 3-D to 2-D, i.e. by preserving the reaction rates. In order to get a 2-D system equivalent to the 3-D system, the leakage rate of the 3-D system in the axial direction has to be added to the absorption cross-section in the 2-D system, both in the fast and thermal group. Furthermore, although the static fluxes and the corresponding eigenvalue are directly available after homogenization from the 3-D static core simulator, they have to be obtained from a 2-D 2-group static core simulator that is compatible with the neutron noise simulator. Otherwise, using the results of the 3-D static core simulator, i.e. results that were calculated using a discretisation scheme possibly different from the one used in the neutron noise simulator would be equivalent to make the 2-D system non-critical. A 2-D 2-group static core simulator compatible with the 2-D 2-group neutron noise simulator was thus developed[3] and is briefly presented in Paper V (and more in detail in Ref. 46). For the sake of simplicity, this static simulator is not reported in this thesis.

The neutron noise simulator was then benchmarked against analytical solutions. All the different types of noise sources that the neutron noise simulator is able to handle were investigated. The accuracy of the neutron noise simulator was found to be completely independent of the type of noise source investigated. Therefore, in the following, only the case of a macroscopic removal cross-section noise is presented. The layout of the core in this benchmark is representative of the Swedish BWR Forsmark-1. The core was assumed to be a two-region system (core + reflector), in which each region was spatially homogeneous. The material constants, the point-kinetic parameters and the flux data were obtained from a generic General Electric BWR/6. At that time, the flux was not recalculated with a discretisation scheme compatible with the finite difference scheme, so that the macroscopic cross-sections were slightly adjusted in each node in order to fulfil the balance equations in each node with respect to the finite difference scheme (this adjustment was only noticeable close to and in the reflector region). The noise source was located in the middle of the core. Since the core was thus roughly homogeneous, an analytical solution could be estimated and was used as a reference solution. The results of this benchmark are presented in Figs. 9 and 10. Since the noise source was located in the middle of the core, the results are rotational-invariant around the *z*-axis crossing the core centre. Therefore only the radial dependence of the fluxes is plotted in the Figures.

It can be noticed that the agreement between the numerical solution and the analytical one is very good for both the magnitude and the phase of the induced neutron noise. Since the noise simulator calculates a spatially-averaged flux noise over each node, the analytical solution was also averaged over each node, so that both solutions could be directly compared. The first point of the numerical solution (from the core centre) therefore represents the flux noise in the node where the noise source is located. The analytical solution gives a different solution in this node, since in the analytical solution the noise

---

3. An adjoint core simulator (see Ref. 46) was also developed together with the static core simulator, so that one has a full package of codes compatible with each other: a 2-D 2-group static core simulator, a 2-D 2-group adjoint core simulator, and a 2-D 2-group neutron noise simulator. All these simulators rely on the diffusion approximation. As will be seen in Section 3.3, the adjoint flux is actually required to calculate the weighting function $w(r)$ used for the estimation of the core average moderator temperature noise δT_m^{ave} in the 2-group diffusion theory.
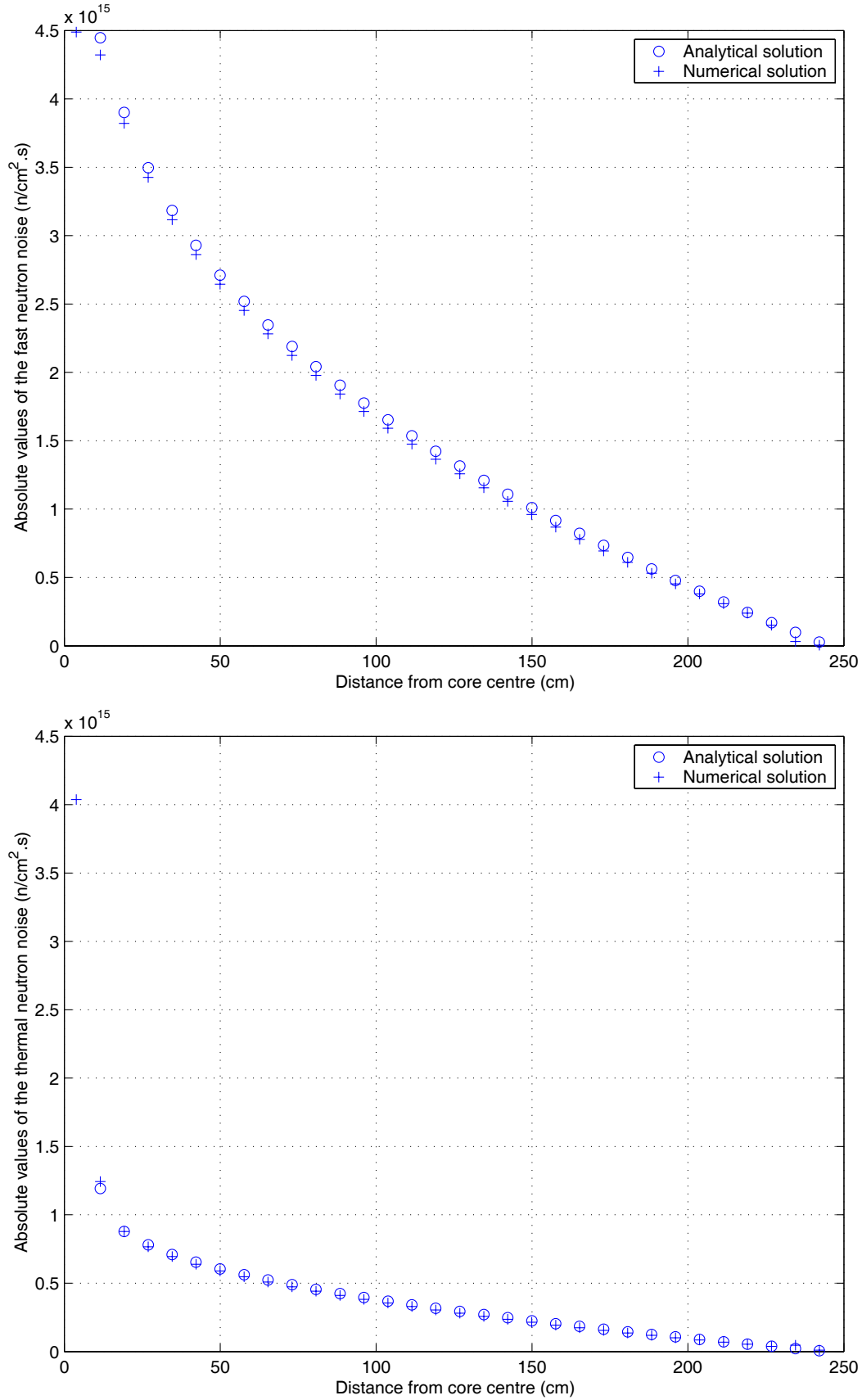
Fig. 9. Comparison between the amplitude of the flux noise calculations of the neutron noise simulator and the reference solution in the benchmark case; the comparison of the fast noise is given in the upper figure, and the comparison of the thermal noise is given in the lower one.
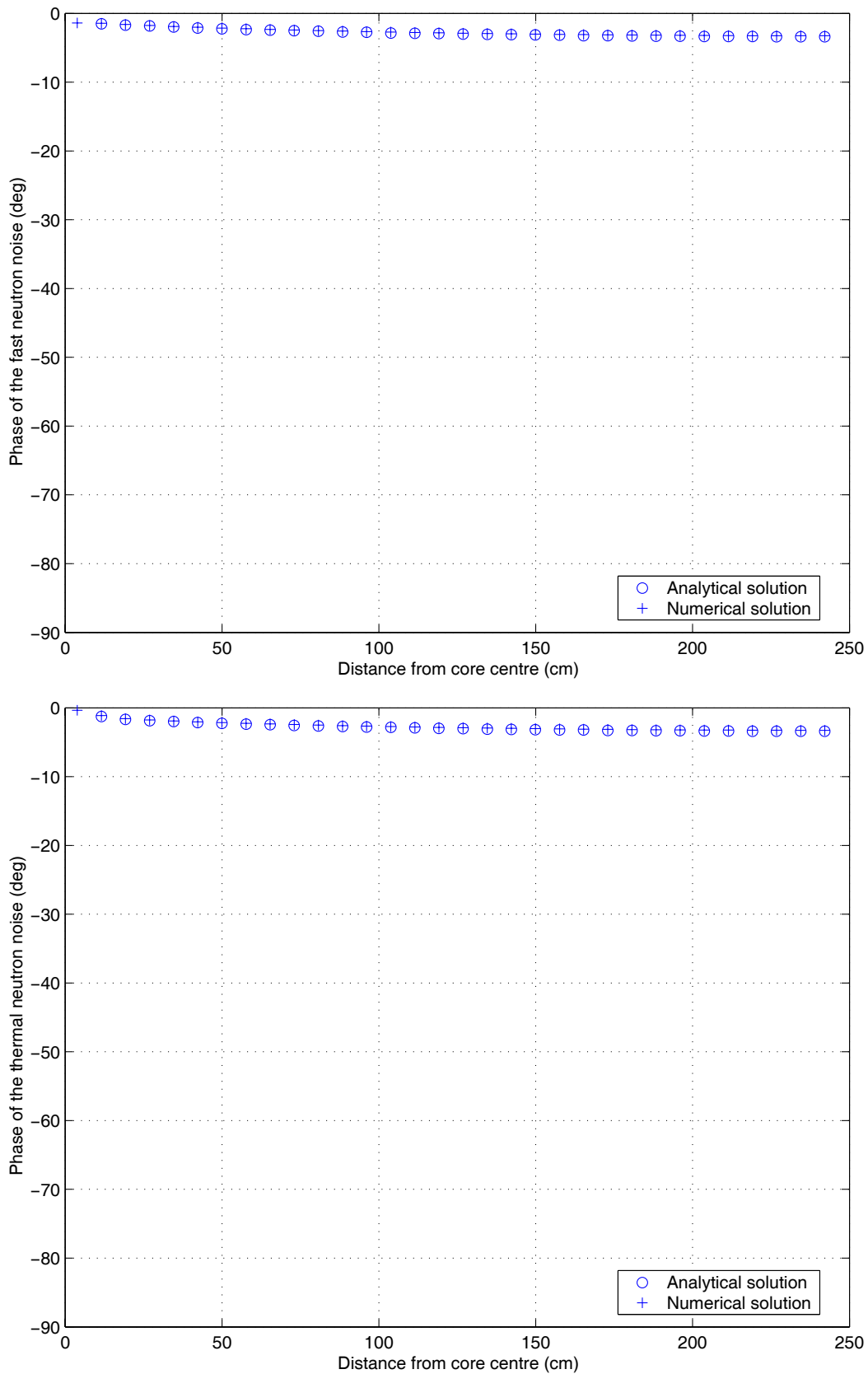
Fig. 10. Comparison between the phase of the flux noise calculations of the neutron noise simulator and the reference solution in the benchmark case; the comparison of the fast noise is given in the upper figure, and the comparison of the thermal noise is given in the lower one.

source is assumed to be a point-source located at the core centre, whereas in the numerical solution the noise source is spatially distributed over the nodes directly neighbouring the centre of the core. The noise source in the numerical case was nevertheless corrected in order to have the same importance as in the analytical case. Consequently, the first point of the analytical solution was systematically disregarded in the benchmark of the neutron noise simulator. Finally, due to the cross-section adjustment, which is noticeable only around the reflector region, the accuracy close to the reflector deteriorates slightly, but remains still acceptable. Another reason explaining this discrepancy lies with the fact that the analytical solution does not take any reflector into account.

This neutron noise simulator was then applied to two practical cases, both related to the Forsmark-1 channel instability event recorded in January 1997. Before applying the simulator to the MTC, these two cases will be reported here briefly. In 1996, during the start-up tests of the Forsmark-1 BWR (Sweden) for the fuel cycle 16, local instabilities were detected at reduced power and reduced core-flow. Although BWRs are known to become less stable at reduced power/core flow, the appearance of this instability event could not be understood and was not predicted by the stability calculations. The corresponding operating point in the power/flow map was therefore avoided. In January 1997, at approximately Middle Of Cycle (MOC) conditions (core average burnup of 22.887 GWd/tHM), stability measurements were carried out in order to study the local instability discovered previously. The core was thus brought to 63.3% of power and to a core flow of 4298 kg/s (41% of the nominal core flow). Again local instability conditions were encountered, at a frequency of roughly 0.5 Hz. An examination of all the LPRM signals available during this measurement campaign clearly shows a peak in the APSD of the LPRMs at this given frequency.

During this stability measurement, the lower plane of the core was rather well equipped with LPRMs, where 27 of the 36 available detector strings were actually recorded at a sampling frequency of 12.5 Hz. One parameter that is relevant for characterising the stability of BWRs is the Decay Ratio (DR), which is defined as the ratio between two consecutive maxima of the ACF of the normalized neutron density, or alternatively two consecutive maxima of the Impulse Response Function (IRF) as calculated by using an Auto-Regressive Moving-Average model (ARMA) or an Auto-Regressive model (AR) to fit the behaviour of the system. Although the DR was always assumed to be a 0-D parameter of the core, i.e. independent of the position where the DR is estimated in the core, the Forsmark-1 measurement revealed that the DR was actually strongly space-dependent, as can be seen on the following Fig. 11 (see Ref. 47). This Figure shows that one half of the core exhibits a DR close to instability (higher than 0.9) and the other half has a DR close to 0.6. A closer look at the phase of the measured flux noise indicated that the neutron noise was driven by a local noise source, similar to the effect of an absorber of variable strength (reactor oscillator), rather than a moving absorber, such as a vibrating control rod. This means that the neutron noise simulator presented previously is able to handle this kind of noise sources. The simulator was first used in an inverting task, namely the determination of the location of the noise source in the core from the LPRM signals, and then the simulator was used to reconstruct the space-dependence of the DR.

3.2.1 Application to an anomaly localisation algorithm

This Section presents a localisation algorithm reported in Paper IV. The localisation algorithm, i.e. the inverting task, was actually developed by Karlsson and Pázsit
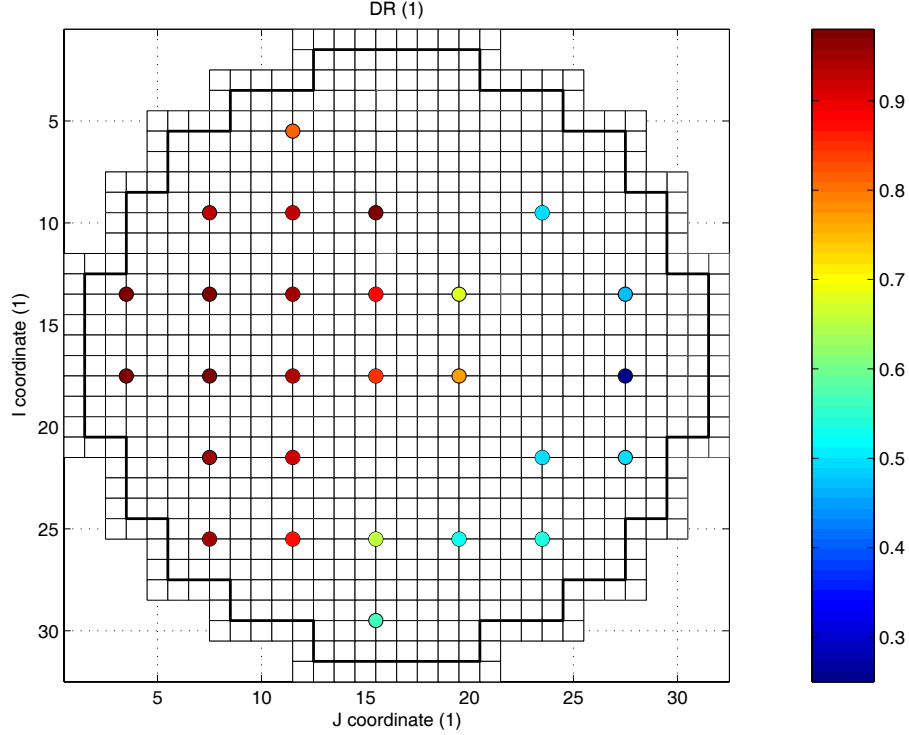
Fig. 11. Measured radial space-dependence of the Decay Ratio in Forsmark-1 (derived from Ref. 48).

(see Ref. 49), where the authors assumed that the reactor was homogeneous in order to be able to calculate analytically the Green's function in the 1-group diffusion approximation. The same algorithm was used in this thesis, but this time with a realistic set of data describing the heterogeneous Forsmark-1 core. The 2-D discretised Green's function was calculated via the neutron noise simulator in the 2-group diffusion approximation. The localisation procedure gives the location of the noise source (if any) existing in the core, not its strength. Actually the localisation is achieved by searching of the minimum of the following function

$$\Delta(\boldsymbol{r}) = \sum_{A, B, C, D} \Delta_{A, B, C, D}^{2}(\boldsymbol{r}) \tag{65}$$

with

$$\Delta_{A, B, C, D}(\boldsymbol{r}) = \frac{CPSD_{A, B}(\omega)}{CPSD_{C, D}(\omega)} - \frac{G_{\delta\Sigma_{rem}\to 2}(\boldsymbol{r}_A, \boldsymbol{r}, \omega) \times G_{\delta\Sigma_{rem}\to 2}^{*}(\boldsymbol{r}_B, \boldsymbol{r}, \omega)}{G_{\delta\Sigma_{rem}\to 2}(\boldsymbol{r}_C, \boldsymbol{r}, \omega) \times G_{\delta\Sigma_{rem}\to 2}^{*}(\boldsymbol{r}_D, \boldsymbol{r}, \omega)}, \tag{66}$$

where (A, B, C, D) represents a quadruplet of neutron detectors. For this investigation, the transfer function between the removal cross-section noise and the thermal flux noise was used. Simply speaking, the $\Delta(\boldsymbol{r})$ function compares the detector readings, or more precisely their CPSDs, to the corresponding calculated flux noise induced by a noise source located in $\boldsymbol{r}$. The ratio between the CPSDs is taken so that the noise source strength, which is unknown, can be eliminated. The function $\Delta(\boldsymbol{r})$ has to be calculated for every possible location $\boldsymbol{r}$ of the noise source in the core. Therefore, the minimum of this function should correspond to the location of the actual noise source in the core. Such a localisation algo-

rithm was tested on simulated data and was demonstrated to give the correct location of the noise source regardless of how many detectors are used, of the actual location of the noise source in the core, and of the contamination of the detector readings by extraneous noise as long as there is one single noise source present in the core. The localisation algorithm was designed for locating one single noise source, and therefore if two or more noise sources are present at the same time in the core, the location of the noise source returned by the algorithm is biased. Nevertheless, simulations carried out with two noise sources revealed that choosing a set of detectors positioned close to one of the noise sources allows detecting successfully the corresponding one, as long as the two noise sources are not close enough. An attempt to locate the noise source in the Forsmark-1 channel instability event was then carried out. The $\Delta(r)$ function is plotted in Fig. 12, where the detectors are positioned via crosses ('X'), the white ones indicating the detectors used in the localisation, the black ones the detectors not used. A difference compared to the localisation reported in Paper IV is the set of data used during the localisation procedure. In Paper IV, the material constants and point-kinetic parameters required by the noise simulator correspond to a generic model of a General Electric BWR/6 reactor (equilibrium core at EOC). Furthermore, the fuel nodes were spatially averaged and so were the reflector nodes. Although a two-region reactor was thus obtained, the cross-sections were adjusted in each node in order to have a critical system, as mentioned previously. In the calculations shown in Figs. 12 and 13, realistic data corresponding to the Forsmark-1 core were obtained, the static flux and the corresponding eigenvalue were recalculated with the 2-D 2-group static core simulator, and the localisation algorithm was used with this new set of data. Therefore, the results presented in Fig. 12 differ slightly from the ones presented in Paper IV.

During the core outage following this instability event, a fuel assembly was found to be unseated close to the location pointed out by the localisation algorithm (see Refs. 50 and 51). Consequently, a noise source of variable strength seems to be responsible for the local instability encountered in Forsmark-1. The localised character of the noise source is in favour of a channel thermal-hydraulic instability, i.e. a self-sustained Density Wave Oscillation (DWO) (see Ref. 52). As pointed out in Ref. 49, when a fuel element is unseated, some of the coolant flow bypasses the fuel element and this might render the channel thermal-hydraulically unstable. Nevertheless, choosing a different set of detectors gives results which are sometimes different, i.e. the noise source is not always located at a position close to the unseated fuel element. This suggests that there are probably two (or maybe even more) noise sources located inside the core. This can be seen in Fig. 13, where the $\Delta(r)$ function obtained with all the available detectors is plotted. As pointed out previously, limiting the number of detectors to a region where a noise source is suspected to be located, as was done in Fig. 12, allows successfully locating this specific noise source, as long as the other noise sources are not in the same vicinity. This is why the region around the unseated fuel element was pointed out by the localisation algorithm. Taking more detectors into account than the one used in Fig. 12 is equivalent to take the effect of several other possible noise sources into account, whereas the algorithm has been designed for a single noise source. It is also worth mentioning that only the region pointed out by the localisation algorithm was visually inspected during the core outage, i.e. other unseated fuel elements might have remained undetected.
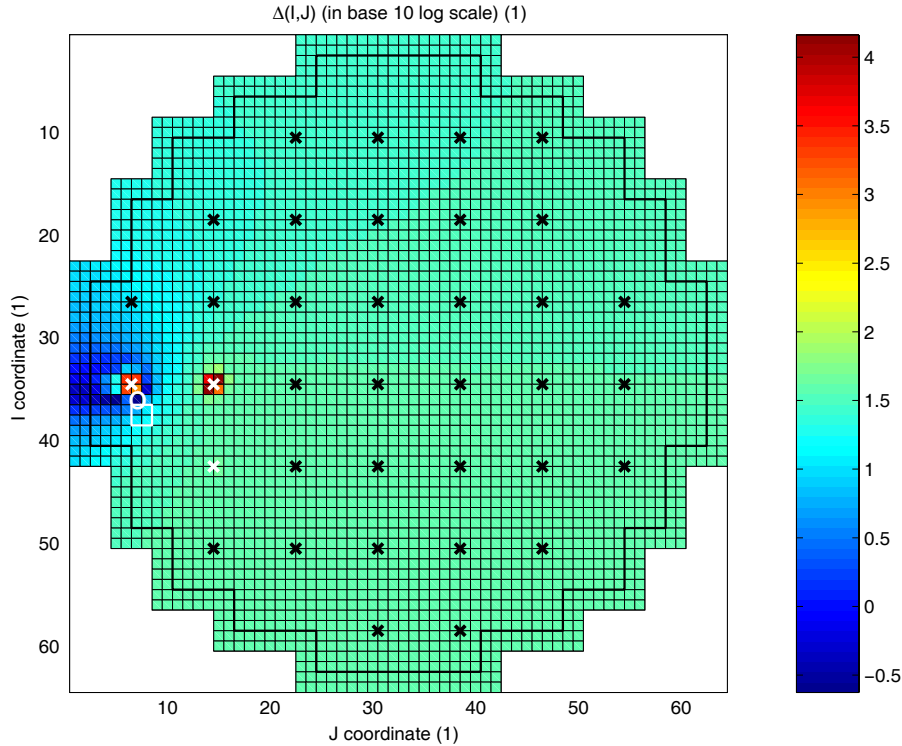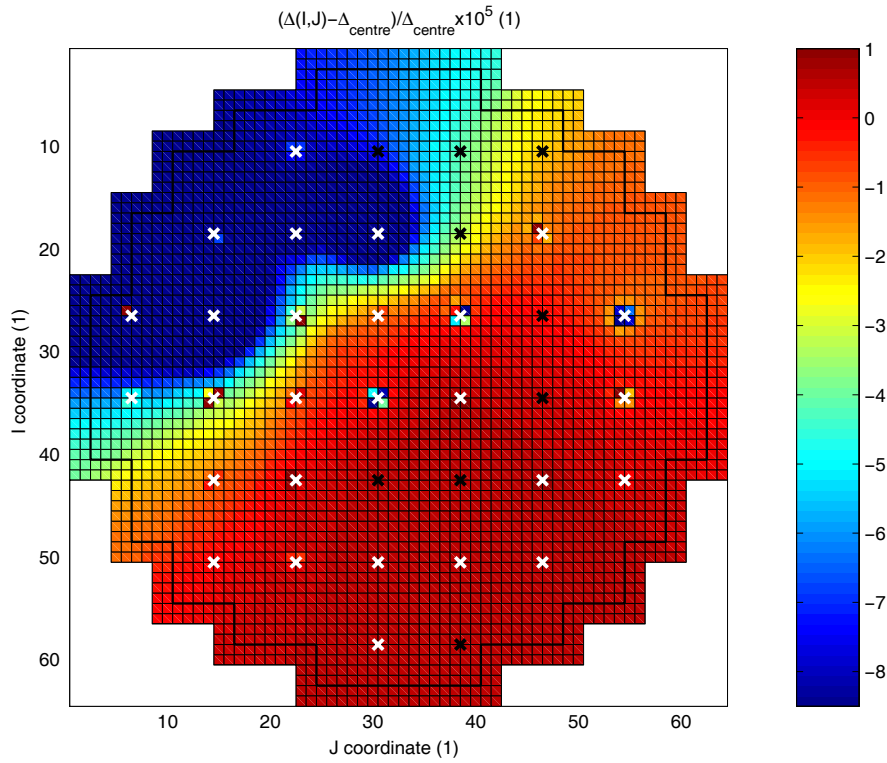
Fig. 12. Result of the localisation algorithm in the Forsmark-1 case (local instability event) when using a reduced set of detectors; the unseated fuel element is marked with a square, and the noise source identified by the localisation algorithm with a circle.

3.2.2 Application to the explanation of the strong space-dependence of the Decay Ratio in Forsmark-1

This Section presents another application of the neutron noise simulator reported in Paper V, namely the explanation of the space-dependence of the DR in the Forsmark-1 channel instability event, via a phenomenological model developed by Pázsit (see Ref. 53). This model was developed to simulate the discontinuous character of the DR when the operating point was changed smoothly on the power-flow map, and relied on dual oscillations according to which the DR was calculated. From this model, it was easy to understand that the DR becomes space-dependent if more than one type or source of instability is present in the core. Otherwise, the DR would be space-independent. The same model was used in Paper V to study exclusively the space-dependence of the DR. The only difference with the previous investigation is the type or source of instability investigated. In Pázsit's paper, only the case of a regional (out-of-phase) type of oscillations coexisting with a global (in-phase) type of oscillations was investigated. This model was extended to any type or source of instability existing simultaneously in the core. In this thesis, only the case of two local noise sources is presented. Nevertheless, in Paper V, the case of a local noise source coexisting with a global type of oscillations is also reported.

If one writes the flux fluctuations as a sum of the contributions of the two local noise sources i ($i = 1, 2$), each of them being factorized into a temporal part only and spatial part only, one can show that the DR, defined in that case as the ratio of the first and the second maxima of the ACF, is given by:

Fig. 13. Result of the localisation algorithm in the Forsmark-1 case (local instability event) when using all the available detectors. Two noise sources (represented by the dark-blue regions) seem to be present in the core.

$$DR(\boldsymbol{r}) = \sum_{i=1}^{2} a_i(\boldsymbol{r}) \cdot DR_i \tag{67}$$

with

$$a_i(\boldsymbol{r}) = \cfrac{1}{1 + C \cdot \cfrac{\left| G_{\delta\Sigma_{rem} \to 1}(\boldsymbol{r}, \boldsymbol{r}_{0,j}, \omega_0) + G_{\delta\Sigma_{rem} \to 2}(\boldsymbol{r}, \boldsymbol{r}_{0,j}, \omega_0) \right|}{\left| G_{\delta\Sigma_{rem} \to 1}(\boldsymbol{r}, \boldsymbol{r}_{0,i}, \omega_0) + G_{\delta\Sigma_{rem} \to 2}(\boldsymbol{r}, \boldsymbol{r}_{0,i}, \omega_0) \right|} \cdot \cfrac{\ln(DR_i)}{\ln(DR_j)}}, \ i \neq j \tag{68}$$

This expression was obtained assuming that each local noise source i had the same resonance frequency ω_0 but different stability properties, i.e. DRs. Furthermore, it was supposed that the CPSD between the two noise sources was negligible, and that the DR of any of the two noise sources was larger than 0.4. The coefficient C represents the ratio between the strength of the noise sources, and was chosen so that the DR calculated throughout the core matched the measured DR given in Fig. 11. The results of the simulation are presented in Fig. 14. A local noise source with a DR of 0.99 was located at the position pointed out by the noise source localisation algorithm reported previously. Another local noise source with a DR of 0.4 was positioned on the opposite side from the other noise source. As can be seen in the Figure, the DR calculated by using the phenomenological model given by Eqs. (67) and (68) is strongly spatially dependent, and reproduces rather well the behaviour of the measured DR in Forsmark-1. The reason of the

sharp boundary between the two stability regions is the fast spatial decay of the amplitude of the local oscillations.
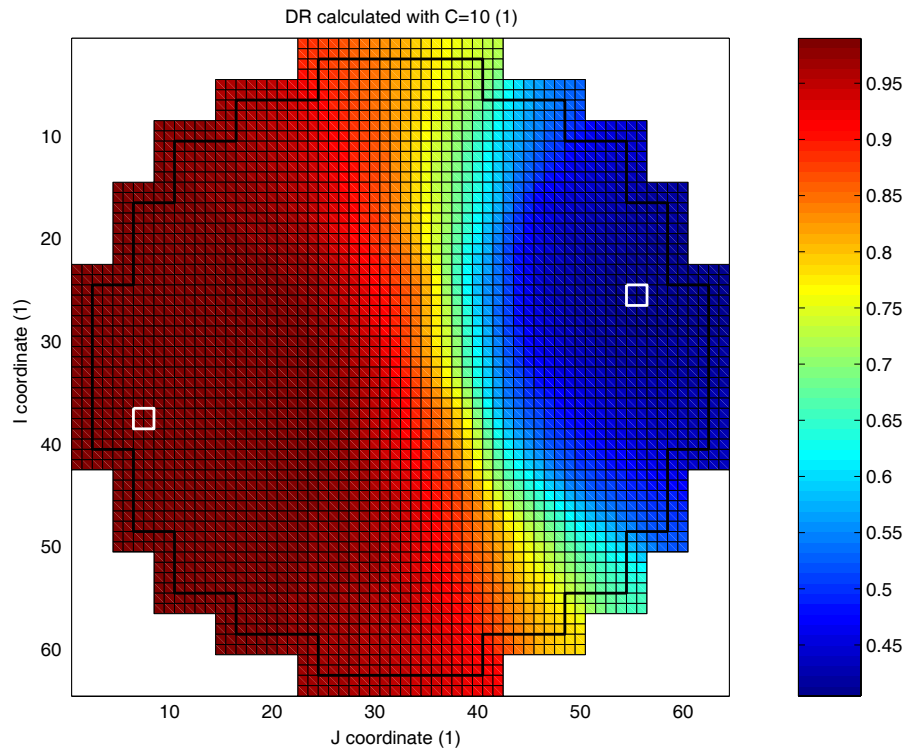
Fig. 14. Simulated radial space-dependence of the Decay Ratio in Forsmark-1 in case of two local noise sources (the white squares represent the location of the local noise sources).

Consequently, the 2-D 2-group neutron noise simulator seems to work properly since it was benchmarked successfully, and was checked against the real case of the Forsmark-1 channel instability event. In this respect, the localisation algorithm relying on the neutron noise simulator was able to find one noise source located close to a fuel assembly that was discovered to be unseated and that was responsible for the reactor instability. The fact that the DR was not spatially homogeneous in the core suggests, according to the phenomenological model based on the neutron noise simulator, that at least two types or sources of instability were present in the core. In the thesis, only the case of two local noise sources was presented, but one might consider also a local noise source coexisting with a global (in-phase) type of oscillations. The neutron noise simulator can therefore be used for other applications, and this is precisely what will be investigated in the next Section. More specifically, the MTC noise estimators presented in Section 3.1 will be estimated numerically in the 2-D 2-group diffusion approximation via the use of the neutron noise simulator.

3.3. MTC noise estimation in 2-D heterogeneous systems

This Section presents the theoretical investigation reported in Paper VI. In this Paper, a 2-dimensional heterogeneous reflected reactor is considered. The space variable is thus assumed to describe the radial position in the core. As in Section 3.1, we will consider that there is proportionality between the fluctuations of the moderator temperature and the fluctuations of the macroscopic cross-sections. In the thesis, the cases of the removal

macroscopic cross-section noise and the thermal absorption macroscopic cross-section noise are considered. This can be written in a generic form as follows:

$$\delta XS(\boldsymbol{r}, t) \;=\; \frac{1}{K} \times \delta T_m(\boldsymbol{r}, t) \tag{69}$$

As before, $\delta XS(\boldsymbol{r}, t)$ is supposed to be stationary and ergodic in time with a zero expected value:

$$\langle \delta XS(\boldsymbol{r}, t) \rangle \;=\; 0 \qquad \forall \; \boldsymbol{r}, t \tag{70}$$

Furthermore, the space-dependent noise sources are defined directly from their statistical properties, as in the 1-D case:

$$CPSD_{\delta XS}(\boldsymbol{r}, \boldsymbol{r}', \omega) \;=\; R(\boldsymbol{r}, \boldsymbol{r}') \;=\; \sigma^2(\hat{\boldsymbol{r}}) e^{-\frac{|\boldsymbol{r} - \boldsymbol{r}'|}{l}} \tag{71}$$

It is also easy to show that in case of a homogeneous distribution of the moderator temperature noise throughout the core, i.e. for an infinite correlation length, the MTC is simply given in the first-order 2-group perturbation theory by:

$$MTC \;=\; \frac{1}{K} \frac{\int w_{\delta XS}(\boldsymbol{r}) d\boldsymbol{r}}{\int [\nu\Sigma_{f, 1 \to 1}(\boldsymbol{r})\phi_1^+(\boldsymbol{r})\phi_1(\boldsymbol{r}) + \nu\Sigma_{f, 2 \to 1}(\boldsymbol{r})\phi_1^+(\boldsymbol{r})\phi_2(\boldsymbol{r})] d\boldsymbol{r}} \tag{72}$$

where $w_{\delta XS}(\boldsymbol{r})$ is a weighting function which depends on the noise source type. For a macroscopic removal cross-section noise source, one gets the so-called W3 weighting function (see Ref. 62 and Paper VIII):

$$w_{\delta\Sigma_{rem}}(\boldsymbol{r}) \;=\; w_3(\boldsymbol{r}) \;=\; -[\phi_1^+(\boldsymbol{r})\phi_1(\boldsymbol{r}) - \phi_2^+(\boldsymbol{r})\phi_1(\boldsymbol{r})] \tag{73}$$

For a macroscopic thermal absorption cross-section noise source, one gets the so-called W4 weighting function (see Ref. 62 and Paper VIII):

$$w_{\delta\Sigma_{a, 1}}(\boldsymbol{r}) \;=\; w_4(\boldsymbol{r}) \;=\; -\phi_2^+(\boldsymbol{r})\phi_2(\boldsymbol{r}) \tag{74}$$

Due to the heterogeneous character of the core, the static and adjoint fluxes cannot be calculated analytically, but only via the 2-D 2-group static and adjoint core simulators mentioned previously and which are compatible with the 2-D 2-group neutron noise simulator. By definition, the MTC given by Eq. (72) represents the actual and reference MTC value of the system, irrespective of the space dependence of the temperature fluctuations. If the temperature fluctuations are not homogeneous in space, one needs to define the average temperature variation, i.e. the weighting function $w(\boldsymbol{r})$ in Eq. (5) that leads to the same MTC. Starting with the reactivity noise in a 2-group heterogeneous model relying on the first-order 2-group perturbation theory as

$$\delta\rho(\omega) \;=\; \frac{\int \delta XS(\pmb{r}, \omega) w_{\delta XS}(\pmb{r}) d\pmb{r}}{\int [\nu\Sigma_{f,\,1\to 1}(\pmb{r})\phi_1^+(\pmb{r})\phi_1(\pmb{r}) + \nu\Sigma_{f,\,2\to 1}(\pmb{r})\phi_1^+(\pmb{r})\phi_2(\pmb{r})]d\pmb{r}} \;, \tag{75}$$

Eqs. (69) and (2) allow writing the core average moderator temperature noise as follows:

$$\delta T_m^{ave}(\omega) \;=\; \frac{1}{MTC} \times \frac{\int \delta T_m(\pmb{r}, \omega) w_{\delta XS}(\pmb{r}) d\pmb{r}}{K\int [\nu\Sigma_{f,\,1\to 1}(\pmb{r})\phi_1^+(\pmb{r})\phi_1(\pmb{r}) + \nu\Sigma_{f,\,2\to 1}(\pmb{r})\phi_1^+(\pmb{r})\phi_2(\pmb{r})]d\pmb{r}} \tag{76}$$

Using the reference MTC value given by Eq. (72), one gets

$$\delta T_m^{ave}(\omega) \;=\; \frac{\int \delta T_m(\pmb{r}, \omega) w_{\delta XS}(\pmb{r}) d\pmb{r}}{\int w_{\delta XS}(\pmb{r}) d\pmb{r}} \;, \tag{77}$$

so that the weighting function that has to be used to calculate the average moderator temperature noise is:

$$w(\pmb{r}) \;=\; w_{\delta XS}(\pmb{r}) \tag{78}$$

More precisely, the weighting function $w(\pmb{r})$ should be equal to the one given by Eq. (73) or Eq. (74) depending on which type of noise source one considers:

$$w(\pmb{r}) \;=\; w_3(\pmb{r}) \text{ or } w_4(\pmb{r}) \tag{79}$$

Regarding the MTC estimation by noise analysis, it has first to be pointed out that the ideal MTC noise estimator, i.e. the noise estimator that should give the correct MTC, is also known in the 2-D 2-group heterogeneous case and is given in the frequency band 0.1 - 1.0 Hz by Eq. (50), which leads to the correct value of the MTC given by Eqs. (72)-(74). This ideal MTC noise estimator has to be compared to the one used in all the experimental work so far, i.e. the one given by Eq. (31). As pointed out previously, if the moderator temperature noise is not radially spatially homogeneous in the core, the noise estimator in Eq. (31) will deviate from the reference MTC value given by both Eqs. (72)-(74) and (50). This deviation is due to the fact that the local moderator temperature noise is different from the core average one, and due to the fact that the reactor does not consequently behave in a point-kinetic way, as explained in Section 2.6. Both effects can be studied separately in a theoretical manner by calculating the space-dependent neutron noise through the use of the Green's function, which is derived in a 2-D model from Eq. (56) and which can be written as:

$$[\bar{\bar{D}}(\pmb{r})\nabla^2 + \bar{\bar{\Sigma}}(\pmb{r}, \omega)] \times \begin{bmatrix} G_{\delta\Sigma_{rem}\to 1}(\pmb{r}, \pmb{r}', \omega) \\ G_{\delta\Sigma_{rem}\to 2}(\pmb{r}, \pmb{r}', \omega) \end{bmatrix} \;=\; \bar{\phi}_{rem}(\pmb{r})\delta(\pmb{r} - \pmb{r}') \tag{80}$$

or

$$[\bar{\bar{D}}(r)\nabla^2 + \bar{\bar{\Sigma}}(r,\omega)] \times \begin{bmatrix} G_{\delta\Sigma_{a,2}\to 1}(r,r',\omega) \\ G_{\delta\Sigma_{a,2}\to 2}(r,r',\omega) \end{bmatrix} = \bar{\bar{\phi}}_a(r) \begin{bmatrix} 0 \\ \delta(r-r') \end{bmatrix} \tag{81}$$

depending on what type of noise source is considered, either a removal macroscopic cross-section noise source or a thermal absorption macroscopic cross-section noise source. The induced flux noise is thus given by:

$$\delta\phi(r,\omega) = \int [G_{\delta XS \to 1}(r,r',\omega) + G_{\delta XS \to 1}(r,r',\omega)]\delta XS(r',\omega)dr' \tag{82}$$

The definition of the Green's function differs slightly between the 1-D 1-group homogeneous case [given by Eq. (51)] and the 2-D 2-group heterogeneous case [given by Eqs. (80) and (81)]. Consequently, the way of calculating the induced neutron noise also differs slightly between the 1-D 1-group homogeneous case [given by Eq. (52)] and the 2-D 2-group heterogeneous case [given by Eq. (82)], so that the calculated induced neutron noises are consistent with each other.

If one first assumes that one could measure the core average moderator temperature noise, one can study the effect of the approximation of the reactivity noise by $\delta\rho^{approx}(\omega)$ defined by Eq. (28). Replacing the exact reactivity noise $\delta\rho(\omega)$ by its approximated expression $\delta\rho^{approx}(\omega)$ in the ideal MTC noise estimator given by Eq. (50) leads to a biased MTC noise estimator that can be numerically evaluated in the present case via the use of the Green's function technique as follows:

$$\tilde{H}_1^{biased}(r,\omega) = \frac{1}{G_0(\omega)}\frac{CPSD_{\delta\phi/\phi_0,\delta T_m^{ave}}(r,\omega)}{APSD_{\delta T_m^{ave}}(\omega)} = \frac{\int w(r')dr'}{G_0(\omega)[\phi_1(r)+\phi_2(r)]K} \tag{83}$$

$$\times \frac{\iint [G_{\delta XS \to 1}(r,r',\omega) + G_{\delta XS \to 2}(r,r',\omega)]CPSD_{\delta XS}(r',r'',\omega)w(r'')dr'dr''}{\iint CPSD_{\delta XS}(r',r'',\omega)w(r')w(r'')dr'dr''}$$

Due to the above approximation, the $\tilde{H}_1^{biased}$ MTC noise estimator is both space- and frequency-dependent. This new $\tilde{H}_1^{biased}$ MTC noise estimator is compared to the actual MTC value in Fig. 15. In this Figure and in Eq. (53), the horizontal coordinates and the space variable r respectively represent the radial location of the neutron noise measurement only, since for the temperature noise measurement the core average is supposed to be taken. The calculations were performed with a set of cross-sections/point-kinetic parameters typical of a PWR core at near EOC conditions. The chosen frequency for the calculations was 1 Hz. The radial correlation length of the moderator temperature noise was chosen to be equal to 50 cm [to be compared to the core radius R (including the reflector) equal to approximately 200 cm][4] and the same shape function as in the 1-D case was used, but discretised here on a 2-D mesh:

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4. The correlation length used for the calculations in Paper VI was chosen to be equal to 150 cm. Nevertheless, such a correlation length was considered to be larger than one would expect. The results of the calculations presented in Figs. 15 and 16 seem to indicate that a correlation length of 50 cm is more realistic.

$$\sigma(\hat{\boldsymbol{r}}) = \frac{1}{1 - \left(\dfrac{\hat{\boldsymbol{r}}}{R + R/5}\right)} \tag{84}$$

As can be seen in Fig. 15, the $\tilde{H}_1^{biased}$ MTC noise estimator gives a relatively good estimation of the actual value of the MTC, wherever the neutron noise is measured in the core. Consequently, the deviation of the reactor response from point-kinetics does not play a significant role regarding the MTC noise estimation. One can therefore consider that approximating the reactivity noise $\delta\rho(\omega)$ by $\delta\rho^{approx}(r, \omega)$ does not induce any appreciable discrepancy in the MTC noise estimation.
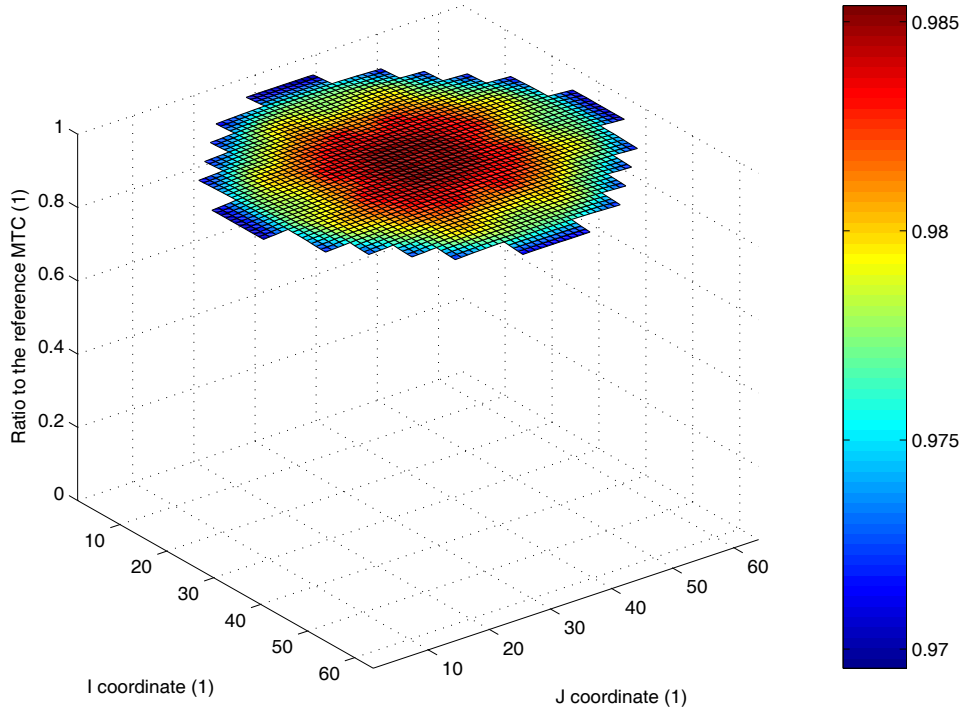
Fig. 15. Ratio between the MTC noise estimate $\tilde{H}_1^{biased}$ and its actual value (due to the deviation from point-kinetics of the reactor response solely) in a 2-D 2-group heterogeneous model.

Assuming that this approximation is valid regarding the MTC noise estimation, one can then study the effect of using the local moderator temperature noise instead of the core average moderator temperature noise. This is accomplished by using in Eq. (83) $\delta T_m(r, \omega)$ instead of $\delta T_m^{ave}(\omega)$. This leads to the usual MTC noise estimator H_1^{biased} given by Eq. (31). As before, this biased MTC noise estimator can be numerically estimated in the present case via the use of the Green's function technique as follows:

$$H_1^{biased}(r, \omega) = \frac{1}{G_0(\omega)} \frac{CPSD_{\delta\phi/\phi_0, \delta T_m}(r, \omega)}{APSD_{\delta T_m}(r, \omega)} = \frac{1}{G_0(\omega)[\phi_1(r) + \phi_2(r)]K}$$
$$\times \frac{\int [G_{\delta XS \to 1}(r, r', \omega) + G_{\delta XS \to 2}(r, r', \omega)]CPSD_{\delta XS}(r', r, \omega)dr'}{APSD_{\delta XS}(r, \omega)} \tag{85}$$

This MTC noise estimator is, as before, both space- and frequency-dependent. Using the same parameters as for the $\tilde{H}_1^{biased}$ MTC noise estimator, i.e. set of cross-sections, point-

kinetic parameters, correlation length, and shape function, the H_1^{biased} MTC noise estimator can be compared to the actual MTC value. Such a comparison was done at a frequency of 1 Hz and is represented in Fig. 16. In this Figure and in Eq. (85), the horizontal coordinates and the space variable *r* respectively represent the radial location of the measurement of both the neutron noise and the moderator temperature noise. As can be seen in this Figure, there is a very strong underestimation of the actual value of the MTC, and this underestimation is noticeably space-dependent. Close to the core centre, the MTC is roughly underestimated by a factor 5, which corresponds to what was noticed in the experimental investigations relying of the usual H_1^{biased} MTC noise estimator.
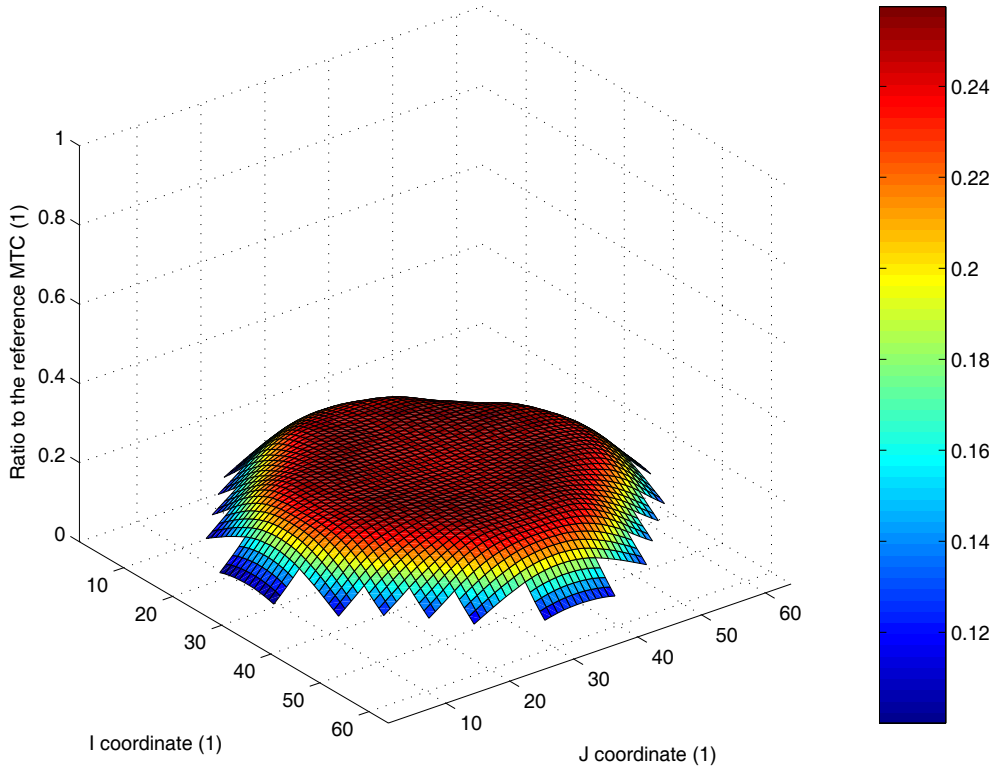
Fig. 16. Ratio between the MTC noise estimate H_1^{biased} and its actual value (due to the deviation from point-kinetics of the reactor response *and* the approximation of the average temperature noise by the local one) in a 2-D 2-group heterogeneous model.

Consequently, both the 1-D 1-group homogeneous model and the 2-D 2-group heterogeneous model indicate that the main reason for the MTC underestimation by noise analysis relying on the so-called H_1^{biased} noise estimator is the radial non-homogeneous structure of the moderator temperature noise throughout the core. The resulting deviation of the reactor response from point-kinetics is negligible with respect to the MTC estimation. A new MTC estimator, the so-called $\tilde{H}_1^{biased}$ MTC noise estimator, which still assumes a point-kinetic behaviour of the reactor but which is able to account for the heterogeneity of the moderator temperature noise, was proposed. It was shown that this new MTC noise estimator was always able to give the actual MTC value within an accuracy of 3%. In the next Chapter, the possibility of using this new MTC noise estimator in practice is investigated.

Chapter 4

Improvement of the noise analysis technique

The previous Chapter demonstrated via core modelling that one of the reasons for the MTC underestimation by noise analysis could be the radial non-homogeneous structure of the moderator temperature noise throughout the core. Furthermore, a noise measurement briefly described in Section 2.5 shows that the moderator temperature fluctuations are indeed radially loosely coupled. Therefore, in view of these findings, the noise analysis technique can be improved if the MTC noise estimator is able to account for the heterogeneous character of the moderator temperature noise throughout the core, so that the correct MTC value could be obtained. This Chapter explains how the noise-based method can be improved. Finally, an MTC noise measurement performed in the Ringhals-2 PWR based on these reflections is analysed.

4.1. Definition of a new MTC noise estimator

Regarding the MTC estimation by noise analysis, the first point to emphasize in the hypothesis of spatially non-homogeneous moderator temperature fluctuations throughout the core is that the ideal MTC noise estimator is actually known and is given by Eq. (50), which is recalled below:

$$H_1^{ideal} = \frac{1}{G_0(\omega)} \frac{CPSD_{\delta\phi^{pk}/\phi_0, \delta T_m^{ave}}(\omega)}{APSD_{\delta T_m^{ave}}(\omega)} \tag{86}$$

In this ideal MTC noise estimator, one should measure the point-kinetic component of the neutron noise and the core average moderator temperature noise. Due to the orthogonality between the fluctuation of the shape function, i.e. $\delta\psi(r, \omega)$, and the static flux $\phi_0(r)$ [see Eq. (21)], the point-kinetic component of the flux noise can be determined from

$$\delta P(\omega) = \frac{\delta\phi^{pk}(r, \omega)}{\phi_0(r)} = \frac{\int \delta\phi(r, \omega)\phi_0(r)dr}{\int \phi_0^2(r)dr} \tag{87}$$

If one has access to many radial in-core neutron detectors, Eq. (87) can be approximated by replacing the integrals by a sum over a number N of detectors, as follows:

$$\delta P(\omega) = \frac{\delta\phi^{pk}(r, \omega)}{\phi_0(r)} \approx \frac{\sum_{i=1}^{N} \delta\phi(r_i, \omega)\phi_0(r_i)}{\sum_{i=1}^{N} \phi_0^2(r_i)} \tag{88}$$

Usually, only the flux noise in one radial location of the core is measured, so that Eq. (88) is not applicable in practice. All one can estimate is therefore the total flux noise. Furthermore, only the local moderator temperature noise is usually measured at the same

radial location. In such a case, one obtains the usual H_1^{biased} MTC noise estimator, which was used in all the experimental work so far. This MTC noise estimator is defined in Eq. (31) and recalled below:

$$H_1^{biased}(\boldsymbol{r}, \omega) = \frac{1}{G_0(\omega)} \frac{CPSD_{\delta\phi/\phi_0, \delta T_m}(\boldsymbol{r}, \omega)}{APSD_{\delta T_m}(\boldsymbol{r}, \omega)} \tag{89}$$

The only case where this MTC noise estimator should give a correct MTC estimation is when the moderator temperature noise is spatially homogeneous. In such a case, the local moderator temperature noise is equal to the core average one, and the reactor behaves in a point-kinetic way, i.e. both assumptions used in the derivation of this MTC noise estimator are fulfilled. In case of non-homogeneous moderator temperature noise, the reactor behaviour will deviate from point-kinetics, and none of these aforementioned hypotheses is valid.

It was shown that the deviation of the reactor response from point-kinetics was nevertheless insignificant with respect to the MTC estimation. The main problem is the use of a local temperature value without knowledge of the temperature correlations. If the correlations were known, a calibration factor could be calculated from Eqs. (72) and (85) as follows:

$$\frac{H_1^{biased}(\boldsymbol{r}, \omega)}{MTC} = \frac{\int [\nu\Sigma_{f, 1 \to 1}(\boldsymbol{r})\phi_1^+(\boldsymbol{r})\phi_1(\boldsymbol{r}) + \nu\Sigma_{f, 2 \to 1}(\boldsymbol{r})\phi_1^+(\boldsymbol{r})\phi_2(\boldsymbol{r})]d\boldsymbol{r}}{G_0(\omega)[\phi_1(\boldsymbol{r}) + \phi_2(\boldsymbol{r})]}$$

$$\times \frac{\int [G_{\delta XS \to 1}(\boldsymbol{r}, \boldsymbol{r}', \omega) + G_{\delta XS \to 2}(\boldsymbol{r}, \boldsymbol{r}', \omega)]CPSD_{\delta T_m}(\boldsymbol{r}', \boldsymbol{r}, \omega)w(\boldsymbol{r}'')d\boldsymbol{r}'}{APSD_{\delta T_m}(\boldsymbol{r}, \omega) \times \int w(\boldsymbol{r})d\boldsymbol{r}} \tag{90}$$

This can be performed without the need for a simplified form of the correlations as in Eq. (41), but requires performing core calculations in order to determine the discretised 2-D 2-group Green's function, the static flux, and the adjoint flux. If the correlation function of the temperature fluctuations is mapped once, it can be used for a longer period and with arbitrary core positions to calculate a calibration function. This method then would work as long as the temperature correlations do not change significantly. In principle, this method bears resemblance with the present way of handling the situation where the calibration factor is determined empirically (comparing the measured value to the true one, obtained from traditional measurements or core calculations) (see Ref. 54).

The modelling presented in the previous Chapter also revealed that if one could measure the core average moderator temperature noise throughout the core, the new $\tilde{H}_1^{biased}$ MTC noise estimator, defined in Eqs. (53) and (83) and recalled below, would give a fairly good estimation of the MTC wherever the neutron noise measurement is performed in the core:

$$\tilde{H}_1^{biased}(\boldsymbol{r}, \omega) = \frac{1}{G_0(\omega)} \frac{CPSD_{\delta\phi/\phi_0, \delta T_m^{ave}}(\boldsymbol{r}, \omega)}{APSD_{\delta T_m^{ave}}(\omega)} \tag{91}$$

This new MTC noise estimator still relies on a point-kinetic behaviour of the reactor. As

explained previously, this approximation does not hold, so that this new MTC noise estimator is also biased. Nevertheless, this bias was proven to be negligible, so that the correct MTC value was obtained within an accuracy of 3% in the 2-D 2-group heterogeneous model. The only quantity that needs to be properly estimated is the core average moderator temperature noise. If the temperature noise could be measured simultaneously at many different radial positions throughout the core, the average temperature noise could be calculated according to Eq. (5) via the use of a proper weighting function. The problem is that first the moderator temperature is usually not measured inside the core in commercial PWRs, and second the weighting function might not be so straightforward to estimate and might itself require advanced core calculations [see Eqs. (49), (73), or (74)], which is from a measurement viewpoint a major drawback. We will see in the next Sections how one can cope with these problems for the case of the Ringhals-2 PWR.

4.2. Use of Gamma-Thermometers (GTs) as moderator temperature noise indicators

Estimating the core average temperature noise can be done by measuring the temperature noise in several points of the reactor core and by using an integration formula so that the integrals in Eq. (5) can be numerically approximated. As pointed out previously, commercial PWRs are badly instrumented with respect to in-core temperature detectors. Nevertheless the Ringhals-2 PWR contains 108 permanent Gamma-Thermometers (GTs) installed in the core (12 strings containing each 9 GTs located at different axial levels). Paper VII investigates the possibility of using GTs for measuring the temperature noise throughout the core, and this Section briefly summarises the results of these investigations. As a matter of fact, the use of GTs for measuring temperature noise has already been proven in the Halden Reactor Project, but in a different setting (see Refs. 55 and 56). The purpose of the work reported in Paper VII was to perform the same investigation regarding measurements taken at an operating power plant.

There are several types of GTs. The Ringhals-2 unit is instrumented with the so-called RADCAL type GTs, derived from the Halden type GTs (see Ref. 57). In the Halden type GT design, a metallic pin is heated up by the gamma flux mostly (photoelectric and Compton interactions) and to a lesser extent by the neutron flux [(n, γ), (n, α), (n, p) reactions, elastic collisions]. The pin is encapsulated into an outer body or housing, and the thermal insulation between the pin and the outer body is made by xenon gas. One of the tips of the pin is therefore thermally insulated, whereas the other tip is in direct contact with the body of the GT. A differential chromel/alumel thermocouple measures the temperature drop along the pin with the hot junction located at the insulated tip, and the cold junction in direct contact with the coolant outside the GT body. Fig. 17 gives an overview of the Halden type GT. Even if GTs are designed to measure the gamma flux (or more practically the power), the cold junction of the differential thermocouple is in direct contact with the coolant and could be therefore used for monitoring the moderator temperature fluctuations as explained in the following.

As mentioned previously, the hot junction of the thermocouple measures the temperature increase due to the heating along the pin with the cold junction acting as a heat sink and which is at the same temperature as the moderator. In case of static measurements, the voltage delivered by the differential thermocouple is therefore directly proportional to the heating produced mostly by the gamma reactions, and consequently proportional to the
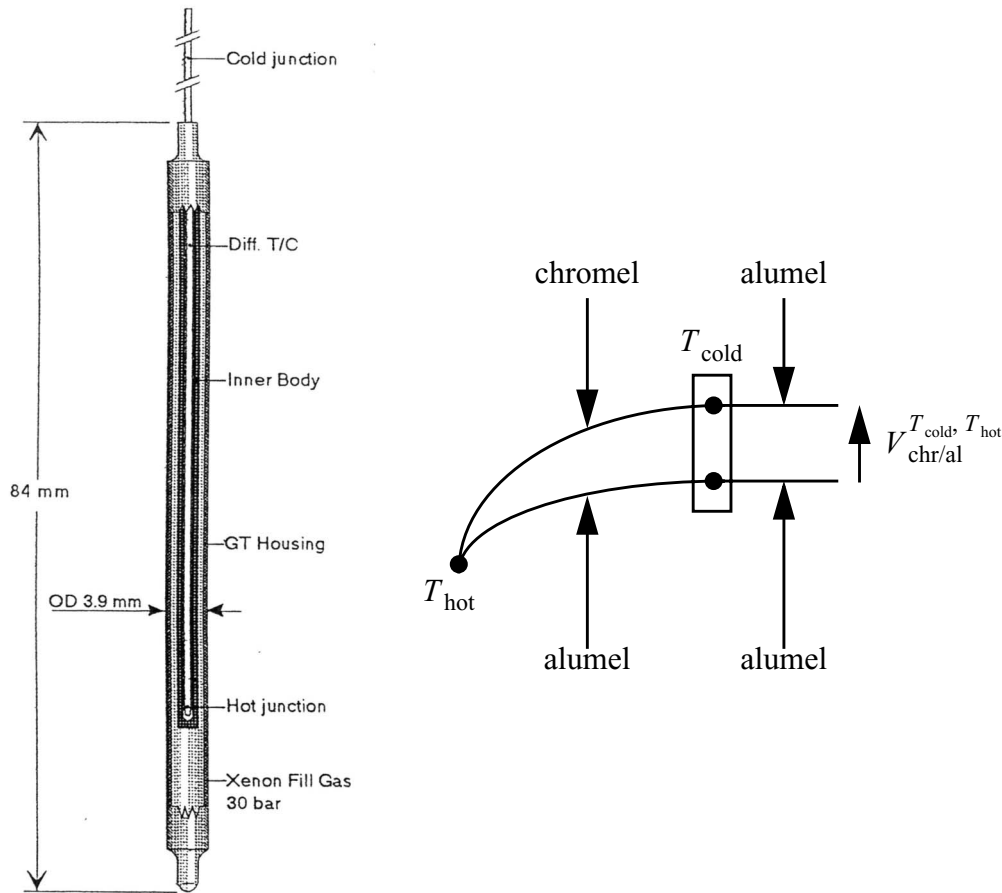
Fig. 17. Sketch of the Halden type GTs (from Ref. 55) on the left, and the corresponding schematic description of the differential thermocouple on the right.

gamma flux. Another interesting feature that will be exploited in Section 4.3 is the fact that the static gamma flux is itself proportional to the static neutron flux under given circumstances.

In case of dynamical measurements, because of the different thermal characteristics of the cold and hot junctions, the thermocouple will respond differently, i.e., with different delays to phenomena associated to the hot junction and to phenomena associated to the cold junction. Since the cold junction is located above the GT body and is in direct contact with the coolant, it responds very quickly to coolant temperature oscillations (with a thermal time constant typically around 0.1 - 1.0 s). This means that the cold junction acts as a low-pass filter of the coolant temperature noise with a cut-off frequency of a few hertz. On the contrary, the hot junction is thermally insulated within the inner body and has therefore a significantly more sluggish response (with a thermal time constant typically around 10 - 100 s). Consequently, the hot junction acts a low-pass filter of the gamma/neutron flux with a cut-off frequency of a few hundredth of hertz. One can write the transfer function from the cold and hot junctions temperature noise ($\delta T_{cold} = \delta T_m$ and δT_{hot} respectively) to the GT voltage in the frequency domain as follows:

$$\delta V_{chr/al}^{T_{cold}, T_{hot}}(\omega) = \frac{\alpha_{chr/al}}{1 + j\tau_{hot}\omega} \times \delta T_{hot}(\omega) - \frac{\alpha_{chr/al}}{1 + j\tau_{cold}\omega} \times \delta T_m(\omega) \tag{92}$$

where the parameters τ_{hot} and τ_{cold} refer to the time constants of the hot and cold junctions respectively, and where $\alpha_{chr/al}$ is the thermocouple constant. Then one notices that in the frequency range of interest for the MTC estimation by noise analysis, i.e. typically from 0.1 to 1.0 Hz, only the signal due to the cold junction remains. The fluctuations due to the gamma/neutron heating are consequently completely filtered out by the hot junction. Only the very low frequencies contain the contribution of both the hot and cold junctions. The GTs could therefore be used to measure the average temperature noise in the Ringhals-2 PWR.

The ability of the GT to measure the moderator temperature noise was checked via a noise measurement performed in the Ringhals-2 PWR during the fuel cycle 24 (core average burnup of 8.5 GWd/tHM). The RADCAL type GTs were used for that purpose and 3 channels only (J10, L11, and M03) were at that time recorded (see Fig. 6). For each detector string, all 9 different axial levels were measured. The different levels are roughly equally spaced and cover the whole core active height. The numbering of the levels (1-9) is done from the bottom of the core active height to the top of the core active height. The ability of the GTs to monitor the temperature fluctuations of the coolant was first checked qualitatively by cross-correlating two GTs within the same fuel channel. If the GTs are sensitive to the temperature noise, then the temperature fluctuations recorded by the lowermost detector should also be present in the uppermost detector. Due to the separation distance between the two detectors, the noise recorded by the uppermost one is simply time-shifted compared to the noise recorded by the lowermost one (if one assumes that no noise is added between the two detectors). In the frequency range where the GTs detect temperature fluctuations, the phase of the CPSD between the two detectors should therefore be linear. Further, the coherence function between the two detectors will present a sink frequency that indicates the upper limit of the frequency range for which the two detectors monitor the same phenomenon. This can be noticed in Fig. 18, where one possible pair of detectors within the fuel channel L11 was chosen as a representative example. This shows that GTs in the frequency range 0.1 to 1.0 Hz are able to detect the moderator temperature fluctuations travelling upwards through the core. GTs were even used successfully to measure flow velocities within the core and benchmarked against core calculations (see Ref. 58).

Another possibility to check if the GTs work as thermocouples in the frequency range of interest for the MTC estimation by noise analysis is to try to numerically estimate the transfer function of the GTs. This can be achieved by the so-called Auto-Regressive Moving Average (ARMA) modelling. If one assumes that the GTs measure a noise e, then an ARMA(n,m) model consists of trying to write the GTs transfer function as:

$$\sum_{k=0}^{n} a_k \delta V_{chr/al}^{T_{cold},\, T_{hot}}(t - kT) = \sum_{l=0}^{m} c_l e(t - lT) \qquad (93)$$

where n and m are the order of the model, and T is the sampling interval. The a_k and c_l coefficients are estimated from the measured output signal $\delta V_{chr/al}^{T_{cold},\, T_{hot}}$ using a minimization procedure (see Ref. 59). The noise e is still unknown and assumed to be completely uncorrelated, i.e. white. Once the coefficients a_k and c_l are estimated, the time constants of the corresponding transfer function can be estimated since any real pole p of the transfer function defines a specific system dynamics first-order mode and can therefore be associated to a
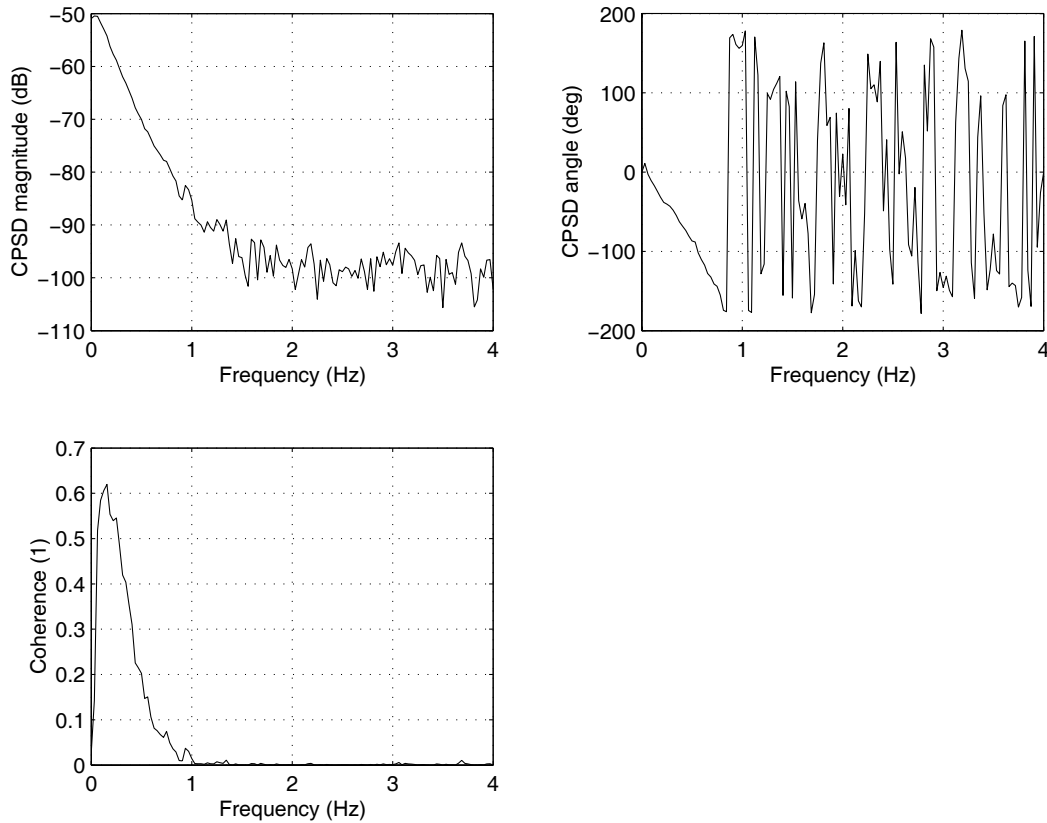
Fig. 18. CPSD, phase, and coherence of the detector string L11, between plane number 4 and plane number 9.

time constant τ as [31]:

$$\tau = -\frac{T}{\ln p} \tag{94}$$

For each GT, an ARMA model was established, and the different time constants corresponding to the cold and hot junctions were determined from the poles of the transfer function resulting from the ARMA models. Simulation cases revealed that the estimation of the time constants might be sometimes biased for different reasons (see Ref. 61). Despite the bias, the simulations showed that the previous procedure was able to indicate rather well a time constant around which the true one actually lies. When this procedure was applied to real measurement signals, one of the time constants was found in the range 0.1 to 1.0 s, and the other time constant in the range 2.0 to 20.0 s. The first one corresponds therefore to the cold junction, whereas the second one corresponds to the hot junction.

Consequently, the transfer function proposed in Eq. (92) seems to model correctly the GT behaviour. Furthermore, in the frequency range of interest for the MTC estimation by noise analysis, i.e. typically from 0.1 to 1.0 Hz, only the signal induced by the cold junction remains, due to the relatively large time constant of the hot junction. This was shown both in a qualitative manner and a quantitative manner in the foregoing. Therefore, GTs work as ordinary thermocouples in the frequency band of interest for the MTC investigation by noise analysis. More practically, this means that the GTs can be used to map the radial structure of the moderator temperature noise throughout the core. This radial

mapping then allows estimating, via the use of an integration formula, the core average moderator temperature noise, so that the new MTC noise estimator $\tilde{H}_1^{biased}$ could be used.

4.3. Test of the new noise estimator in Ringhals-2

A noise measurement was carried out at the Ringhals-2 PWR during the fuel cycle 26, (core-averaged burnup of 7.30 GWd/tHM). This measurement is briefly analysed in Paper VIII, and presented in more detail in this thesis (see also Ref. 62). The measurement setup is presented in Fig. 19 below. In this noise measurement, all the available detectors [12 GTs and 2 Neutron Detectors (NDs)] on one plane of the reactor, located at 30% of the core active height from its bottom (plane 7 in Fig. 19), were used by the data acquisition system. Likewise, a fuel assembly (assembly J10 in Fig. 19) was axially fully monitored with all the 9 GTs. In this Section, the numbering of the levels (1-9) was done from the top to the bottom of the core active height. This axial numbering is actually reverse compared to the one used in Section 4.2. The GTs were of the RADCAL type, design derived from the Halden type GTs described previously, whereas the NDs were ordinary fission chambers. The NDs were chosen so that they were located as close as possible to a GT. For the purpose of comparison, the signal of a Thermocouple (TC) at the core-exit (assembly J10 in Fig. 19) was also recorded. This core-exit TC was an ordinary K type TC, i.e., chromel/alumel, and was located at the top of a fuel assembly containing a GT, and next to a fuel assembly containing a ND.
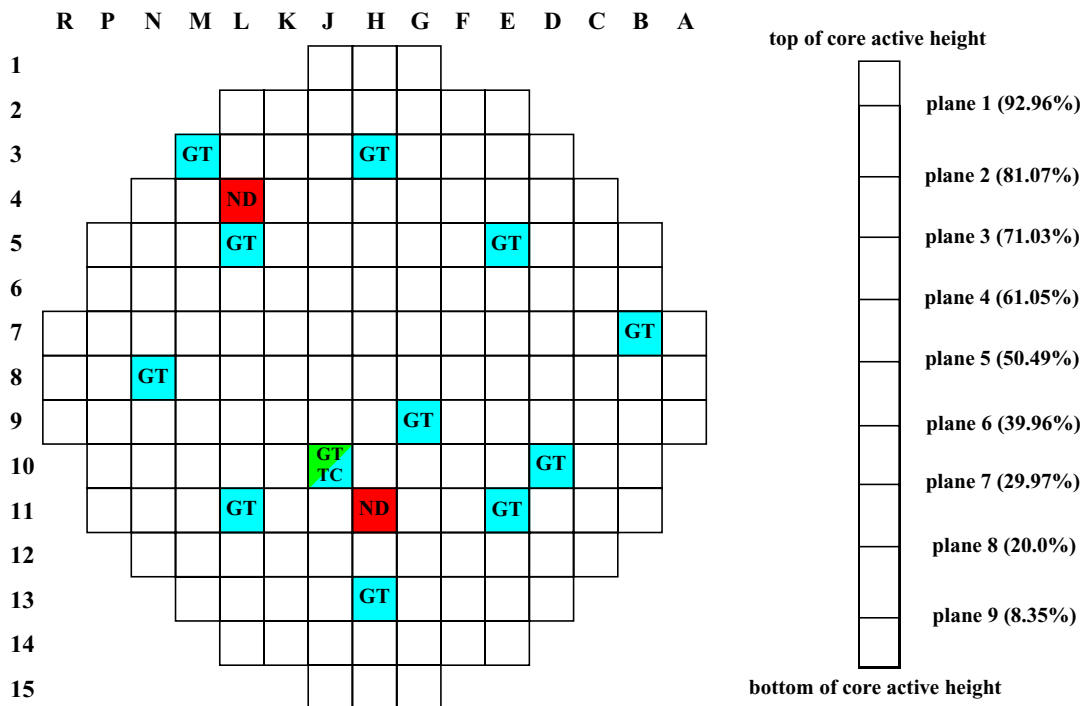
Fig. 19. Position of the detectors for the noise measurement in Ringhals-2, cycle 26 (on the left hand side the radial position on the 7th axial plane; on the right hand side the axial position in the J10 fuel assembly).

Regarding the hardware processing of the signals, only the noise content of the NDs and the core-exit TC were monitored by manually offsetting the mean values. These signals were then amplified. No offset and no amplification were applied to the signals of the GTs. These recorded signals were thus digitally converted. The software processing of the signals was carried out via MATLAB (see Ref. 63). The time-signals were detrended (if

a trend was found), and data analysis was performed in the frequency domain. In order to evaluate the APSDs and CPSDs of the different signals, the Welch's averaged, modified periodogram method was used. The time-signals were divided into overlapping sections of n points, then windowed by using a Hanning window. The sections were assumed to overlap by $n/2$ points. As explained in the following, several values for n were tested: 512, 256, and 128 points.

As pointed out previously, the GTs offer a unique opportunity to map the structure of the temperature noise throughout the core, both radially and axially. For that purpose, the APSDs, CPSDs, and coherence of all the available GT signals were calculated. Since the frequency band of interest for the MTC noise estimation is from 0.1 to 1.0 Hz, all the spectra were averaged on that frequency band. Furthermore, the number of points used for the Fast Fourier Transform (FFT) calculations was chosen to be 256 (this number of FFT points was proven to give better MTC estimations by noise analysis, as will be seen in the following).

The APSD plots are given in Fig. 20. As can be seen in this Figure, the strength of the moderator temperature noise, i.e. the $\sigma^2(\hat{r})$ function in Eq. (41), is spatially non-homogeneous in the frequency interval 0.1 - 1.0 Hz. Nevertheless, this does not say anything about the temperature correlations, and thus about the MTC underestimation. If the correlation length of the temperature fluctuation was infinite, the MTC would be misestimated, i.e. overestimated or underestimated depending on the location of the moderator temperature/neutron noise measurement in the core, but not necessarily underestimated. This can be demonstrated by using for instance Eq. (85) with an infinite correlation length, all the other parameters being identical to the ones in Section 3.3. In such a case, the usual MTC noise estimator H_1^{biased} reads as:

$$H_1^{biased}(r, \omega) \tag{95}$$

$$= \frac{1}{G_0(\omega)[\phi_1(r) + \phi_2(r)]K} \times \frac{\int [G_{\delta XS \to 1}(r, r', \omega) + G_{\delta XS \to 2}(r, r', \omega)]\sigma^2\left(\frac{r' + r}{2}\right)dr'}{\sigma^2(r)}$$

The numerical comparison between this MTC noise estimator and the actual MTC value given by Eqs. (72)-(74) would reveal that the MTC given by noise analysis is either overestimated or underestimated, depending on the location of the measurement of the moderator temperature noise and neutron noise. Therefore, only a finite correlation length can explain the systematic MTC underestimation. Regarding the axial structure of the moderator temperature noise, one notices an axial damping of the temperature noise with core elevation. This suggests that the temperature noise is probably created outside the core, most likely at or before the core-inlet. The fact that there is no moderator temperature noise source present inside the core is essential since the MTC noise estimators given by Eqs. (86), (89), and (91) all rely on this assumption. This also means that the axial direction can be completely disregarded while evaluating the MTC by noise analysis, i.e. the axial dimension has a second-order effect compared to the radial one. Due to the axial damping, it has nevertheless to be noticed that the accuracy of the MTC noise estimation would probably be higher at the bottom of the core, i.e. where the temperature noise is larger, than

at the top of the core, i.e. where the temperature noise is damped. Finally, it can be seen that the temperature noise monitored by the core-exit TC (located at the top of the core active height in Fig. 20) is much higher than the temperature noise recorded inside the core. The mixing of the coolant flow above the fuel assemblies is probably responsible for this effect, which is equivalent to the presence of an extraneous noise source. As will be explained in the following, this could be a contributing reason why the MTC was systematically underestimated by using a core-exit TC while evaluating the MTC by the noise analysis technique.

The coherence plots are given in Fig. 21. The different points represent all the possible combinations of detectors one can correlate, either radially (upper Figure) or axially (lower Figure). As can be seen in this Figure, there is very little radial coherence between the different GTs. On the other hand, the axial coherence is much higher than the radial coherence. Although there is some scattering in the results, one can notice that the axial coherence decreases with increasing distance between two GTs. The damping of the moderator temperature noise travelling upwards is probably responsible for this effect. Regarding the dependence of the CPSD between two detectors with their separation distance depicted in Fig. 22, it could be interesting to try to estimate the correlation length of the moderator temperature noise as defined in Eq. (41). In this Figure, the different points represent, as before, all the possible combinations of detectors one can correlate, either radially (upper Figure) or axially (lower Figure). Unfortunately, such a determination seems to be impossible since one needs to know the full spatial dependence of the APSD of the moderator temperature noise throughout the core [which is the $\sigma^2(\hat{r})$ shape function in Eq. (41)], whereas only the APSD in a few discrete points is actually measured. One can nevertheless clearly see that there is no radial dependence of the CPSD between two GTs with their separation distance. On the other hand, it seems that the CPSD between two GTs depends on their axial separation distance in an exponential manner. It has to be emphasized that the new MTC noise estimator given by Eq. (91) is able to provide a correct MTC estimation whatever the spatial structure of the moderator temperature noise is, i.e. the model given by Eq. (41) being valid or not. Regarding the radial dependence of the CPSD, the correlation length is probably so short that it is not possible to notice it in Fig. 22, where the shortest separation distance between two GTs is roughly 30 cm, i.e. already too large to see any exponential behaviour. This means that all the points are probably located on the tail of the distribution, thus explaining why there is almost no spatial dependence.

Radial dependence of the moderator temperature noise (frequency band 0.1 – 1.0 Hz)
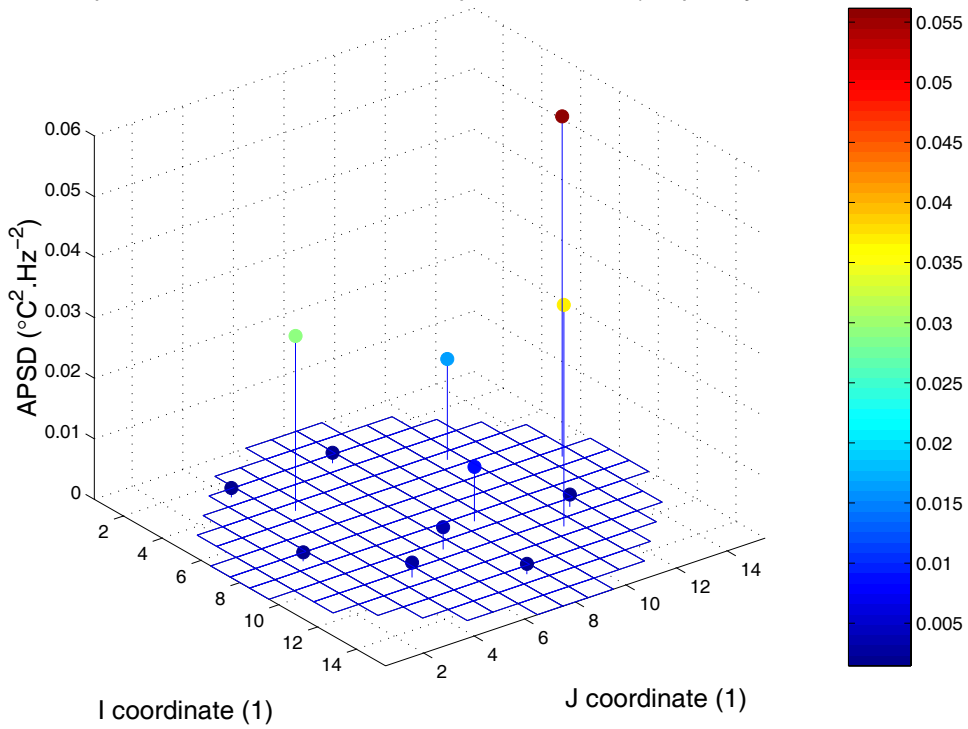
Axial dependence of the moderator temperature noise (frequency band 0.1 – 1.0 Hz)
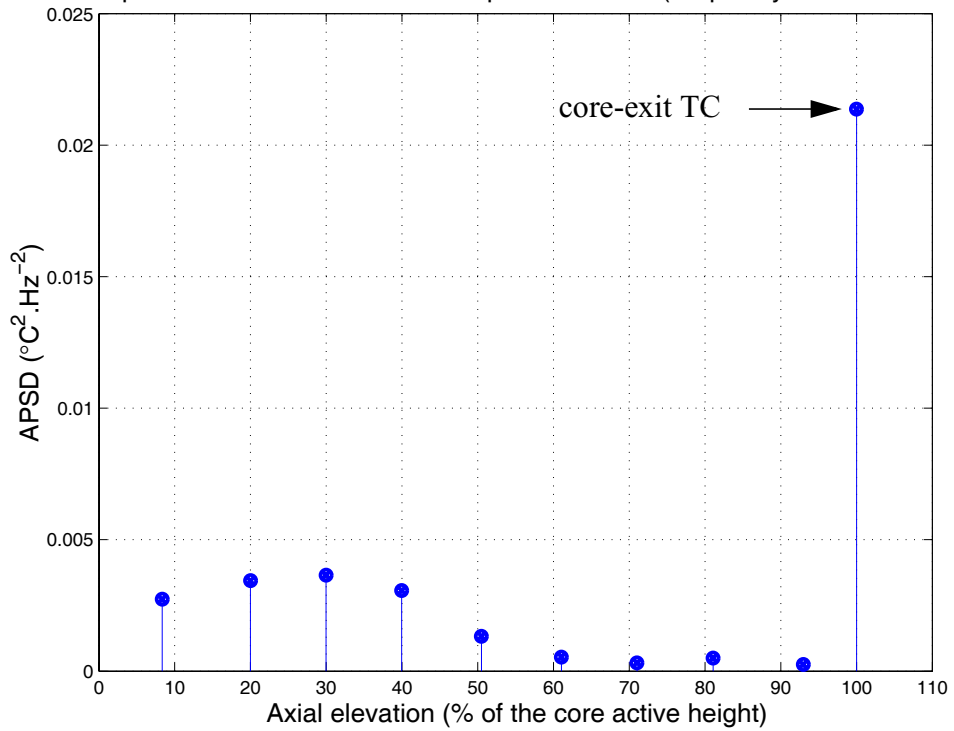
Fig. 20. Spatial dependence of the APSD of the moderator temperature noise in the frequency band 0.1 - 1.0 Hz.

Radial dependence of the coherence between two GTs (frequency band 0.1 – 1.0 Hz)
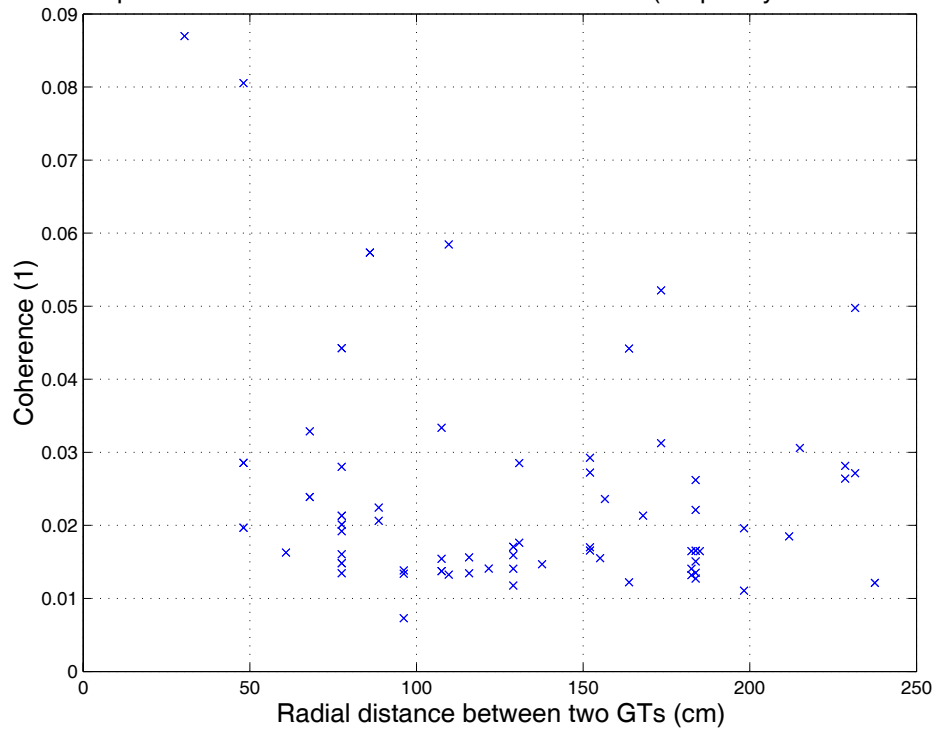
Axial dependence of the coherence between two GTs (frequency band 0.1 – 1.0 Hz)
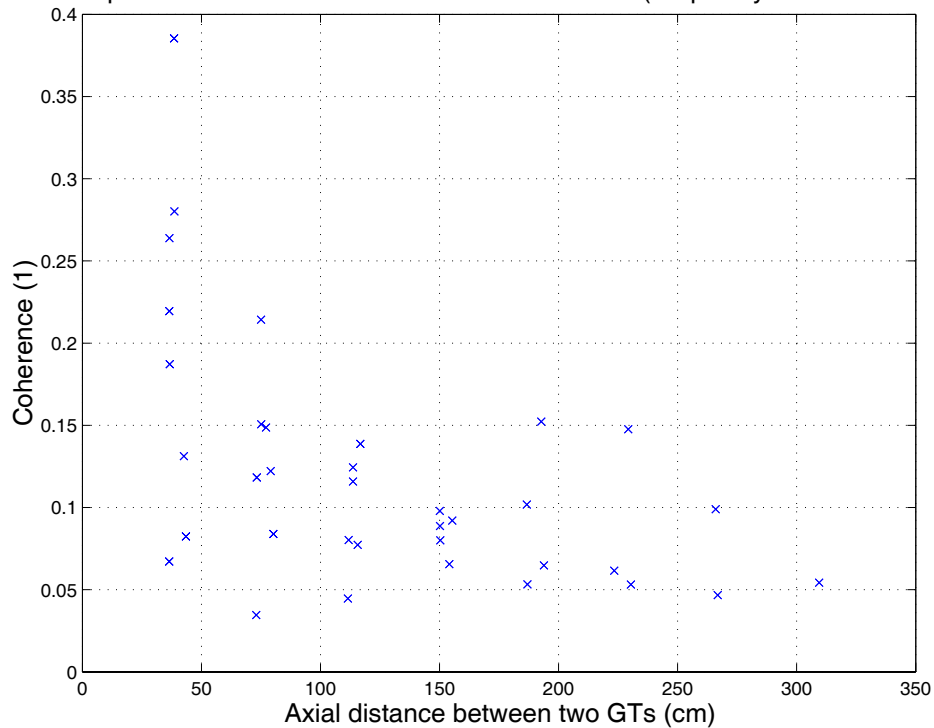
Fig. 21. Dependence of the coherence between two GTs with their separation distance in the frequency band 0.1 - 1.0 Hz.

Radial dependence of the CPSD between two GTs (frequency band 0.1 – 1.0 Hz)

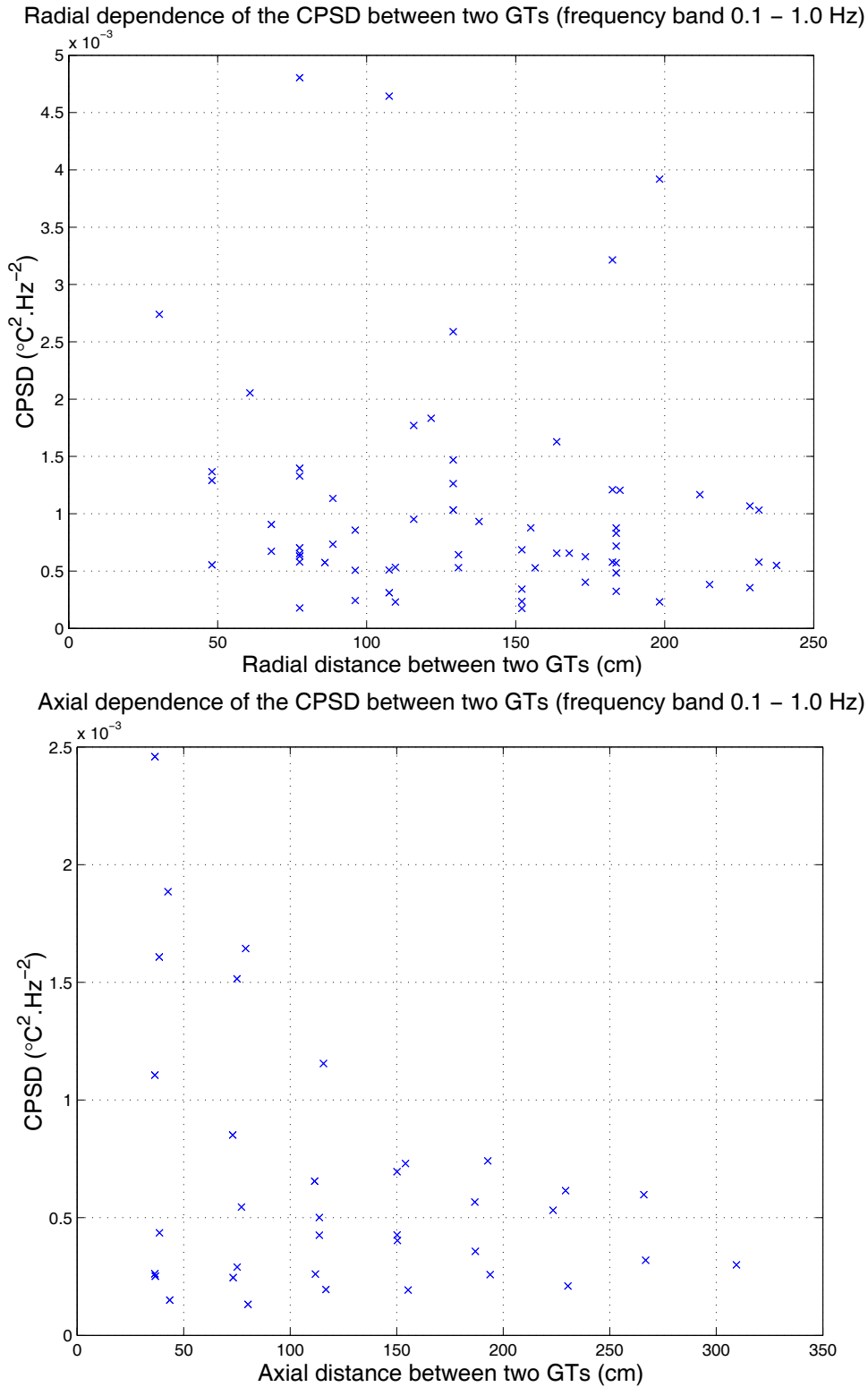Axial dependence of the CPSD between two GTs (frequency band 0.1 – 1.0 Hz)

Fig. 22. Dependence of the CPSD between two GTs with their separation distance in the frequency band 0.1 - 1.0 Hz.

As mentioned previously, the new MTC noise estimator relies on the core average moderator temperature noise that has to be calculated properly. Several possibilities investigated along this thesis are considered in the following and are summarised in Table 1. In this Table, the W2 weighting function assumes that the static gamma flux is directly proportional to the static neutron flux. Since only the static neutron flux relative to its core-averaged value is required in Eq. (5), the knowledge of the corresponding proportionality factor between the static gamma and neutron fluxes is not required[5]. Therefore, the GT could be used not only to monitor the spatial distribution of the moderator temperature noise, but also the spatial distribution of the static flux noise (see Ref. 64). This is particularly interesting since a good measurement technique should rely on as few as possible calculated parameters. The GTs allow thus avoiding calculating the static/adjoint neutron flux via a static core simulator.

Table 1: Possible weighting functions to be used for the core averaging of the moderator temperature noise

Weighting functions	Names	Comments
$\phi_0^2(r)$ obtained from SIMULATE-3 calculations	W1	Weighting function based on the 1-group first-order perturbation theory
$\phi_0^2(r)$ obtained from the square of the mean value of the GTs	W2	Weighting function based on the 1-group first-order perturbation theory
$[\phi_1^+(r)\phi_1(r) - \phi_2^+(r)\phi_1(r)]$ obtained from SIMULATE-3 calculations	W3	Weighting function based on the 2-group first-order perturbation theory; effect of a change in the moderator temperature having the greatest effect on the macroscopic removal cross-section
$-\phi_2^+(r)\phi_2(r)$ obtained from SIMULATE-3 calculations	W4	Weighting function based on the 2-group first-order perturbation theory; effect of a change in the moderator temperature having the greatest effect on the macroscopic thermal absorption cross-section

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5. Since the GTs are located within fuel assemblies that have different burnup, the ratio between the static gamma flux and the static neutron flux might be space-dependent. In such a case, the space-dependence has to be taken into account in the evaluation of the core-averaged temperature noise. Preliminary CASMO-4 (see Ref. 65) modelling of a single typical PWR assembly at different burnup showed that the standard deviation of the ratio between the static gamma and neutron fluxes is less than 10% of the average value for burnup up to 60 GWd/tHM. In the case of a full core, this figure is probably lower since a GT is sensitive to the gamma flux of several neighbouring fuel assemblies with different burnup. From one GT location to another, the average burnup of the fuel assemblies that the GT is sensing is roughly the same, due to the reloading pattern. Nevertheless, using more heterogeneous loading patterns, for instance a partial mixed UOX/MOX loading (UOX - Uranium Oxide), would probably make this figure bigger (see Ref. 66). Further investigation is needed in this respect.

The different weighting functions were tested by using the new MTC noise estimator $\tilde{H}_1^{biased}$. For the purpose of comparisons, the usual MTC noise estimator H_1^{biased} was also evaluated. In the latter case, the local temperature noise was used, recorded either inside the core via the closest GT to the ND, or outside the core via the core-exit TC. All the MTC estimations were therefore carried out for both of the in-core NDs H11 and L04. The point-kinetic parameters of the core, i.e. the effective fraction of delayed neutrons, the prompt neutron lifetime, and the 1-group precursors decay constant, which are required in the MTC noise estimators, were estimated by SIMULATE-3. The MTC was also directly evaluated by SIMULATE-3 and was found to be equal to -51 pcm/°C. This value was considered as the reference value in the rest of this Section.

The MTC noise evaluations showed that the MTC was frequency dependent with rather huge variation of the MTC magnitude in the frequency range 0.1 - 1.0 Hz. Therefore the following methodology was applied. In this frequency range, the maximum of the coherence between the ND and the temperature noise (estimated either from the W1, W2, W3, or W4 weighting functions, or directly from the GT J10, the GT L05, or the core-exit TC) was first determined. Then all the frequencies for which the coherence was larger than half this maximum were used for the MTC evaluation. The final MTC value was simply obtained by averaging these values at the corresponding frequencies.

It was found that the MTC estimated via the previous procedure was strongly dependent on the number of points used for the Fast-Fourier Transform (FFT). This suggests that the quality of the measured signals is relatively poor and could be probably improved in later measurements. Such dependence can be seen in Figs. 23 and 24, where the W2 weighting function was used for the calculations (since this weighting function is the most practical one to use from a measurement viewpoint). In these Figures, the points used for the final MTC evaluation are circled in bold.

The resulting MTC values are depicted in Fig. 25, where the standard deviation associated with each MTC evaluation is also represented. As can be seen in this Figure, the 256 FFT points evaluation seems to be the most realistic one, both with respect to the reference MTC value given by SIMULATE-3 and to the relatively flat behaviour of the MTC for the selected frequencies (the peaks in the 512 FFT points MTCs are clearly non-realistic). Assuming therefore that the spectral analysis of the signals has to be carried out with 256 FFT points, one can compare the MTCs given by the different noise estimators and the different weighting functions. Such a comparison can be seen in Fig. 26, where the standard deviation associated with each MTC estimation is also represented.

The main conclusion from this MTC noise measurement is that using the new MTC noise estimator $\tilde{H}_1^{biased}$ gives an MTC value that is very close to the reference value given by SIMULATE-3, if one takes the confidence intervals into account. This new MTC noise estimator relies on the core-averaged moderator temperature noise, which can be evaluated in different ways (by using either the W1, the W2, the W3, or the W4 weighting functions). As Fig. 26 shows, if one uses the core-exit TC located above the fuel assembly J10, and consequently uses the traditional MTC noise estimator H_1^{biased}, then the MTC is strongly underestimated by a factor of approximately 10. Likewise, replacing the core-exit TC by a single GT (the nearest one to the ND used in the evaluation) systematically underestimates the MTC value by a factor of 3 to 5. The reason why the MTC is underestimated at the core-exit is that the temperature fluctuations are larger at the core-exit than inside the core. Similarly, the fact that the MTC is still underestimated when using one

single GT instead of using all the signals of the GTs and the corresponding core average temperature noise means that the temperature noise recorded in this specific point of the reactor is larger than the core average one. The underestimation of the MTC by using one single temperature detector (either the core-exit TC or a GT) can be directly seen on Fig. 27, which shows the square root of the ratio between the APSD of the average temperature and the APSD of one in-core GT and that of the core-exit TC, respectively. By virtue of the Wiener Khinchin theorem, this square root represents actually the ratio between the traditional and the new MTC noise estimators as follows:

$$\sqrt{\frac{APSD_{\delta T_m^{ave}}(\omega)}{APSD_{\delta T_m}(r, \omega)}} \approx \left| \frac{\delta T_m^{ave}(\omega)}{\delta T_m(r, \omega)} \right| \approx \left| \frac{H_1^{biased}(r, \omega)}{\tilde{H}_1^{biased}(r, \omega)} \right| \tag{96}$$

Assuming a model for the moderator temperature noise as the one given by Eqs. (69)-(71), this square root can be rewritten using the Wiener Khinchin theorem as:

$$\sqrt{\frac{APSD_{\delta T_m^{ave}}(\omega)}{APSD_{\delta T_m}(r, \omega)}} \approx \sqrt{\frac{\iint \sigma^2\left(\frac{r' + r''}{2}\right) e^{-\frac{|r' - r''|}{l}} w(r')w(r'')dr'dr''}{\sigma^2(r)\iint w(r')w(r'')dr'dr''}} \tag{97}$$

As mentioned previously and as can be seen in Eq. (97), it is the finite correlation length that is responsible for the MTC underestimation by noise analysis. The non-homogeneous character of the APSD of the moderator temperature noise, i.e. the fact that $\sigma^2(r) \neq$ constant, does not allow explaining the MTC underestimation alone. If the correlation length was infinite with $\sigma^2(r) \neq$ constant, the MTC estimated via the H_1^{biased} MTC noise estimator would be either overestimated or underestimated compared to the actual MTC value, depending on the radial location of the moderator temperature/neutron noise measurement. As Fig. 27 shows, the square roots estimated from the measured data are both smaller than unity and, interestingly, quite flat in the frequency range 0.1 - 1.0 Hz. One reason for the MTC underestimation in all the experimental work so far is therefore the overestimation of the temperature noise outside the core. But using a single in-core TC, i.e. a GT in the case of Ringhals-2, does not provide either the actual MTC value, since the temperature noise appears to be radially loosely coupled in the core. On the other hand, using the core average moderator temperature noise gives the correct MTC value.

The fact that the results using the core average moderator temperature noise do not depend strongly on the radial position of the ND used in the MTC evaluation and give the actual MTC value suggests that the deviation of the reactor response from point-kinetics does not play a significant role on the MTC estimation by noise analysis. Consequently, the conclusions drawn by the theoretical work performed in Chapter 3 are consistent with the experimental one: the main reason for the MTC underestimation by the traditional noise analysis method lies with the fact that the moderator temperature noise is radially strongly heterogeneous in the core. The resulting deviation of the reactor response from point-kinetics is nevertheless not significant.

As can be seen on Fig. 25 and on Figs. 23 and 24, the MTC evaluation by using 256 FFT points seems to give the most realistic results. Taking the standard deviation into

account gives an MTC estimated by SIMULATE-3 lying in the confidence interval of the measurement. The way the final MTC is calculated, i.e. detecting the frequency having the highest coherence and taking all the frequencies between 0.1 and 1.0 Hz having a coherence higher than half this maximum into account, is very subjective. Having a more restrictive way of choosing the frequencies for the final MTC evaluation would narrow the confidence interval and give a better MTC estimation. It was for instance suggested that only the frequencies for which the coherence is higher than 10% should be retained (see Ref. 67). In such a case, the confidence interval narrows considerably to become about ± 4 pcm/$^\circ$C, a confidence interval that still includes the MTC value given by SIMULATE-3.

As can be seen in Fig. 26, the MTC depends to some extent on the weighting function used to calculate the core-averaged temperature noise throughout the core. The weighting functions using the square of the static flux, either calculated by SIMULATE-3 (W1) or measured via the GTs (W2), give the best results. The W3 weighting function gives somewhat underestimated MTC values (but still much higher than using a single GT or a single TC), whereas the W4 weighting function also gives acceptable results. This means that the hypothesis on which the W4 weighting function was derived is better than the one on which the W3 weighting function was derived, i.e. the moderator temperature noise has a bigger effect on the macroscopic thermal absorption cross-section than the removal cross-section with respect to the MTC.

Using the W2 weighting function has many practical aspects, the most important one being that the static flux does not need to be calculated but can be directly measured via the GTs. The GTs are therefore very versatile tools since they can provide both the moderator temperature noise and the static neutron flux throughout the core. These are required for an accurate estimation of the core-averaged moderator temperature noise. This core average can then be used in the new MTC noise estimator that was proven, both theoretically and experimentally, to give an accurate MTC estimation, wherever the neutron noise is measured in the core. The only parameters that are needed for the MTC estimation are the effective fraction of delayed neutrons, the prompt neutron lifetime, and the 1-group precursors decay constant, which can be easily predicted by any static core simulator.
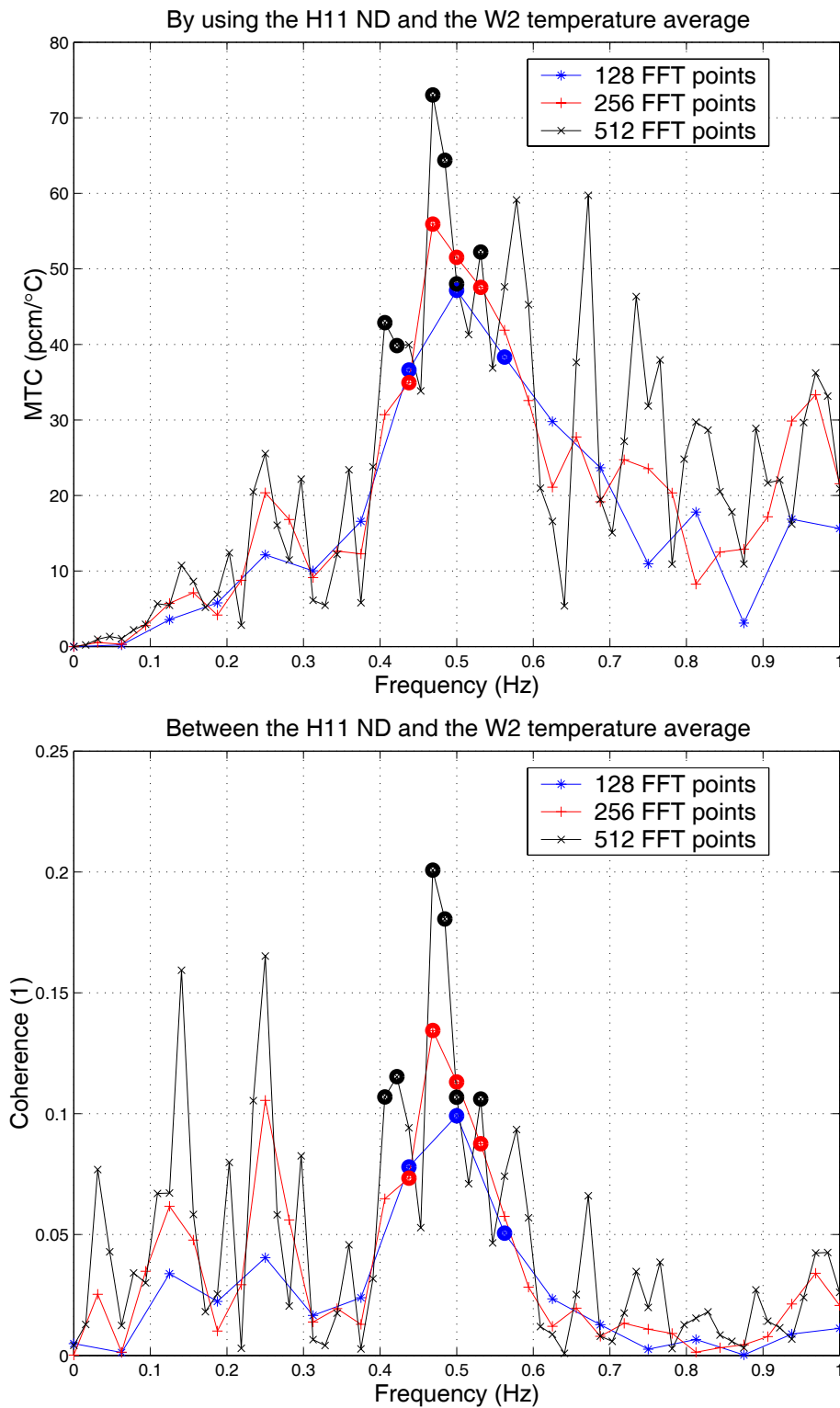
Fig. 23. Frequency dependence of the MTC noise estimation with respect to the number of FFT points used (neutron noise measured in the H11 assembly and temperature noise evaluated by using W2).
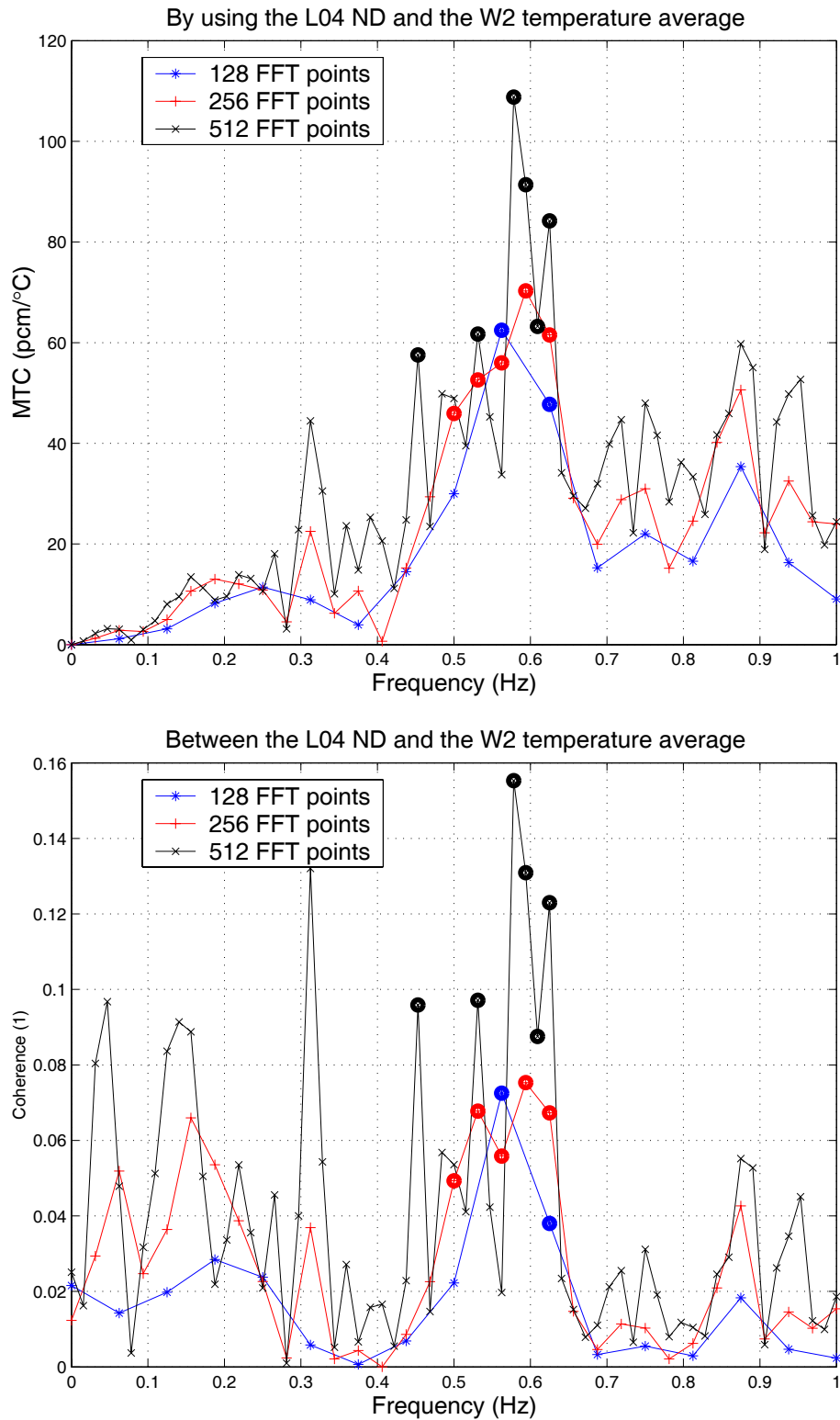
Fig. 24. Frequency dependence of the MTC noise estimation with respect to the number of FFT points used (neutron noise measured in the L04 assembly and temperature noise evaluated by using W2).
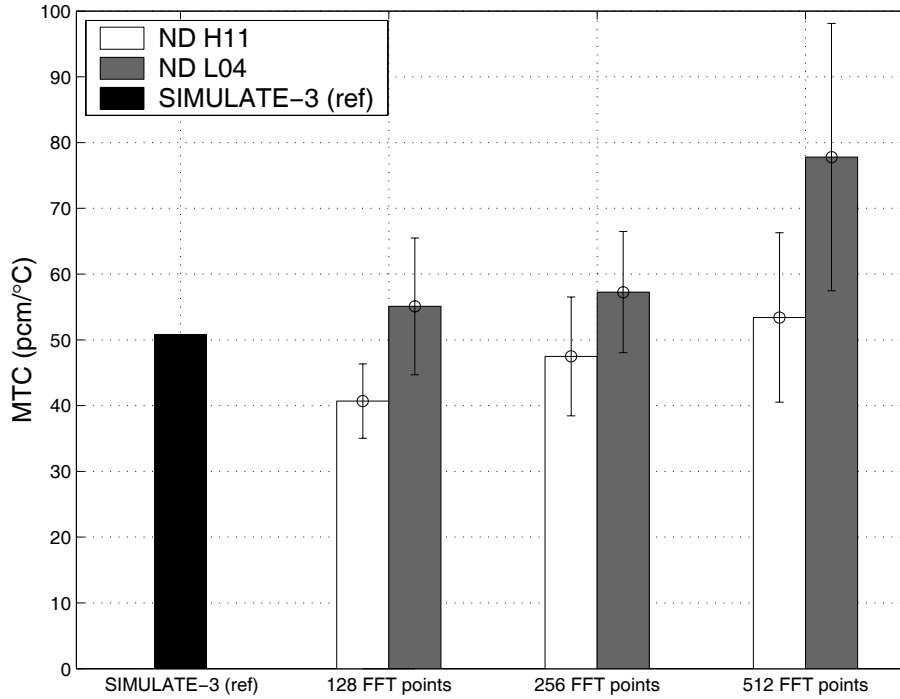
Fig. 25. Comparison of the MTC noise evaluations with respect to the number of FFT points used (temperature noise evaluated by using W2).
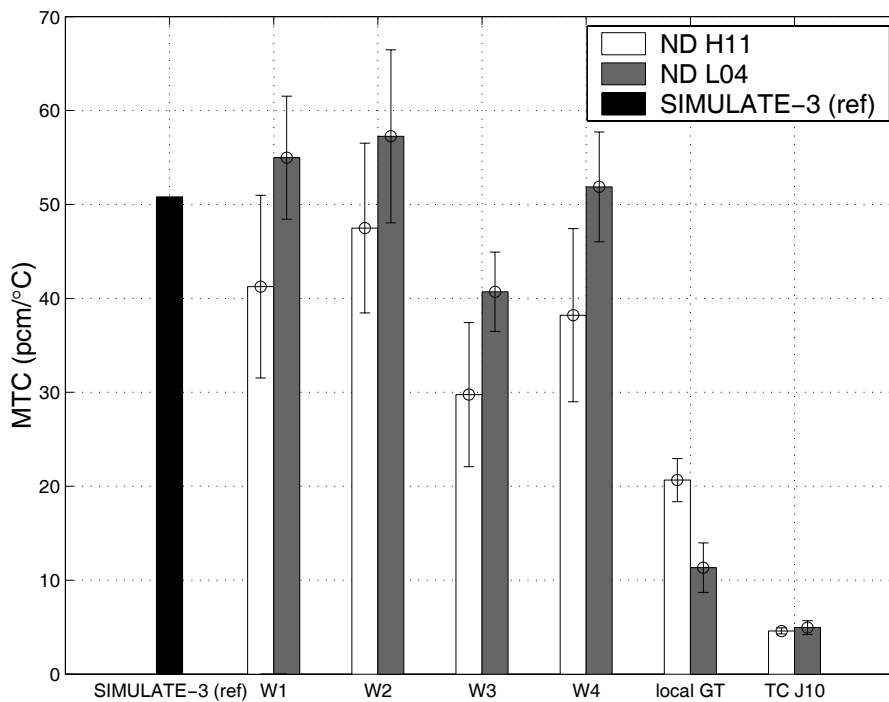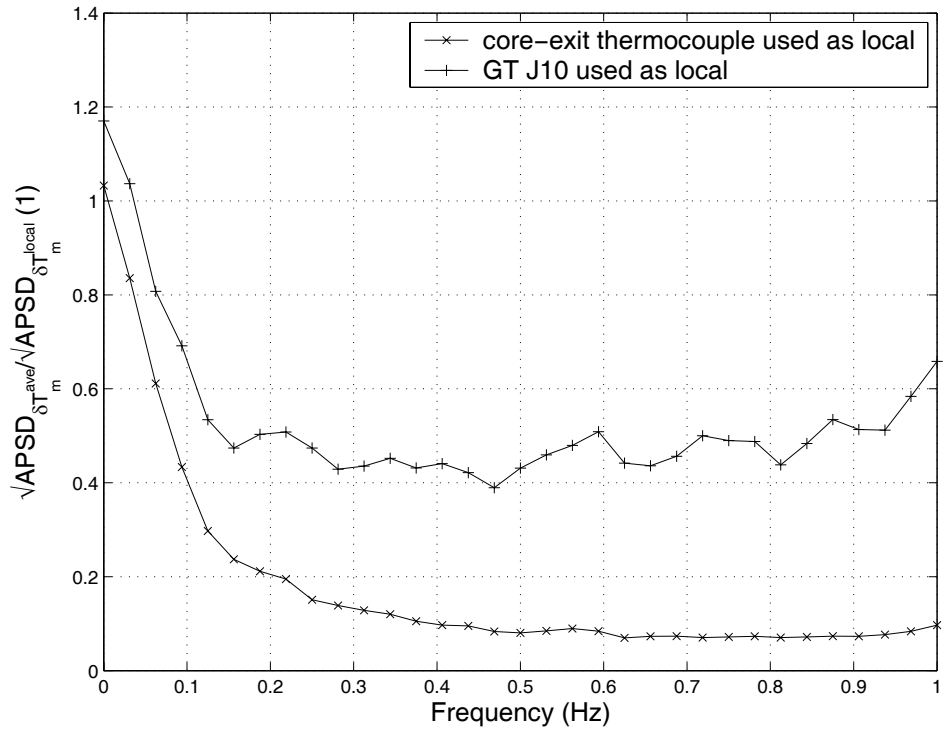

Fig. 26. Comparison of the different MTC noise estimators (estimations carried out with 256 FFT points).

Fig. 27. Comparison of the noise levels recorded by the core-exit thermocouple, a GT, and the planar average of all the GTs.

Chapter 5

Conclusions

In this thesis, the at-power MTC measuring techniques have been reviewed. Two main methods can be distinguished depending on the necessity of disturbing or not disturbing the reactor when performing the MTC measurement. For the former, the most commonly used method worldwide is the boron dilution method, whereas for the later noise analysis offers such a possibility.

For the boron dilution method, it was shown that many calculated parameters are required to estimate the MTC. But because these calculations represent a rather difficult task, it was pointed out that some of the reactivity corrections could have a much larger uncertainty than previously expected, and the confidence interval of the measured MTC could thus be wider. In such a case, the usefulness of the at-power MTC estimation might be questioned since the lower bound of the confidence interval makes it possible to have an MTC more negative than what the Technical Specifications usually allow and since the goal of the measurement if to verify that this threshold is not exceeded. Regarding the measurement technique itself, one might also wonder why the MTC is such an important parameter to estimate since, in case of accidental situations, several reactivity effects, among which the MTC effect, will condition the behaviour of the reactor. Removing the reactivity effects other than the ones associated to the MTC during an MTC measurement is therefore trying to characterise a small part of the feedback chain, whereas taking all the other reactivity effects would be probably more interesting in an accident perspective. Regarding the measurement technique itself, the boron dilution initiates also a plant transient that the operators need to monitor for at least 24 h. In this respect, noise analysis does not require any perturbation of the reactor status and is consequently very well suited to an on-line monitoring of the MTC. In this technique, the MTC is inferred from the neutron and temperature noise, measured traditionally via an in-core neutron detector and a core-exit thermocouple located in the same fuel assembly or in neighbouring fuel assemblies.

It was demonstrated, both via modelling and measurements, that the deviation of the usual MTC noise estimator from its expected value was mostly due to the spatially radially heterogeneous structure of the moderator temperature noise, rather than the resulting deviation from point-kinetics of the reactor response. The underestimation of the MTC by the usual MTC noise estimator is actually due to the overestimation of the moderator temperature noise measured locally, compared to the core average moderator temperature noise, which has to be used according to the American Nuclear Society standard. It was also noticed experimentally that the local moderator temperature noise was overestimated compared to the core average one both inside the core and at the core-exit. The reason for the overestimation of the local moderator temperature noise is the radially loosely coupled character of the moderator temperature noise throughout the core. The moderator temperature noise measured at the core-exit was also found to be much more overestimated than inside the core, even when the local measurements are both performed in the same fuel assembly. This suggests that there is an extraneous moderator temperature noise source at the core-exit, probably due to the coolant mixing above the fuel assemblies (before the core-exit). Regarding the axial dependence of the moderator temperature noise

inside the core, it was noticed that a very slight damping was taking place in the frequency band of interest for the MTC investigation by noise analysis. Consequently, the assumption of inlet temperature noise source, i.e. temperature noise created outside the core, on which all the MTC noise estimators presented in this thesis rely, is valid. This means that only the radial incoherent character of the moderator temperature noise has to be accounted for in order to obtain a correct MTC estimation.

A new MTC noise estimator relying on the core average moderator temperature noise was then proposed. This estimator still assumes a point-kinetic behaviour of the reactor, but this approximation was proven, both theoretically and experimentally, to have a negligible effect on the MTC estimation, so that this new MTC noise estimator always gives an acceptable MTC value. Although it was found that calibrating the usual MTC noise estimator to a known MTC at a given burnup will lead to correct MTC estimations for the remaining part of the cycle, the new MTC noise estimator is completely calibration free. The measurements required to apply this new MTC noise estimator are the local neutron noise (via an in-core neutron detector for instance), and as many as possible in-core thermocouples (GTs in the case of Ringhals-2, which were proven to work as ordinary thermocouples in the frequency range of interest for the MTC estimation). The radial location of the neutron noise measurement inside the core was also demonstrated to have no effect on the noise-based MTC value, thus reinforcing the fact that the deviation of the reactor response from point-kinetics is negligible with respect to the MTC evaluation. Regarding the Ringhals-2 case, the GTs are very versatile tools since they both measure the moderator temperature noise in the frequency range 0.1 - 1.0 Hz and the spatial structure of the static neutron flux. Both of these are required to estimate a proper core average moderator temperature noise to be used in the new MTC noise estimator. A major advantage of using the GTs for both is that the core average moderator temperature noise can be evaluated without the need of core calculations. The new MTC noise estimator relying on the use of the GTs is therefore a very good estimator from a measurement viewpoint, since this measurement technique does not rely too much on parameters that can only be determined via core calculations. The only quantity that is actually needed is the zero-power reactor transfer function, which in the frequency band of interest for the MTC can be very well approximated by the reciprocal of the effective fraction of delayed neutrons. The estimation of this parameter is an easy task with today's reactor codes, and can be considered as reliable and accurate.

New noise measurements are expected to be carried out at the Ringhals-2 PWR so that the reproducibility of these results can be demonstrated. More specifically, several specific points have to be checked, and among them the independence of the results with burnup and even with the fuel cycle. Another characteristic of the noise-based technique that has to be understood is the dependence of the MTC magnitude with frequency in the frequency band 0.1 to 1.0 Hz, whereas the MTC magnitude was expected to be roughly frequency independent. In this respect, the way to estimate the final MTC from its spectrum has to be discussed in detail and possibly improved. Finally, and most importantly, one should also consider the possibility of using the core-exit thermocouples instead of the GTs, since western-type PWRs usually do not have any in-core thermocouples.

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Acknowledgements

Nomenclature

ACF	Auto-Correlation Function
APSD	Auto-Power Spectral Density
AR	Auto-Regressive
ARMA	Auto-Regressive Moving Average
BOC	Beginning Of Cycle
BWR	Boiling Water Reactor
CCF	Cross-Correlation Function
CEA	Commissariat à l'Energie Atomique
CPSD	Cross-Power Spectral Density
DR	Decay Ratio
DWO	Density Wave Oscillation
EOC	End Of Cycle
EOFP	End Of Full Power
FFT	Fast-Fourier Transform
GT	Gamma-Thermometer
HFP	Hot Full Power
HZP	Hot Zero Power
IRF	Impulse Response Function
ITC	Isothermal Temperature Coefficient
LPRM	Local Power Range Monitor
MOC	Middle Of Cycle
MOX	Mixed-Oxide
MTC	Moderator Temperature Coefficient
ND	Neutron Detector
PWR	Pressurized Water Reactor
RCS	Reactor Coolant System
RMS	Root-Mean-Square
SKC	Swedish Centre for Nuclear Technology
SKI	Statens Kärnkraftinspektion (Swedish Nuclear Power Inspectorate)
TC	Thermocouple
UOX	Uranium Oxide

PAPER I

EVALUATION OF THE BORON DILUTION METHOD FOR MODERATOR TEMPERATURE COEFFICIENT MEASUREMENTS

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A measurement of the at-power moderator temperature coefficient (MTC) at the pressurized water reactor Unit 4 of the Ringhals Nuclear Power Plant (Sweden) during fuel cycle 16 is analyzed. The measurement was performed when the boron concentration decreased under 300 ppm in the reactor coolant system, by using the boron dilution method. Detailed calculations were made to estimate all reactivity effects taking place during such a measurement. These effects can only be accounted for through static core calculations that allow calculating contributions to the reactivity change induced by the moderator temperature change. All the calculations were performed with the Studsvik Scandpower SIMULATE-3 code.

Analysis of the measurement showed that the contribution of the Doppler effect (in the fuel) was almost negligible, whereas the reactivity effects due to other than the Doppler fuel coefficient and the boron change were surprisingly significant. It was concluded that due to the experimental inaccuracies, the uncertainty associated with the boron dilution method could be much larger than previously expected. The MTC might then be close to -72 pcm/°C, whereas the main goal of the measurement is to verify that the MTC is larger (less negative) than this threshold. The usefulness of the boron dilution method for MTC measurements can therefore be questioned.

I. INTRODUCTION

The moderator temperature coefficient (MTC) of reactivity is an important safety parameter in pressurized water reactors (PWRs). It is defined as the change of reactivity induced by an inlet temperature change of the coolant, divided by the core average coolant temperature change. This reactivity change must be due solely to the modification of the moderator properties. As a matter of fact, a change in moderator temperature affects the reactivity both directly and indirectly. The direct effects are the modification of the thermal equilibrium temperature of the neutrons due to changes in the thermal scattering of neutrons by water (temperature-only or spectral component of the MTC at a microscopic level), and also the change of the moderator density (density component of the MTC at a macroscopic level).[1] These changes induce

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indirectly a redistribution of the axial flux, and the MTC must account for this as well. Furthermore, if the measurement technique significantly perturbs the axial power shape, the observed change in temperature should accurately reflect the change in the core average moderator temperature, on which the true MTC is dependent according to the standard.[2,3]

The MTC is part of the feedback mechanism and should therefore be negative in most circumstances in order to have a stable reactor. Nevertheless, a positive MTC can be allowed at beginning of cycle (BOC) if transient analyses have proven that there is no safety issue in accident situations. A positive MTC might occur for instance with high-burnup fuel bundles. Because of the high excess of reactivity of such fresh fuel bundles, the corresponding required amount of boron will be larger at BOC. The boron content only affects the density component of the MTC, not the spectral one since ^{10}B has a $1/v$ absorption cross-section behavior. Consequently, if

high-burnup fuel is used, the MTC could become positive at BOC since the change of the thermal utilization factor with the moderator temperature, which is normally positive, is so large that it cannot be compensated any longer by the decrease of the resonance escape probability. This is equivalent to saying that increasing the boron concentration too much will reduce the probability of neutrons being scattered/captured by water, therefore minimizing the effect of decreasing reactivity when the moderator temperature is raised, which could lead to a positive MTC. As a matter of fact, power utilities nowadays are willing to use high-burnup fuel bundles, so that the fuel economy could be improved, i.e., the cycle length could be increased. They argue that a positive MTC at BOC could be allowed since the Doppler effect will still ensure a negative feedback. In case of mixed-oxide fuel, the ^{239}Pu resonance might also render the MTC positive, due to the spectral effect only. The presence of the 0.3-eV resonance of ^{239}Pu implies that a coolant temperature increase will increase both the thermal utilization factor and the thermal fission factor. Consequently, monitoring of the MTC might become of prime importance in a near future.

Although measuring the at-power MTC is a very common practice worldwide, the MTC measurement was given new attention some years ago with the development of a new measuring technique, namely, the MTC estimation by noise analysis.[4,5] In this technique, an in-core neutron detector and a core-exit thermocouple located above the same fuel channel or one of the neighboring fuel channels, respectively, are used. The noise signals provided by these two detectors contain some information about the MTC, which can be extracted by using an appropriate noise estimator. The main advantage of this technique is that the reactor does not need to be perturbed in order to be able to estimate the MTC. Nevertheless, several attempts to monitor the MTC by noise analysis revealed that the MTC was systematically underestimated by a factor of 2 to 5 (Refs. 4 through 23). Several reasons could explain the underestimation of the MTC by noise analysis. Some of them were investigated in the past, such as the deviation of the reactor response from point kinetics, the effect of the axial separation distance between the in-core neutron detector and the core-exit thermocouple, the possible generation of temperature fluctuations at all axial elevations during coolant flow, the fact that several other noise sources may coexist at the same time, and finally the presence of the Doppler effect.[12,13,22,24–27] These corrections are nevertheless usually small, cannot be easily estimated in practice, and cannot either explain solely the strong deviation of the MTC noise estimate from the true value. It is generally accepted that the traditional techniques give a good MTC estimation, whereas it is believed that the noise method systematically gives an incorrect MTC evaluation.[4–23] Such a belief was strengthened recently when it was suggested that a radial nonhomogeneous

structure of the temperature noise throughout the core could explain the MTC underestimation by noise analysis.[28] There is also some experimental evidence that the temperature noise is not spatially homogeneous.[29]

However, the traditional measurement techniques might be questionable as well since their accuracy is relatively poor. Furthermore, in contrast to the noise analysis technique, they do not induce solely a change of the moderator temperature but also other reactivity effects that can be accounted for only through static core calculations. It is important to understand that all the traditional measuring methods make use of calculated parameters. Moreover, one judges the accuracy of the MTC measurement by comparing it to the MTC calculation performed by static codes. Consequently, even if both methods seem to coincide, comparing the results of the measurements to the MTC calculation might hide some inaccuracy since the same calculational tools are used in both cases. Only the zero-power isothermal temperature coefficient (ITC) calculation—ITC, which comprises both moderator and fuel reactivity effects—has been benchmarked against measurements.[30] At the Ringhals nuclear power plant, this measurement is carried out by using a digital reactivity meter and the core-exit thermocouples. The digital reactivity meter uses the neutron flux signal as input and evaluates the corresponding reactivity by adopting the one-point reactor kinetics model. This ITC measurement, of which the precision is within 0.5 pcm/°C (Ref. 31), can be considered as reliable for two main reasons. First, the temperature change is uniform both in the fuel and the moderator and can therefore be measured accurately. Second, today's reactivity meters can measure the reactivity with a high level of accuracy.

Consequently, not only the noise method but also the traditional measurement techniques need to be studied in detail. The aim of this paper is to analyze the traditional measurement techniques. The study of the noise analysis technique, which will not be reported in the following, is also in progress, and some preliminary results can already be found in Ref. 28. As the boron swap is the most commonly used method worldwide and since a measurement performed at the PWR Unit 4 of the Ringhals Nuclear Power Plant was available to us, it was decided to investigate how the MTC is determined by the boron dilution method and how reliable the reactivity coefficient is. First, the boron dilution method is described in detail, with emphasis on both its advantages and weaknesses. Then, the Ringhals measurement is analyzed. The available data are presented, and the steps that need to be estimated by core calculations are highlighted. The Studsvik Scandpower SIMULATE-3 code was used in all the core calculations throughout this study.[32] The MTC and its associated uncertainty are estimated using these sets of available and calculated data. Finally, the MTC estimated from the measurement is compared to the MTC calculated directly by

SIMULATE-3, and some conclusions are drawn regarding the reliability of the MTC measurement using the boron dilution method.

II. THE AT-POWER MTC MEASUREMENT METHODS

Several measurement techniques exist for the measurement of the MTC that are based on the perturbation of the reactivity in the reactor and then compensating for it by a change of the coolant temperature of the core. The main traditional methods are as follows [2,33–35]:

1. the power change or xenon transient method, in which the power level of the reactor is changed (and so is the xenon concentration). The corresponding reactivity effect is then compensated by a modification of the inlet temperature of the core in order to keep the reactor critical.

2. the depletion or stretch-out method, in which the boron concentration of the core is maintained constant during about 1.5 days. As a result of the fuel depletion, the coolant average temperature needs to be decreased.

3. the control rod swap method, in which the inlet temperature of the core is modified. The corresponding reactivity effect is compensated by a modification of the insertion of some of the control rods (the control rod worth can be either calculated or measured previously).

4. the boron dilution method, in which the inlet temperature of the core is increased. The reactor is kept critical by diluting the boron content.

In the following, the boron dilution method is described in detail. After recalling the MTC definition, the measurement procedure used at the Ringhals Nuclear Power Plant in Sweden is presented. Finally, the advantages and drawbacks of this measuring technique are discussed.

II.A. Definition of the MTC

The MTC is defined as the partial derivative of the reactivity ρ with respect to the core average moderator temperature T_m^{ave}:

$$\text{MTC} = \frac{\partial \rho}{\partial T_m^{ave}} \ . \tag{1}$$

For a small change in the average moderator temperature, then, the reactivity change would be

$$\Delta \rho \approx \text{MTC} \times \Delta T_m^{ave} \ . \tag{2}$$

The newest American National Standard says that the way of calculating the coolant temperature average does not play a significant role as long as the same methodology is used in both the measurement and the calculations.[3] The standard thus implicitly proposes to simply use a volume average of the temperature change:

$$\Delta T_m^{ave} = \frac{\int \Delta T_m(\boldsymbol{r}) \, d\boldsymbol{r}}{\int d\boldsymbol{r}} \ . \tag{3}$$

Nevertheless, as pointed out in Ref. 28, a more suitable core average moderator temperature change is obtained if the average is calculated by using the square of the static flux as a weighting function as explained in the following.

In principle, a temperature change affects very much the removal cross section and to a lesser extent the absorption cross section. The change induced by the removal cross-section change can only be accounted for in a two-group representation. On the other hand, a shift of the thermal spectrum of the moderator and thus that of the thermal neutrons will lead to increased absorption in the fuel. This phenomenon can be modeled in a one-group model, and this is what we shall use here. Therefore, if one assumes that in a one-group model the space-dependent temperature change is directly proportional to the space-dependent macroscopic absorption cross section of the homogeneous mixture fuel + moderator via a space-independent coefficient K,

$$\Delta T_m(\boldsymbol{r}) = K \times \Delta \Sigma_a(\boldsymbol{r}) \ ; \tag{4}$$

then for small changes, first-order perturbation theory is applicable, and one can write in one-group theory:

$$\Delta \rho = \frac{-\int \Delta \Sigma_a(\boldsymbol{r}) \phi_0^2(\boldsymbol{r}) \, d\boldsymbol{r}}{\nu \Sigma_{f,0} \int \phi_0^2(\boldsymbol{r}) \, d\boldsymbol{r}} \tag{5}$$

with

$$\nu \Sigma_{f,0} = \frac{\int \nu \Sigma_f(\boldsymbol{r}) \phi_0^2(\boldsymbol{r}) \, d\boldsymbol{r}}{\int \phi_0^2(\boldsymbol{r}) \, d\boldsymbol{r}} \ . \tag{6}$$

If the coolant temperature change is homogeneous throughout the core, then the MTC is directly given by

$$\text{MTC} = -\frac{1}{K \nu \Sigma_{f,0}} \ . \tag{7}$$

Since PWRs are not usually instrumented with in-core thermocouples, but only with core-exit thermocouples (at a few core-exit fuel channels), core-inlet and core-outlet thermocouples, the Standard suggests using the following definition for the temperature average:

$$\Delta T_m^{ave} = \frac{\Delta T_{in} + \Delta T_{out}}{2} \quad , \qquad (8)$$

where *in* and *out* stand for the core-inlet and the core-outlet, respectively. If the temperature change is not homogeneous throughout the core, this definition will obviously not accurately reflect the change in the core average moderator temperature, on which the true MTC is dependent according to the Standard. An average that reflects this distribution must be used. A nonhomogeneous temperature change should nevertheless give the same reference MTC as the one given by Eq. (7) since the MTC is independent of the spatial structure of the temperature change throughout the core. The only possibility to fulfill Eqs. (4) through (7) and Eq. (2) is to define a core average temperature change by using the square of the static flux as a weighting function, as follows:

$$\Delta T_m^{ave} = \frac{\int \Delta T_m(\boldsymbol{r}) \phi_0^2(\boldsymbol{r}) \, d\boldsymbol{r}}{\int \phi_0^2(\boldsymbol{r}) \, d\boldsymbol{r}} \quad . \qquad (9)$$

The approximation of proportionality between the change of the macroscopic absorption cross section and the change of the moderator temperature, i.e., Eq. (4), has been verified via SIMULATE-3 calculations performed at different core-inlet temperatures on a zero-dimensional system, i.e., equivalent to a homogeneous reactor.

Usually, the MTC must be determined twice during each fuel cycle: at BOC at hot zero power, and near end of cycle (EOC) at hot full power. Because of the decrease of the boron content during the cycle to compensate for the fuel depletion, the magnitude of the MTC increases from BOC to EOC. Therefore, the objective of the measurement early in the cycle is to demonstrate that the MTC is negative (preventing the consequences of a positive power feedback), while the objective near the EOC is to show that it remains less negative than some prescribed limit (preventing the consequences of a reactivity increase following a cooldown event).

The ITC, which is actually measured at BOC, is determined by inducing a heatup/cooldown cycle while the reactor is at zero power. Since the ITC measurement is usually considered as accurate and reliable, only the at-power MTC measurement will be discussed in the following. More specifically, the boron dilution method, which is used in Ringhals, will be presented.

II.B. Description of the Boron Dilution Method

The method for determining the at-power MTC used at the Ringhals Nuclear Power Plant (and for most other PWR units) is to measure how much boron one needs to dilute in order to keep the reactor power constant for a given increase of the average coolant temperature. The measurement procedure is initiated by increasing the core-inlet temperature by means of the change of the heat removed from the primary loop (by varying the steam generator load). This is balanced by boron dilution, and the calculated differential boron worth is used both for the predetermination of this dilution and in the estimation of the MTC itself. As will be seen later on (see Sec. III.B), additional core calculations are required in order to be able to estimate the MTC because many reactivity effects are taking place during a boron dilution, and the reactivity change associated with the MTC effect should not include all of these changes. Since these contributions cannot be measured, core calculations are necessary to estimate them.

The following procedure is applied in practice at the Ringhals Nuclear Power Plant[36]:

1. First of all, the reactor is stabilized at steady-state conditions.

2. Immediately before the increase of the core-inlet temperature, the boron concentration is measured three times, with a time interval of about 15 min. The boron concentration is measured by chemical titration. The boron concentration is taken as the mean value over these three samples in order to reduce the effect of the relatively large uncertainty of the chemical titration technique.

3. Since the reactivity compensation has to be carried out with the use of boron dilution solely, the manual operating mode of the control rods must be switched on, thus preventing any automatic modification of the control rod positioning.

4. The core-inlet temperature is then increased slowly (increase of approximately 2.0°C during about 4 h); the reactor power should be kept constant during this transient by means of boron dilution (perfect reactivity compensation).

5. The core average temperature and the power are held constant for 2 h after the temperature rise, and the boron concentration measured accordingly.

6. The core average temperature is finally brought back to its initial level in about 1 h; the axial offset, defined as

$$AO = \frac{P_{up} - P_{down}}{P_{up} + P_{down}} \quad , \qquad (10)$$

where P_{up} and P_{down} are the power averaged on the upper part and the lower part of the core, respectively, is then set to an acceptable value.

7. The control rods are switched back to their automatic operating mode.

During the measurement, the temperature rise induces an increase of the negative axial offset since the moderator density change is not axially linear, i.e., bigger

in the upper part of the core than in the lower one. This fact is due to the axial distribution of the coolant temperature (increase from bottom to top because of the nuclear heating) although the temperature increase is roughly homogeneous throughout the core. If the axial offset becomes larger in magnitude than some prescribed limit, control rods are used to maintain the axial offset in the prescribed operating range. In such a case, the reactivity contributions corresponding to the use of the control rods have to be calculated in order to estimate the MTC solely without any parasitic reactivity effects.

II.C. Advantages and Weaknesses of the Boron Dilution Method

From the viewpoint of power utilities, a good method for measuring the MTC should have an acceptable level of accuracy, be easy to implement and carry out, and not perturb the reactor operation. From the experimental viewpoint, a good measuring technique should rely on as few as possible calculated and precalculated parameters. As will be seen in the following, the boron dilution method, like any other traditional method to estimate the at-power MTC, does not fulfill all these requirements.[2] Depending on which method one uses, the advantages and weaknesses can be different.

The main strength of the boron swap method is probably that the boron dilution induces a relatively small modification in the axial power shape. Therefore, the definition of the average temperature in Eq. (8) is representative of the core average moderator temperature change with reasonable accuracy, if the control rods are not used during the measurement (such as for compensating the axial offset). Otherwise, a relatively simple adjustment is necessary to account for the change of the axial power shape. Such an adjustment is done via core calculations. The second main strength of the boron dilution method is the differential boron worth, which is required for both the predetermination of the boron dilution and the estimation of the MTC itself. Although this parameter is a result from core simulations, it can be easily predicted to a high level of accuracy with today's reactor codes [accuracy of 1 to 2% (Ref. 2)]. Furthermore, the differential boron worth is not particularly sensitive to the range of conditions encountered during the measurement.

The differential boron worth, which is a calculated parameter, constitutes itself one of the drawbacks of the measurement technique since, as written previously, a good measurement technique should rely on as few core-calculated parameters as possible. This method has other weaknesses such as the relatively large uncertainty in the measurement of the boron concentration. Another major concern about this technique is the time required to ensure that the boron concentration is in equilibrium and is not changing. In usual cases, this test can take up to 12 h to perform. The length of this period increases

the probability of change of other core parameters such as power, xenon worth, and fuel burnup. The reactivity contributions of these are thus required in order to correctly estimate the MTC. These contributions can only be evaluated by core calculations, not measured directly. In some cases, although the test is performed at near full power, it is considered inappropriate to perform the measurement at exactly 100% power because of the proximity of the high flux trip setpoints and the effects of changing moderator density on ex-core detector response.[2] Performing the test at reduced power during several hours represents a considerable loss of production for power utilities. Finally, and it is the most important, this test induces a plant transient that the operators must monitor for about 24 h until steady-state conditions are again achieved. Furthermore, another issue with the boron dilution method is the neutronic depletion of the soluble ^{10}B in the reactor coolant, which must be accounted for. The boron content is estimated by chemical methods and cannot therefore identify the depletion of boron in ^{10}B atoms. Assuming a concentration of 19.78 at.% of ^{10}B in natural boron cannot be valid any longer. Codes are available to estimate this depletion.[37–39] They account for the reactor history in boration and boron dilution, from which the amount of depleted ^{10}B can be evaluated. If the reactor has been operated at full power for a relatively long time, the boron is most likely depleted to such an extent that the reactivity contribution of this effect to the differential boron worth is necessary.[2]

III. ANALYSIS OF THE ACTUAL MEASUREMENT

In the following, an MTC measurement performed at the Ringhals Nuclear Power Plant in Sweden is described in detail. The measurement was performed at Unit 4 on May 5, 1999, for the fuel cycle 16. Ringhals-4 is a three-loop Westinghouse-type PWR with a net power of 915 MW(electric) (thermal power of 2775 MW). It started its commercial operation in November 1983. The total coolant flow is $12\,860$ kg·s^{-1} at nominal conditions, the coolant inlet temperature 284°C, and the coolant outlet temperature 323°C. The core contains 157 fuel bundles of the traditional 17×17 PWR design. Only uranium dioxide fuel is used in Ringhals-4. The average core burnup corresponding to the measurement was estimated to be 9.040 GWd/tHM by core calculations using the SIMULATE-3 code.

The methodology presented in the following is exactly the one that was used at that time in Ringhals to evaluate the at-power MTC. Nevertheless, all the calculations were performed at the Department of Reactor Physics, Chalmers University of Technology, with the code SIMULATE-3. Because of different versions of the code in use at Chalmers and at Ringhals, the results might differ slightly. The MTC estimated by Ringhals was found

to be equal to -50 pcm/°C (SIMULATE-3 version 5.08.05), whereas the one calculated by our SIMULATE-3 version was equal to -46 pcm/°C.

III.A. Description of the Available Measurements

According to the technical specifications of the Ringhals Unit 4, one has to check that the MTC at full power is larger (less negative) than -72 pcm/°C seven days equivalent full power after the boron concentration in the core has reached 300 ppm (about 2 to 3 months before the end of full power (EOFP) (Ref. 40). Although this MTC estimation is called in the following the EOC MTC, this MTC measurement does not correspond exactly to the EOC MTCs (and not even the EOFP MTCs) since the boron concentration is about 300 ppm in the reactor coolant system (RCS). The EOFP is defined as the moment in the fuel cycle when the boron concentration in the RCS approaches 0 ppm. At that point, it is still possible to operate the reactor by decreasing the power level. The positive reactivity gained by the decrease of the power level allows compensating the reactivity lost by the fuel depletion for about two more months. When it is not possible to operate the reactor any longer, the EOC is reached. The part of the fuel cycle between EOFP and EOC is called the coastdown. Consequently, the MTC measurement is actually performed 4 to 5 months before the actual EOC, i.e., when the reactor has to be shut down. Since the purpose of the MTC measurement is to verify that the MTC will not become lower than some prescribed value during the remaining part of the cycle, the measured MTC needs to be extrapolated. For that purpose, SIMULATE-3 can be used to estimate the MTC change during that part of the cycle. The MTC measurement is therefore used as a calibration or a checking of the SIMULATE-3 ability to correctly predict the MTC at 300 ppm.

In the MTC measurement analyzed below, the available measured parameters were the following:

1. *The average moderator temperature*: For each loop, one calculates an average temperature as the average between the cold leg and the hot leg:

$$T_{m,i}^{ave} = \frac{T_{in,i} + T_{out,i}}{2} \quad , \tag{11}$$

where i denotes the loop number. The core average moderator temperature is then defined as the mean value of the three loops:

$$T_m^{ave} = \frac{1}{3} \sum_{i=1}^{3} T_{m,i}^{ave} \quad . \tag{12}$$

This core average moderator temperature is in accordance with the Standard and with Eq. (8). As pointed out previously, the boron dilution method does not perturb significantly the axial power shape, so that this simple definition of the core average temperature can be used. As a matter of fact, the actual core average coolant temperature might be slightly different from the one given by Eq. (11). But the increase of the core average moderator temperature is satisfactorily given by Eq. (12) since the increase of the coolant temperature is roughly axially homogeneous.

2. *The relative power*: This is calculated via the temperature difference between the cold leg (average of the three loops) and the hot leg (average of the three lops).

3. *The boron concentration*: For each boron measurement, three samples were analyzed and the mean value was taken. As pointed out previously, taking several samples at 15-min intervals allows reducing the relatively large uncertainty associated with the boron chemical titration.

The measurement of these parameters is depicted in Fig. 1. The data were recorded from 07:30 a.m. to 10:00 p.m., whereas the core transient was initiated at 07:51 a.m. For the sake of simplicity, the origin of the time-axis (time = 0) in the following figures (Figs. 1 through 5) corresponds to the beginning of the measurement campaign, i.e., 07:30 a.m.

III.B. Calculation of the Reactivity Contributions

As explained earlier, the boron concentration and the core average temperature are not the only parameters that vary during the measurement. Other phenomena may affect the core as well, such as the Doppler effect (due to a fuel temperature change), the fuel depletion, the xenon redistribution, the change in the neutron leakage, and the change in the axial flux profile (only the change due to the measurement technique itself, not due to the change in the moderator properties, which must be accounted for in the MTC according to the Standard[3]).

Since these changes cannot be measured in practice, they were determined with the use of core calculations via the SIMULATE-3 code. As a matter of fact, three different calculations were carried out:

1. As the first step, one has to calculate the Doppler coefficient and the boron worth. The Doppler coefficient will be used later on to estimate the Doppler reactivity effect, which should be removed from the total reactivity change. Otherwise, the MTC would include the Doppler effect. The boron worth will be used to convert the change of the boron concentration in a reactivity change. The calculations are done in the reactor state prior to the measurement, i.e., before the plant transient. SIMULATE-3 performs the calculation of the Doppler coefficient by changing the fuel temperature solely by $+5$°F ($+2.78$°C); all other parameters remaining unchanged. The same methodology is applied to the calculation of the boron worth, where the boron concentration is increased by $+10$ ppm, with the other
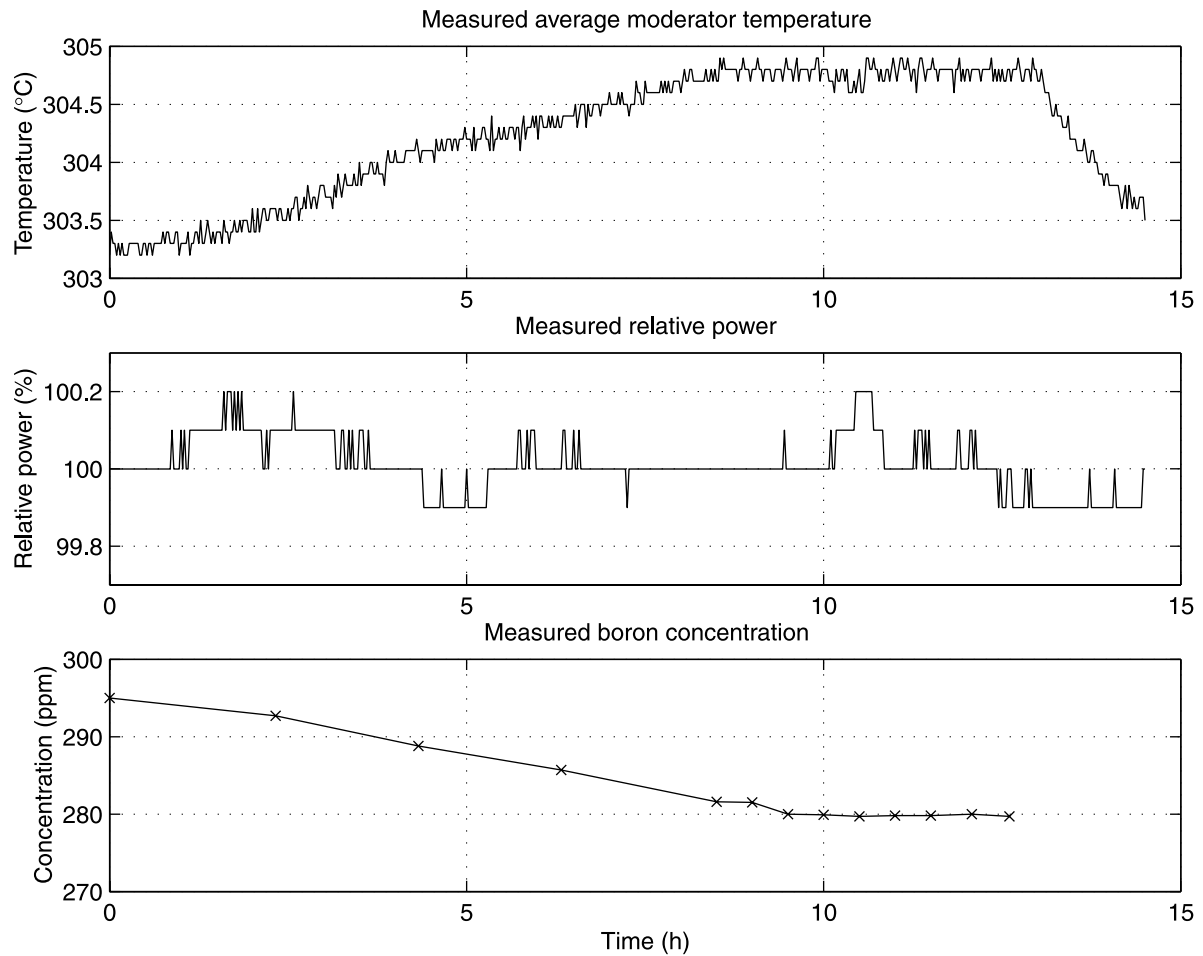
Fig. 1. Available measured parameters for the EOC MTC measurement at Ringhals-4, cycle 16.

parameters remaining constant. The calculation of the MTC coefficient is also performed via SIMULATE-3 directly, in order to be compared to the result of the MTC measurement. In this calculation, SIMULATE-3 increases the inlet moderator temperature by +5°F, with all other parameters kept constant.

2. In the second step, the change of the fuel temperature itself has to be estimated during the measurement so that the Doppler reactivity effect, defined as the product between the Doppler coefficient and the fuel temperature change, could be calculated. By using small depletion steps (between 15 min and 1 h) and the corresponding measured relative power and core-inlet temperature as input parameters to SIMULATE-3, the fuel temperature change can be determined and so can the Doppler reactivity effect (see Fig. 2). What is estimated by SIMULATE-3 is actually a volume average of the fuel temperature at the end of each depletion step (this calculation is carried out by usual thermal-hydraulics/neutronics iterations). The difference between the final

and the initial values gives the fuel temperature change that occurred during the measurement. The calculation is performed by requiring a critical reactor, i.e., the boron concentration is automatically adjusted by SIMULATE-3 to maintain the core critical. In principle, this calculated boron concentration should be very close to the real one, and so should be the calculated average moderator temperature to the measured one. As can be seen in Fig. 3, the calculated data are in very good agreement with the measured ones. Further, this calculation is assumed to reproduce as closely as possible the real measurement, so that the axial offset, defined by Eq. (10), can also be estimated.

3. Finally, it remains to calculate the reactivity contributions other than the Doppler effect. The time steps chosen for this calculation are the ones corresponding to the boron measurements. During each time step, one maintains the power, the inlet temperature, and the boron concentration constant to their value at the beginning of the depletion step (as obtained from the measurements).
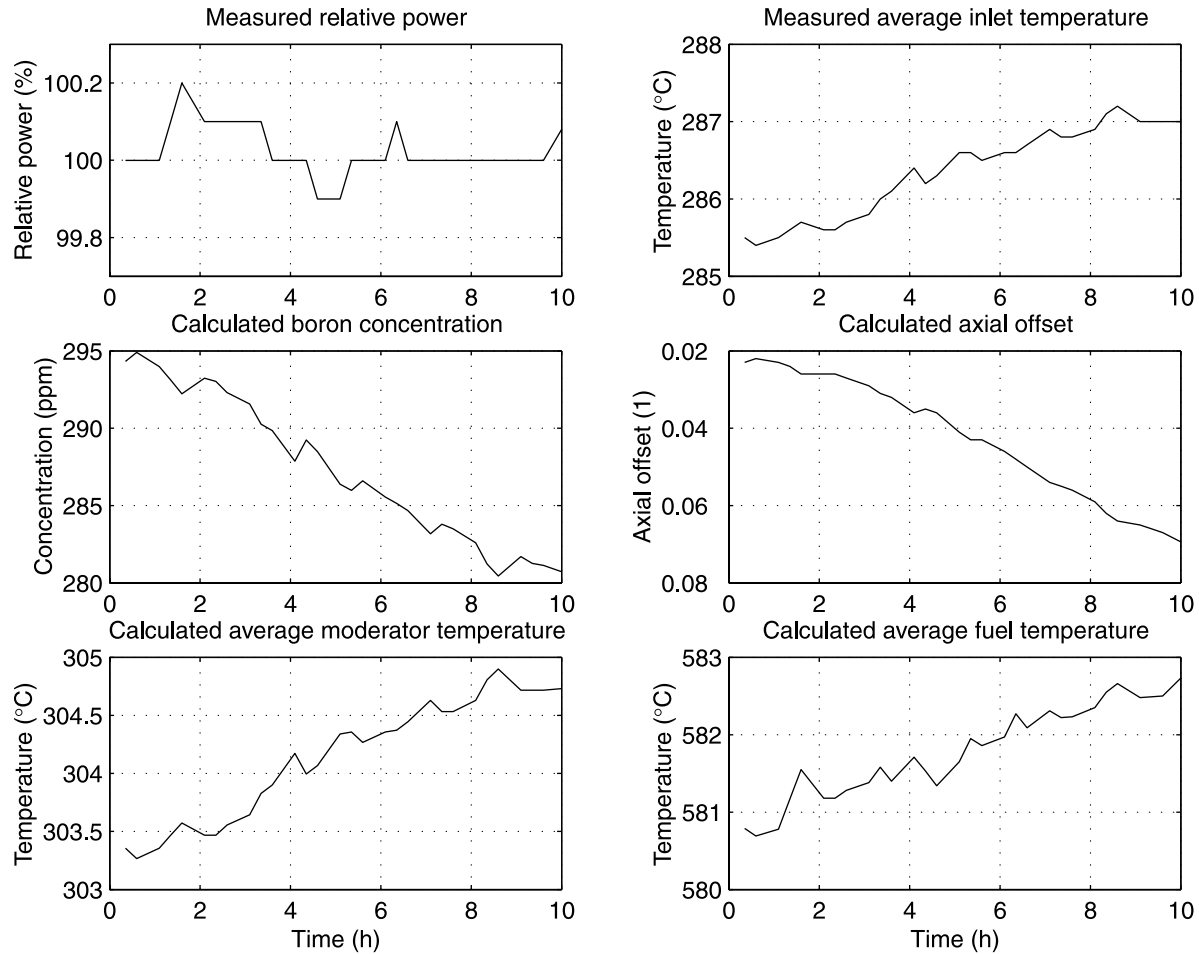
Fig. 2. Calculation of the Doppler contribution for the EOC MTC measurement at Ringhals-4, cycle 16 (the measured parameters were used as input parameters to SIMULATE-3, whereas the calculated parameters are the results of the SIMULATE-3 calculations).

Because of the fuel depletion, the effective multiplication factor varies over each time step. The difference of k_{eff} between the end and the beginning of each step defines the change of reactivity due to effects other than the moderator temperature change, the boron change, and the fuel temperature change, since these quantities have been kept constant (or almost constant) during each time step (as can be seen in Fig. 4, the fuel temperature change calculated by SIMULATE-3 during a time step is negligible). Summing these reactivity changes, i.e., neglecting the jumps in the plot of k_{eff} in Fig. 4, the reactivity contribution to the MTC measurement other than the Doppler effect is obtained. A comparison between the measured and the calculated data and between the measured and input data is also depicted in Fig. 5.

In all these calculations, the input data used for SIMULATE-3 are strictly the ones used by the nuclear engineer in Ringhals who made the MTC evaluation. Therefore, it will not be discussed in the following why and how these input data were chosen.

III.C. Determination of the MTC

In the boron dilution method, assuming that the reactivity does not change during the whole measurement (perfect reactivity compensation), the MTC can be estimated as

$$\text{MTC} = \frac{\partial \rho}{\partial T_m} = \frac{1}{k_{eff,1} \cdot k_{eff,2}} \frac{\partial k_{eff}}{\partial T_m}$$

$$\approx \frac{1}{\Delta T_m} \left(\Delta \rho - \frac{\partial \rho}{\partial T_f} \Delta T_f - \frac{\partial \rho}{\partial C_B} \Delta C_B - \Delta \rho^* \right) ,$$

$$(13)$$

where

k_{eff} = effective multiplication factor (the subscripts 1 and 2 refer to the initial and the final states, respectively)

ρ = reactivity and $\Delta \rho$ = reactivity change during the measurement, if there is any
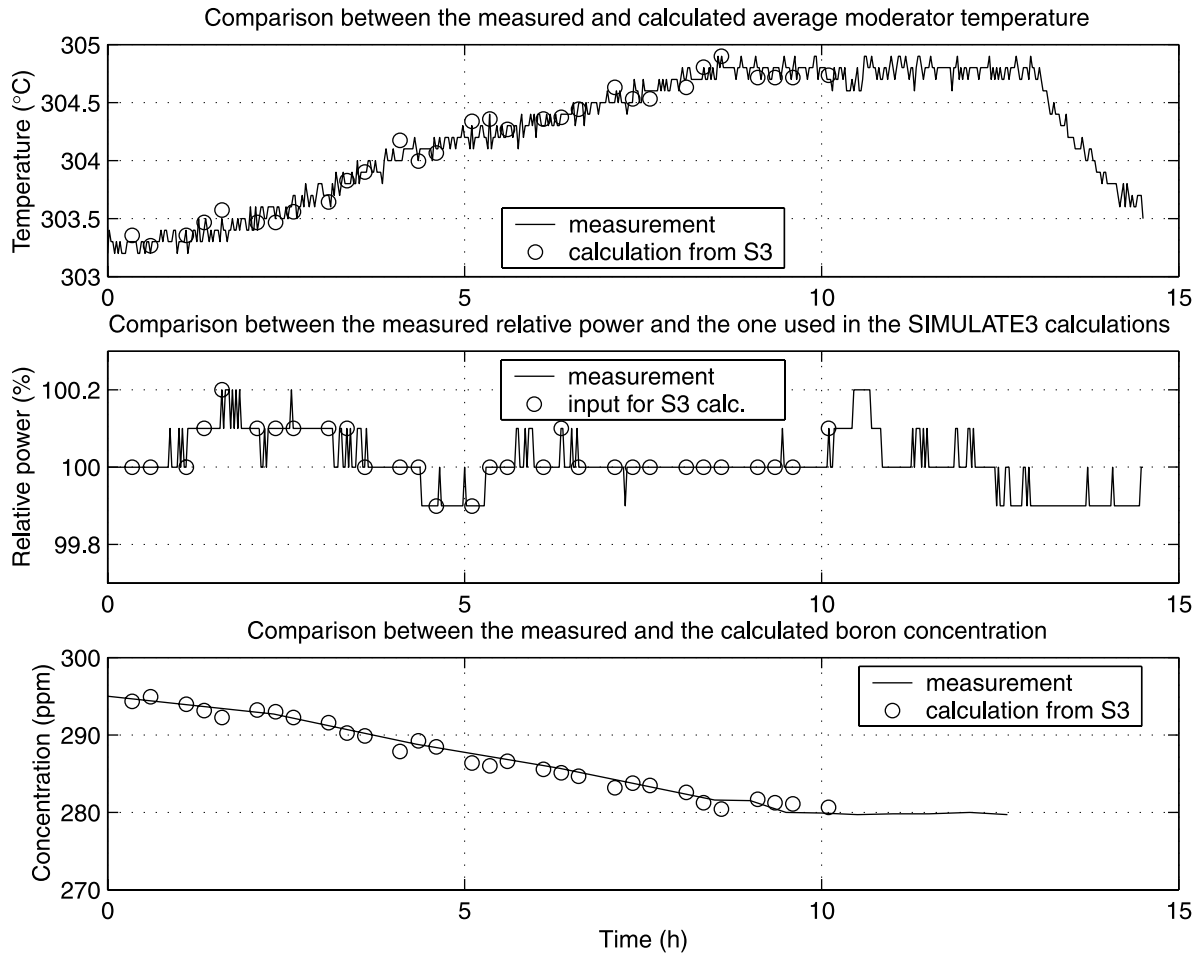
Fig. 3. Comparison between the calculated/input data of SIMULATE-3 and the measured data for the Doppler contribution to the EOC MTC measurement at Ringhals-4, cycle 16.

T_m = core average moderator temperature [the average is defined as in Eq. (8)]

T_f = core average fuel temperature

C_B = boron concentration

$\Delta\rho^*$ = reactivity contribution to the MTC other than the Doppler effect and the boron concentration change.

The power change does not appear explicitly in Eq. (13) but is used to calculate the Doppler contribution and the remaining reactivity contributions according to the methodology described in the foregoing (see Figs. 1 through 5).

The boron concentration, the core average moderator temperature, and the power are the only measured parameters that are accessible during an MTC measurement (the control rods were maintained fully withdrawn during this measurement campaign so that one does not need to take them into account in this MTC determina-

tion). Using the data between 07:30 a.m. and 07:51 a.m. (0 and 0.35 h on the time-axis of the figures) for the initial state, and the data between 05:00 p.m. and 05:30 p.m. (9.5 and 10 h on the time-axis of the figures) for the final state gives the following (see Fig. 1):

1. a boron concentration change of $\Delta C_B \approx -15.05$ ppm

2. an average moderator temperature change of $\Delta T_m \approx 1.53°C$.

As can be seen in Fig. 2, the axial offset becomes more negative during the measurement but remains within the operating range. Consequently, the control rods were not used. Otherwise, the reactivity effect due to the modification of the insertion of the control rods should have been taken into account, and another reactivity contribution in Eq. (13) should have been calculated accordingly.

Since one assumes a perfect reactivity compensation, the reactivity does not change during the whole measurement and thus $\Delta\rho = 0$.

Fig. 4. Calculation of the reactivity contributions other than the Doppler effect for the EOC MTC measurement at Ringhals-4, cycle 16 (the measured parameters were used as input parameters to SIMULATE-3, whereas the calculated parameters are the results of the SIMULATE-3 calculations).

All the other parameters are the results of calculations that were performed at our Department using the SIMULATE-3 code:

1. *Doppler coefficient*: $\partial\rho/\partial T_f \approx -2.61$ pcm/°C

2. *differential boron worth*: $\partial\rho/\partial C_B \approx -8.35$ pcm/ppm. This value assumes that the boron is undepleted. Since in the boron dilution method, one part of the coolant, containing depleted ^{10}B, is replaced by water (dilution), one has to take into account the fact that the reactivity worth of the removed boron is lower than the one determined by core calculations; for a core burnup of 9 GWd/tHM, the differential boron worth given by SIMULATE-3 must be multiplied by 0.935 [correction factor calculated by Vattenfall (in Ref. 40)], so that the differential boron worth is $\partial\rho/\partial C_B$: -7.81 pcm/ppm.

3. *average fuel temperature change*: $\Delta T_f \approx 1.79$°C

4. *other reactivity effects*: $\Delta\rho^* \approx -25$ pcm between the initial and the final stages. This term has been calculated as explained before, i.e., by summing the reactivity changes other than the Doppler reactivity effect on each time step (see Fig. 4).

5. *inlet temperature coefficient*: $\partial\rho/\partial T_{in} \approx -41.35$ pcm/°C (this coefficient will be needed in the following, but it is not required for the MTC estimation directly, see Sec. III.D below).

With all the values listed above, the MTC measured by the boron dilution method can be estimated, and one obtains a value of -58.12 pcm/°C.

III.D. Uncertainties Associated with the MTC Measurement

In this section we shall estimate the inaccuracy of the measured value of the MTC as a function of the

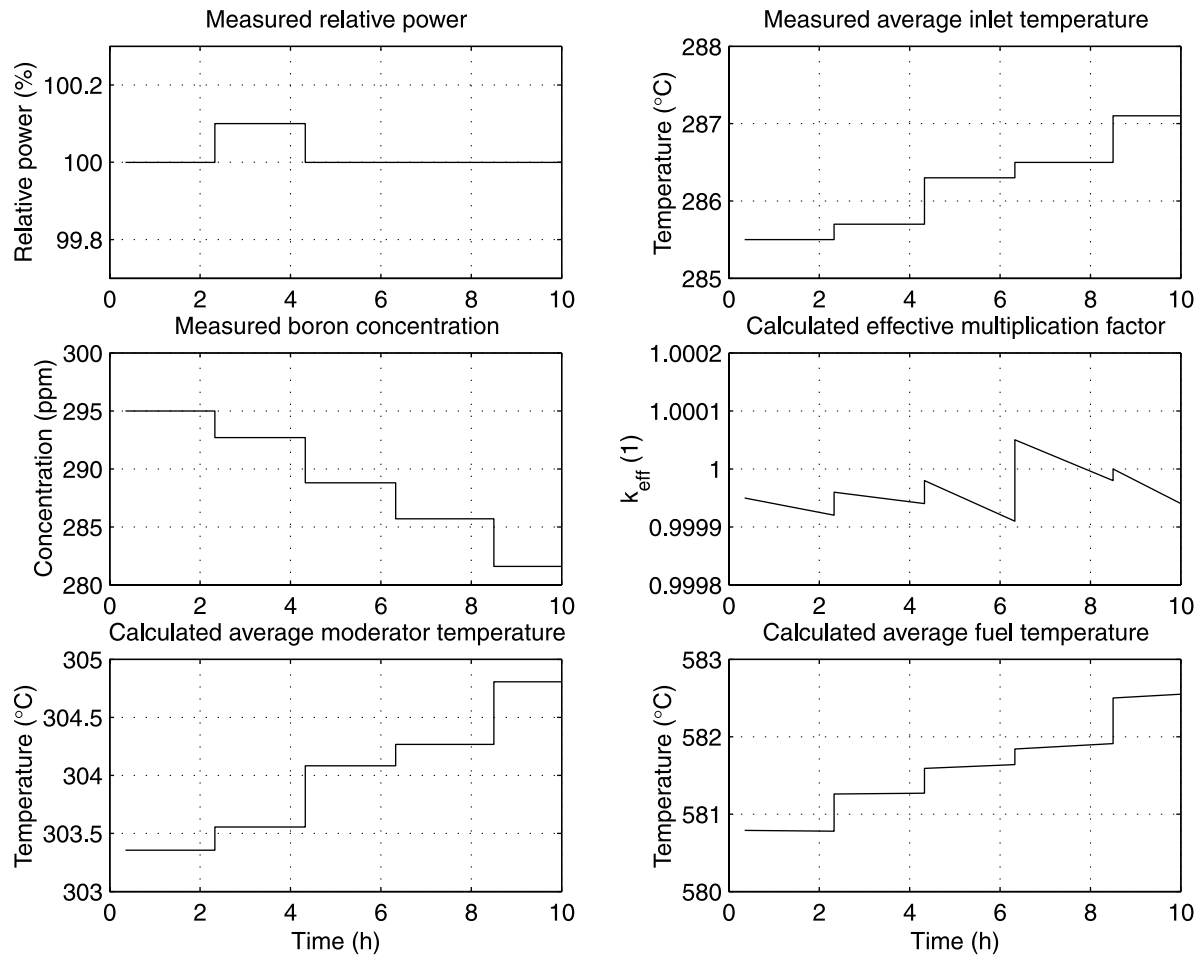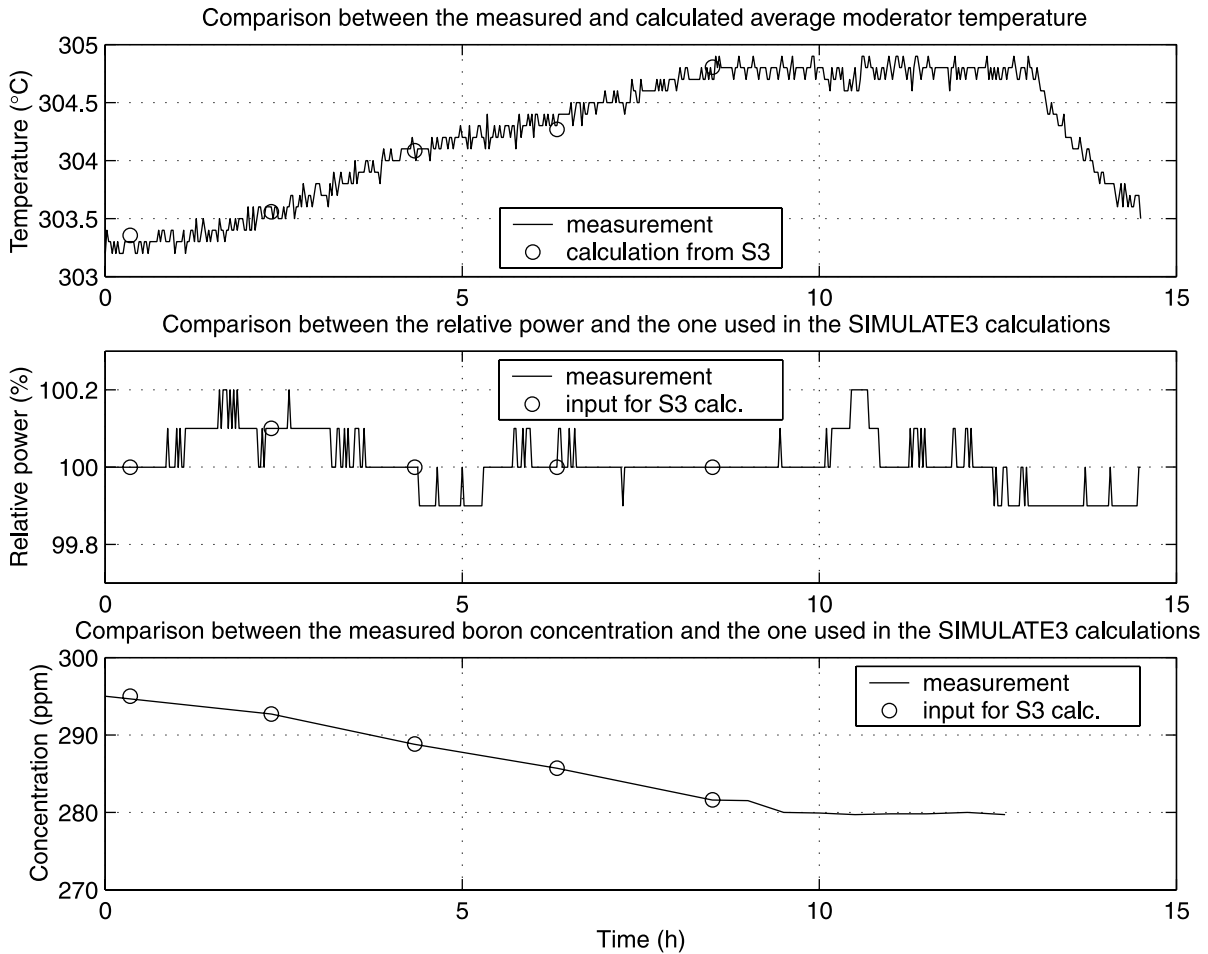Fig. 5. Comparison between the calculated/input data of SIMULATE-3 and the measured data for the reactivity contributions (other than the Doppler effect) to the EOC MTC measurement at Ringhals-4, cycle 16.

measurement uncertainties. A conservative definition of the standard deviation associated with the MTC allows writing the following:

$$
|\delta(\text{MTC})| \leq \left| \frac{\partial \rho}{\partial T_m} \frac{\delta(\Delta T_m)}{\Delta T_m} \right| + \left| \frac{\partial \rho}{\partial T_f} \frac{\delta(\Delta T_f)}{\Delta T_m} \right|
$$

$$
+ \left| \frac{\partial \rho}{\partial C_B} \frac{\delta(\Delta C_B)}{\Delta T_m} \right| + \left| \frac{\delta(\Delta \rho^*)}{\Delta T_m} \right| \quad .
$$

$$(14)$$

The uncertainty of the parameters $\delta(\Delta T_m)$, $\delta(\Delta T_f)$, $\delta(\Delta C_B)$, and $\delta(\Delta \rho^*)$ are determined from the fluctuations in the measured (and consequently calculated) parameters between 07:30 a.m. and 07:51 a.m. and between 05:00 p.m. and 05:30 p.m.. The contribution due to the precision/uncertainty of the measured parameters needs also to be taken into account. Therefore, if one denotes with $\delta(\Delta p)$ the uncertainty associated with the change Δp of any parameter p in Eq. (14) (such as the average

moderator temperature T_m, the average fuel temperature T_f, the boron concentration C_B, or the remaining reactivity effects ρ^*), one can write

$$
|\delta(\Delta p)| \leq \sqrt{(\sigma_{p,1} + |\delta p_{reading}|)^2 + (\sigma_{p,2} + |\delta p_{reading}|)^2}
$$

$$(15)$$

since the measured data are statistically uncorrelated between the initial stage (denoted with the subscript 1) and the final stage (denoted with the subscript 2). In this equation, $\delta p_{reading}$ is the precision of the measured parameter p (due to the finite number of digits used and due to the uncertainty of the measuring technique itself), and $\sigma_{p,1}$ and $\sigma_{p,2}$ are the standard deviations associated with the initial and final stages, respectively.

Regarding the precision/uncertainty of the measured parameters, in the present case one has

1. $\delta T_{m,reading} \approx \delta T_{in,reading} \approx \pm 0.05°C$ for the moderator temperature

2. $\delta C_{B,reading} \approx \pm 0.25$ ppm for the boron concentration.

In principle, the precision in these parameters induces also an uncertainty in the calculated parameters since these SIMULATE-3 calculations are based on measured parameters, which have a given uncertainty. Two parameters required for the reactivity contributions in the boron dilution method are calculated: the fuel temperature change and what was called previously the remaining reactivity contributions.

For the fuel temperature, one can estimate the consequences of the precision of the measured parameters upon the calculated fuel temperature using the variation principle:

$$\Delta T_f = \frac{1}{\frac{\partial \rho}{\partial T_f}} \cdot \left(\frac{\partial \rho}{\partial T_{in}} \Delta T_{in} + \frac{\partial \rho}{\partial C_B} \Delta C_B + \Delta \rho^* \right) \ . \quad (16)$$

Since the boron concentration and the term $\Delta \rho^*$ are calculated, not measured, the uncertainty due to the precision in the measured parameters simplifies into the contribution due to the uncertainty associated with the measurement of the core-inlet temperature, so that one can write

$$|\delta T_{f,reading}| = \frac{1}{\left| \frac{\partial \rho}{\partial T_f} \right|} \cdot \left| \frac{\partial \rho}{\partial T_{in}} \right| \cdot |\delta T_{in,reading}| \ . \quad (17)$$

For the term $\Delta \rho^*$, one can use once again the variation principle, which allows writing

$$\Delta \rho^* = \left(\frac{\partial \rho}{\partial T_f} \Delta T_f + \frac{\partial \rho}{\partial T_{in}} \Delta T_{in} + \frac{\partial \rho}{\partial C_B} \Delta C_B \right) \ . \quad (18)$$

Since the fuel temperature is calculated, not measured, the uncertainty due to the precision in the measured parameters simply becomes

$$|\delta \rho^*_{reading}| \leq \left| \frac{\partial \rho}{\partial T_{in}} \right| \cdot |\delta T_{in,reading}|$$

$$+ \left| \frac{\partial \rho}{\partial C_B} \right| \cdot |\delta C_{B,reading}| \ . \quad (19)$$

Table I summarizes all the uncertainties that are relevant for the MTC estimation by the boron dilution method. These uncertainties need to be converted into reactivity effects, but one can already notice that the uncertainty associated with the term ρ^* is relatively significant.

With all the values listed in Table I, the uncertainty associated with the MTC measured by the boron dilution method can be estimated, and one obtains a value of ± 14.51 pcm/°C.

TABLE I

Uncertainties Associated with the MTC Measurement

Parameter p	Unit	$\sigma_{p,1}$	$\sigma_{p,2}$	$|\delta p_{reading}|$	Maximum of $|\delta(\Delta p)|$
T_m	°C	0.06	0.06	0.05	0.16
C_B	ppm	0	0.07	0.25	0.41
T_f	°C	0	0.14	0.79	1.22
ρ^*	pcm	0	1.41	4.02	6.76

IV. RESULTS AND DISCUSSION

For comparison purposes, it is interesting to calculate the MTC directly via SIMULATE-3 as described previously in Sec. III.B. One finally obtains the following:

1. calculated MTC using SIMULATE-3: -45.68 pcm/°C

2. measured MTC using the boron dilution method: -58.12 pcm/°C with $|\delta(\text{MTC})| \leq 14.51$ pcm/°C.

One notices that the MTC calculated by SIMULATE-3 lies in the confidence interval associated with the measured MTC. This confidence interval is nevertheless so large that the MTC might even be close to -72 pcm/°C. Since the goal of the measurement is to precisely verify that the MTC is larger (less negative) than this threshold, one might question the usefulness of the boron dilution method.

Figure 6 shows the results obtained in the present study of the MTC measurement. In this figure, the contribution of each term in Eq. (13) is plotted. The uncertainties associated with the MTC and with each of its contributing terms in Eq. (13) are also represented. The MTC calculated by SIMULATE-3 is also given on the right side of Fig. 6 as an indicative parameter. One notices that the Doppler contribution is negligibly small, even if its relative error is rather high. The main reactivity effect, which counteracts the moderator temperature change, is due to the change in the boron concentration. Nevertheless, the term $-\Delta \rho^*/\Delta T_m$ (referenced as to "Remaining effects" in Fig. 6) contributes to the MTC by introducing a positive contribution of $+15.70$ pcm/°C. This contribution is relatively significant.

It is questionable for several reasons whether this contribution is accurate or not. First of all, this term has been obtained by summing small reactivity differences (between 2 and 7 pcm) over several time steps. Even if SIMULATE-3 is a reliable code, such a level of accuracy is hard to achieve. Further, the time steps have been chosen so that each beginning of a step matches a boron measurement (and the boron is thus kept constant during
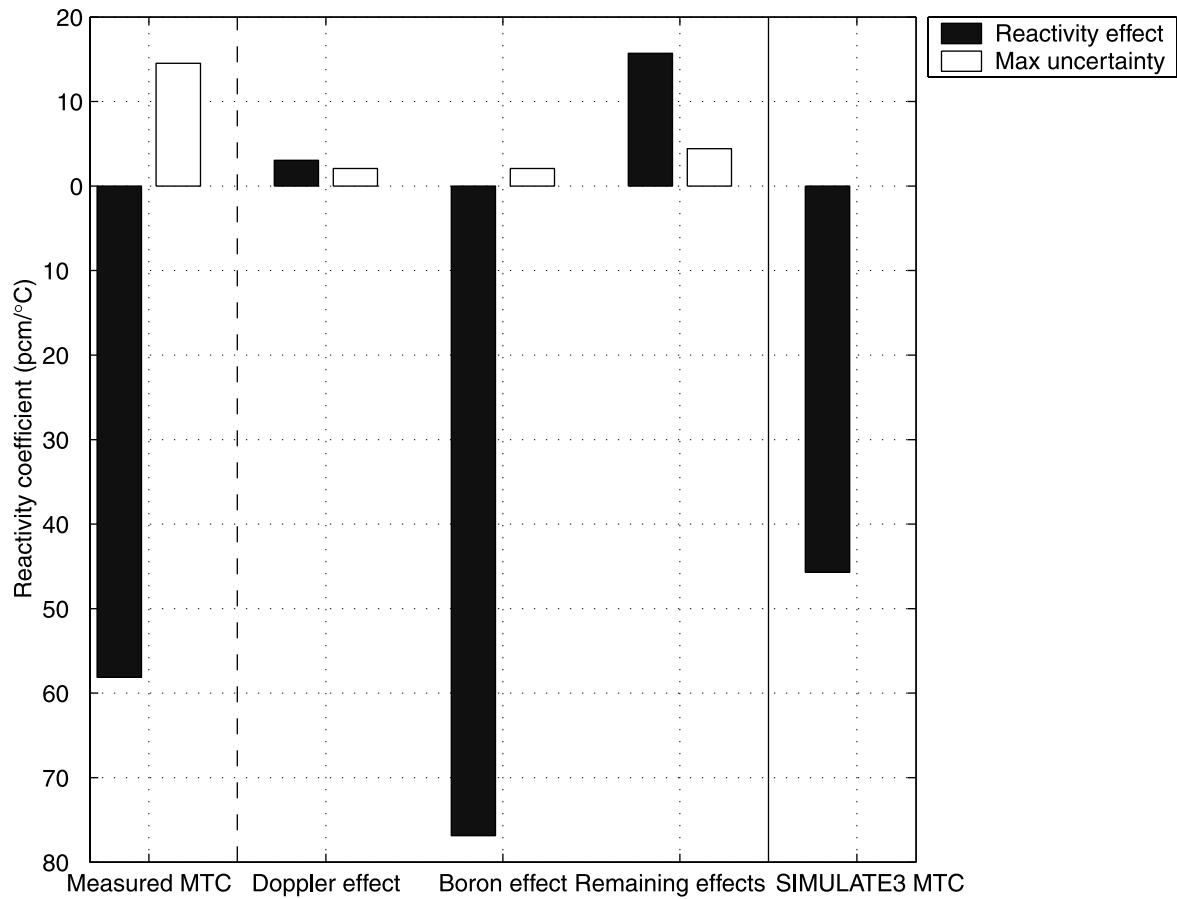
Fig. 6. Measured EOC MTC at Ringhals-4, cycle 16, with the contribution of the different reactivity components and their respective uncertainties (the MTC directly calculated by SIMULATE-3 is also given as an indicative parameter).

the corresponding step). Nevertheless, one has to know also the power level and the inlet temperature in order to be able to run SIMULATE-3 in these cases. Determining the values of these parameters is quite a difficult task. Let us take for instance the power level. The main purpose of the calculation is to estimate the reactivity effects due to effects other than the Doppler effect, the MTC effect, and the boron effect. Consequently, the power level should remain constant during each time step. Choosing an appropriate power level for this purpose is far from trivial. Choosing the one at the beginning of each step or a kind of average value leads to very different results. According to Fig. 5, the first representative point for the power level was chosen to be 100.0% (value at the beginning of the step), whereas an average value over the first step would be 100.1%. Since the power coefficient is -18.21 pcm/% (this reactivity coefficient was also calculated via SIMULATE-3), such a small difference in power level has a 1.8 pcm reactivity effect. A similar reactivity effect can take place with the inlet temperature misestimation. The inlet temperature coefficient of reactivity, computed by SIMULATE-3, is

-41.35 pcm/°C. Consequently, a misestimation of 0.1°C has a 4.1 pcm reactivity effect. Thus, these errors are not negligible when calculating the $-\Delta\rho^*/\Delta T_m$ term.

Consequently, even if it is generally assumed that the differential boron worth is computed accurately, the reliability of the MTC estimate is not very good since it is not granted that the term $-\Delta\rho^*/\Delta T_m$ is estimated precisely and since its contribution to the MTC may be relatively significant.

Because of the significantly large inaccuracy of the boron dilution technique, Ringhals developed a new methodology to estimate the at-power MTC. This methodology completely relies on the MTC calculated by SIMULATE-3 as will be explained in the following. This means that the purpose of the measurement is not any longer to check the ability of SIMULATE-3 to correctly estimate the at-power MTC. Therefore, it is implicitly assumed that SIMULATE-3 is able to accurately estimate the at-power MTC. As written previously, the at-power MTC was never benchmarked (and cannot be benchmarked due to the parasitic reactivity effects that take place during any lengthy measurement).

In this new methodology,[41] the boron dilution is carried out first, using the same recommendations as the ones given in Sec. II.B. Then SIMULATE-3 is used to model the transient with very short time steps. The input parameters necessary to model the transient are the core-inlet temperature, the reactor power level, and the control rod pattern. Since the MTC is very much dependent on the boron concentration, the burnup allowing us to obtain the same boron concentration as the one given by the measurement just before the transient has to be estimated. Such a burnup estimation can easily be done in SIMULATE-3.

One of the SIMULATE-3 results of the transient modeling is the boron concentration change during the transient. Then, the measured MTC is estimated as follows:

$$\mathrm{MTC}_{measured} = \mathrm{MTC}_{calculated}$$

$$+ \frac{\partial \rho}{\partial C_\mathrm{B}} \left(\Delta C_{\mathrm{B},calculated} - \Delta C_{\mathrm{B},measured} \right)$$

$$\times \frac{1}{\Delta T_{m,measured}} \ , \tag{20}$$

where the indices *calculated* and *measured* stand for the SIMULATE-3 calculations and the boron dilution measurement, respectively. The philosophy of this method can be summarized as follows. If one found complete equality between the measured and calculated values of the transient, then one could assume that the MTC is also correctly calculated. The deviation between the measured and calculated boron concentration indicates that this is not the case. The calculated MTC is therefore corrected according to Eq. (20). In that, the coefficient $\partial \rho / \partial C_\mathrm{B}$ is also calculated, and thus includes an uncertainty, but it can be assumed that the error of this contribution is of second order.

Because of the implicit and heuristic character of Eq. (20), it is difficult to estimate the accuracy of this method. We shall therefore here not attempt to analyze the method represented by Eq. (20).

V. CONCLUSION

An analysis of the methodology and accuracy of the determination of the MTC by the boron dilution method was made. This measurement technique does not induce solely a modification of the moderator temperature; therefore, other effects such as the fuel temperature change and the fuel depletion have to be taken into account. Since these effects cannot be measured, core calculations are used for the estimation of the corresponding reactivity effects. The way of determining these contributions and the uncertainties associated with each of these reactivity effects were presented in this paper. It was found that most of the reactivity change during a mea-

surement campaign is due to the boron dilution but that the fuel depletion, i.e., the reactivity effects other than the ones induced by the boron and the fuel temperature changes, contributes noticeably to the MTC. Finally, the measured MTC was compared to the MTC calculated by SIMULATE-3, and it was noticed that the discrepancy between these two results was within the uncertainty associated with the measured MTC, uncertainty that was relatively significant.

Except for the power level, which is measured together with the boron concentration and the core average coolant temperature, all the other effects are calculated. Even if the uncertainty associated with the Doppler effect is relatively large, this effect is negligible in comparison with the reactivity effect due to the boron change. It is not the case for the remaining reactivity contributions, which represent more than 20% of the boron effect. It has been shown that the way of choosing the corresponding measured parameters for the SIMULATE-3 calculation may affect significantly the remaining reactivity contributions, so that the calculation of these cannot be considered as reliable and accurate. For the boron concentration (which is responsible for most of the reactivity balance), it is generally accepted that the differential boron worth is computed accurately.

Although the measured MTC is much lower than the MTC calculated by SIMULATE-3, the discrepancy between these two results is within the uncertainty associated with the measured MTC. As a matter of fact, the reliability of the SIMULATE-3 code cannot be questioned so far, but the way of computing, i.e., of choosing the input parameters for the SIMULATE-3 calculation, is questionable for the determination of the $\Delta \rho^*$ term. Consequently, the uncertainty associated with the measured MTC using the boron dilution method could be much higher than previously expected.

The relatively large inaccuracy of the boron dilution technique was also concluded by Ringhals. Therefore, they developed a new methodology to estimate the at-power MTC. This technique entirely relies on the MTC directly calculated by SIMULATE-3. Thus, the aim of the boron dilution measurement is not any longer to check the SIMULATE-3 calculations. This also means that what is measured does not correspond to the MTC defined by the Standard since the MTC calculated by SIMULATE-3 (which fulfills the definition given by the Standard) is corrected.

It is also questionable whether the MTC according to its traditional definition is a parameter that needs to be estimated at all. First, it was proven that the MTC cannot be measured easily in practice, whatever the measurement technique might be. In the case of the boron dilution method analyzed previously, the confidence interval was so large that the MTC could be equal to -72 pcm/°C, whereas the goal of the measurement was to check that the MTC is larger (less negative) than this threshold. Even if noise analysis could provide a disturbance-free

estimation of the MTC, some tests have to be carried out in the view of the recent theoretical investigations. Second, the main goal of the MTC measurement is to demonstrate that in case of accidental situations, the reactor will remain stable and within its operating limits. Nevertheless, in such a case, many reactivity effects take place, and the MTC is only a part of them. One might therefore question the usefulness of the traditional MTC definition since all the reactivity effects taking place during a reactor transient, and which are removed from the traditional definition, are as interesting from a safety viewpoint as the traditional MTC effect solely.

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PAPER II

DEVELOPMENT AND APPLICATION OF CORE DIAGNOSTICS AND MONITORING FOR THE RINGHALS PWRS

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ABSTRACT
Noise analysis and reactor diagnostics have been applied at the Ringhals PWRs for a long time. Through a collaboration with the Department of Reactor Physics, Chalmers University of Technology, methods for treating new problems were elaborated, and known methods were developed further to make them more effective or to suit specific applications. All these methods were tested in real measurements, and many of them have been used routinely afterwards. In this paper two particular new methods are described in detail: 1) the determination of the axial position of control rods from the axial shape of the neutron flux with neural network methods, and 2) the use of gamma thermometers for the determination of the MTC and for core flow estimation.

1. INTRODUCTION

Development and application of core diagnostic methods have been pursued during a longer period in a co-operation between the Ringhals Power Plant and the Department of Reactor Physics, Chalmers University of Technology. Methods for tackling new problems have been developed as such questions arose, and the suggested solutions and methods were tested in measurements at operating plant. Some new methods have been included into the routine monitoring system of the plant.

Ringhals is the largest nuclear power site in Sweden, with one BWR and 3 PWR units of a total power of 3540 MWe. The core monitoring system SCORPIO (Hval and Andersson, 1990) developed in collaboration with the OECD Halden Reactor Project has been installed in Ringhals-2 and other plants. One particular speciality of the Ringhals system is that the core power distribution is monitored by in-core gamma-thermometers. On the other hand, there are no fixed in-core neutron detectors; in-core neutron noise measurements are performed by movable SPN detectors.

Methods were tested and applied for a large number of diagnostic problems, both in the BWR and PWR units. These include BWR stability measurements, monitoring of core-barrel vibrations, investigation of core internal vibrations such as fuel vibration modes, investigation of ultra-low frequency power and flow oscillations with partial coherences, determining axial control rod position from neutron flux mappings, and finally, developing methods for the determination of the Moderator Temperature Coefficient (MTC) by noise methods. A substantial part of this work has been documented in the literature (Demazière *et al*, 2000), (Demazière *et al*, 2001), (Demazière and Pázsit, 2002a), (Demazière and Pázsit, 2002b), (Garis *et al*, 1998), (Nagy, 2000), (Pázsit, 1999), (Pázsit *et al*, 1999). This paper gives a detailed account of the last two items above.

2. DETERMINATION OF AXIAL CONTROL ROD ELEVATION FROM THE MEASURED FLUX SHAPE WITH NEURAL NETWORKS

This work was motivated by the fact that the electromechanical position indicators, giving the axial elevation of a control rod in a PWR, can get de-calibrated during operation. The control rod position is an important operational variable and thus its accurate value, both for a bank of control rods and for the individual rods separately, must be known accurately to detect de-calibration of the standard rod positioning instrumentation, and to calibrate it, an alternative method was needed. The idea arose to use the information that exists in the axial flux shape regarding the rod elevation. The axial flux shape can be measured within, or in the close vicinity of a fuel assembly containing a control rod assembly. Presence of a control rod affects the axial flux shape, which means that the flux shape contains information regarding the rod elevation. From this information the rod elevation can be determined in principle.

The principle is illustrated in Fig. 1 that shows the axial flux shape for a control rod partially inserted to different elevations. The Figure shows the axial flux profile in assembly G14 as calculated for five different elevations of the control rod bank D at 100% power: 220, 210, 200, 191 and 180 steps (with one step being equal to 1.6 cm). The core master code SIMULATE-3 was used in this calculation, just as throughout the present work.
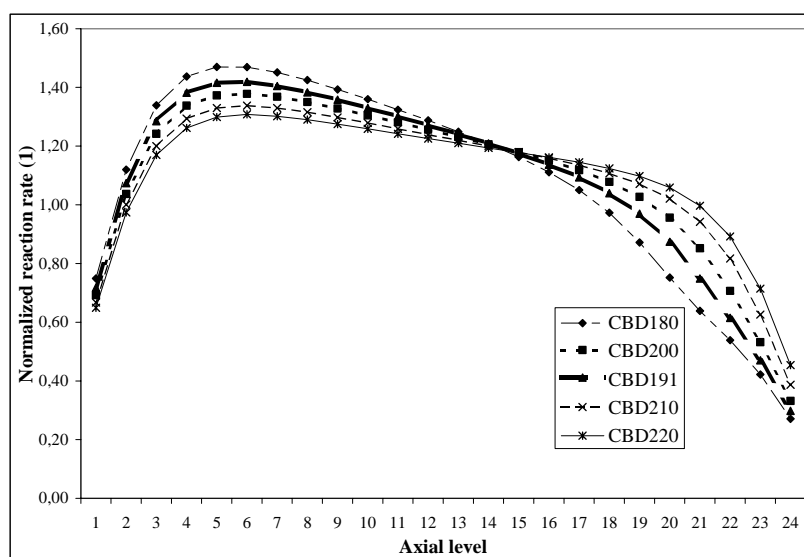

Fig. 1. The axial flux shape in position G14 as a function of elevation of the control rod bank D

However, as Fig.1 shows, the information on the control rod elevation is rather implicit, and its extraction requires advanced unfolding methods. Such implicit inverse problems can be effectively solved by neural network techniques, if the suitable training set can be obtained from measurements or from calculations. Hence neural network methods were used to unfold the rod position from the axial flux shape. A simple three-layered feed-forward network with backward error propagation was sufficient for this purpose. The training set, i.e. axial flux shapes corresponding to a large number of known rod positions, was generated by the core master code SIMULATE-3 (Umbarger and DiGiovine, 1992). The trained network was tested on real measurement data. The measurement was taken in the Swedish PWR Ringhals-4 at the beginning of cycle 13. An accuracy of a few cm was achieved when determining the control rod elevation. This work has been reported in the literature (Garis *et al*, 1998), (Pázsit *et al*, 1999).

For the success of the method, it is essential that the training set be calculated in a core corresponding to the actual core status as close as possible. It was noticed for instance that in case when the control rod bank was inserted just before the measurements, the neutron flux calculation had to be made corresponding to a non-equilibrium Xenon distribution. However all the above tests were made at a certain power level and core burnup. It is clear that doing a calibration of control rod elevation at various points of the fuel cycle (i.e. corresponding to different burnup) or taken at different power levels, requires a different training set in each case. One illustration is given in Fig. 2 where the axial power shape is shown in a given core position and with a constant control rod elevation but at various core burnup values.
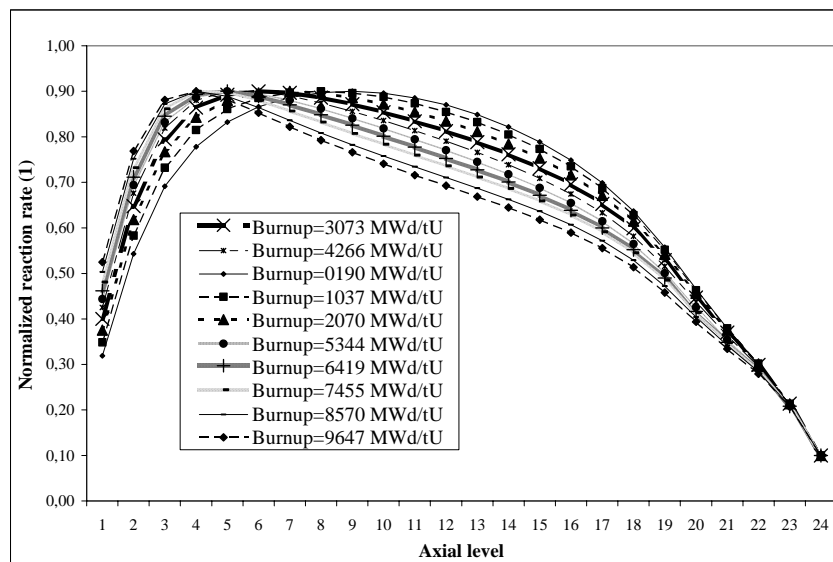
Fig. 2. Axial flux shapes in position H3 and control rod elevation 180 steps, for various burnup values

If the calibration is planned beforehand, the training can also be performed off-line, but in case a calibration needs to be made unplanned, the training and calculation of the training set corresponding to the actual burnup and power level can take too long time. Thus it was investigated whether the method can be extended such that it can be used throughout the fuel cycle or for various power levels. These investigations constitute the essence of this Section.

The cases of different burnup or different power levels were handled separately so far. The extension was made to introduce one more input node, which was either the burnup ("Method 1"), or the reactor power level ("Method 2"). For each value of the extra parameter, a training set was generated with different control rod positions, as before. This way the dimensionality of the training set was significantly increased, as well as the CPU time for training.

Tests were then made regarding the accuracy of the extended method, i.e. for checking the accuracy of Method 1 for various burnup values, and that for Method 2 for various power levels. The tests were only made on simulated data so far, i.e. both the training set and the test set were calculated by SIMULATE-3. It was found that, despite the extension, the same accuracy could be achieved with the extended method as with the previous method, i.e. when using one single training set for a given burnup and a given power

level. Details of these calculations and the results can be found in (Nagy, 2000). A permanent installation of the method is underway and its routine use is planned at the Ringhals PWRs.

A further complication may exist when one control rod of an inserted bank de-calibrates at an unknown time and a calibration is to be performed later. In principle, the training set should be calculated with a core history that is unknown to some extent. Investigation is going on how this case could be handled with the present method.

3. USE OF THE GAMMA THERMOMETERS FOR DETERMINATION OF THE MTC AND FOR DETERMINING CORE FLOW

An important constraint in the safety analysis of Pressurized Water Reactors (PWRs) is the Moderator Temperature Coefficient (MTC). Most European plants require the MTC to be negative at zero power throughout the cycle. At middle of life and full power, the MTC is required to be less negative than typically -72 pcm/°C. The MTC requirements are verified by measurements. The present method of full power MTC measurement is time consuming and gives rise to core perturbations that adds to the measurement uncertainty. It is therefore of great benefit to find a method that does not perturb the core, is quick and accurate and allows for frequent measurements throughout the cycle.

The determination of the Moderator Temperature Coefficient (MTC) from cross-correlation of the in-core neutron noise and the core-exit temperature noise has actually been the matter of interest for quite some time, for the obvious advantages it offers over the traditional intrusive methods. Experience showed, however, that the MTC determined by noise methods severely underestimated the true MTC value. Some current work by the recent authors showed that the main reason for this underestimation is the radial incoherent structure of the temperature fluctuations (Demazière and Pázsit, 2002a). Using a local temperature value in the procedure is equivalent with assuming homogeneous temperature fluctuations, which will overestimate the driving force of reactivity fluctuations, and thus underestimate the MTC.

It was thus suggested that instead of a local core-exit temperature, a flux-square weighted average temperature over the core, or just over a horizontal cross-section of the core, should be used:

$$\delta T_m^{ave}(\omega) = \frac{\int \delta T_m(\boldsymbol{r},\omega)\phi_0^2(\boldsymbol{r})\,d\boldsymbol{r}}{\int \phi_0^2(\boldsymbol{r})\,d\boldsymbol{r}} \tag{1}$$

Such a core average temperature requires measurement of the temperature in several points in the core. Westinghouse type PWR cores are usually not equipped with in-core thermocouples. However, some core monitoring systems use in-core Gamma Thermometers (GTs). In the frequency range of interest for the MTC, the GTs act as thermometers. Hence, the core-averaged temperature, necessary for the correct determination of the MTC, can be extracted from the GT signal.

The unit Ringhals-2 is equipped with such GTs, whose original use was to serve as part of the core monitoring system SCORPIO (Hval and Andersson, 1990). There are altogether 108 GTs in Ringhals-2, in 12 strings with 9 GTs in each (see Fig. 3). It was decided that these would be used for the determination of the MTC by noise methods.

Two measurement series have been conducted so far. In the first, performed in 2000, three strings were used in core positions J10, L11 and M03 (see Fig. 3). The results of this measurement were reported in (Demazière *et al*, 2000) and (Demazière and Pázsit, 2002b). No in-core neutron detectors were used in this measurement. The purpose was to get experience with the frequency response of the GTs, and to investigate the useful information content of the temperature fluctuations. Since for the MTC, the average temperature basically means an averaging over a horizontal cross-section of the core, the three radial positions in this measurement did not make it possible to estimate and investigate the average temperature. An inspection of the coherence between three detectors in the three different strings at the same axial level

showed that the radial temperature correlations decay fast with increasing radial distance. The maximum coherence values in the frequency band 0.1 - 0.5 Hz between the three detector pairs are marked in Fig. 3. There is practically no coherence for the large distances, whereas there is a clear coherence between the two close strings. One can conclude that the radial correlation length is unlikely to exceed 50 cm in this case.
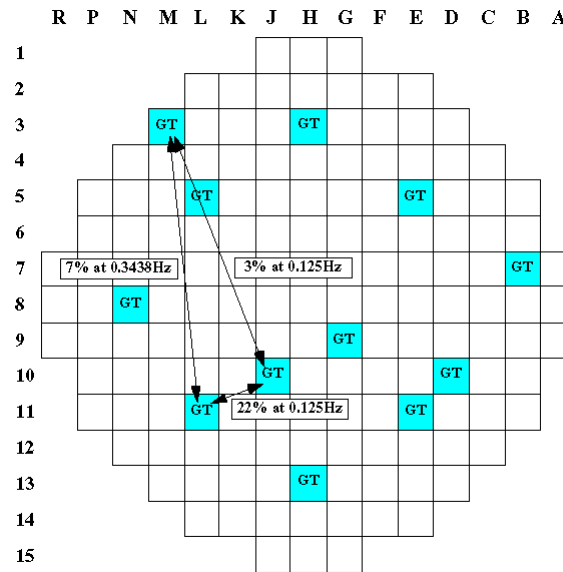
Fig. 3. Radial location of the Gamma Thermometers in Ringhals-2 (the figures represent the maximum of the coherence between the strings measured in 2000, on the third axial plane from the core bottom)

Since in all three strings there were 9 GTs, it was possible to attempt to measure the transit time of flow for the instrumented assemblies by cross-correlating the signals of the GTs within one string. The transit time between the GTs was determined from the slope of the phase between the lowermost GT and all other GTs in the same string in the frequency range 0.1 - 0.7 Hz (see Fig. 4). The flow velocities so obtained for the various elevations were then compared to estimates obtained from SIMULATE-3. Reasonably good agreement was obtained, in view of the fact that the frequency transfer of the GTs is somewhat narrow (the time constant too large) for this purpose. Such a comparison is shown in Fig. 4 for the string M03. A later investigation (Demazière *et al*, 2001) showed that there were appreciable differences in the time constants of the GTs in this measurement, which leads to biases in the transit time determination. The conclusion is that core flow can be successfully mapped with the GTs, which offers a very effective monitoring tool, but both the hardware and the signal processing needs to be developed further if the accuracy is to be improved.
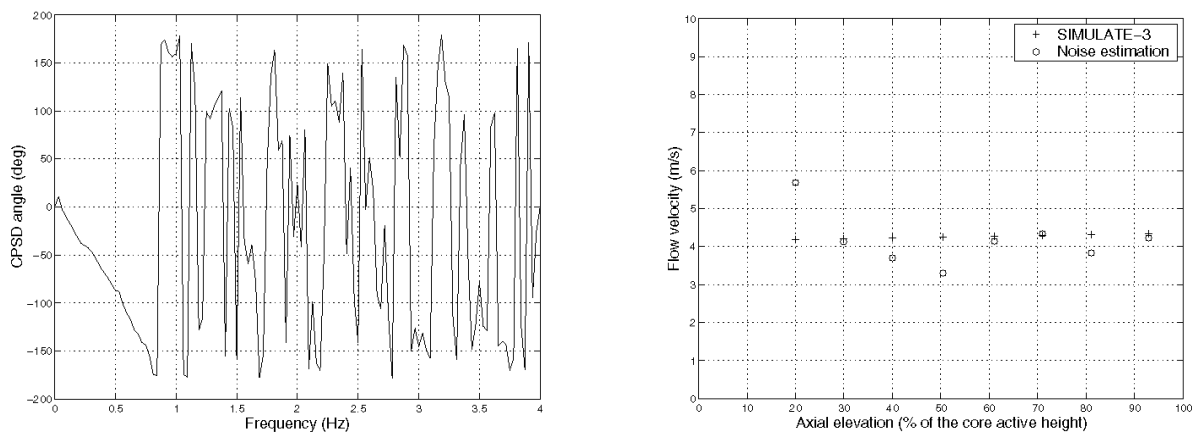
Fig. 4. Some characteristics of the M03 channel (on the left-hand side, the CPSD angle between the fourth and the ninth planes from the core bottom; on the right-hand side, the flow velocities estimations)

A second measurement was performed recently that was designed to test the possibility of using the average temperature for determining the MTC. In this measurement, all GTs from only two axial levels were used, but in all 12 core positions at one level, and 10 positions at the other axial level. Neutron noise was measured simultaneously with 2 movable detectors. For purposes of comparison, the signal of one core-exit thermocouple was also recorded.

Some preliminary results are shown in Fig. 5, which shows the square root of the ratio between the APSD of the average temperature and the APSD of one in-core GT and that of one core-exit thermocouple, respectively. As the Figure shows, both of them are smaller than unity and, interestingly, quite flat in the frequency range 0.1-1.0 Hz. The fact that they are smaller than unity is equivalent with the overestimation of the driving force, mentioned previously, which is in turn the reason for the underestimation of the MTC. It is very interesting however to note that the ratio is much lower for the core-exit thermocouple than for the GT, which thus leads to a much more significant underestimation of the MTC. In other words, the temperature fluctuations are larger at the core exit than inside the core, which is somewhat surprising. Using one single in-core GT in the procedure will already improve the estimate, although would still not give the correct value. It is also seen that the MTC underestimation by the core-exit thermocouple, indicated by Fig. 5, is in the same order as the one experimentally observed with the traditional method in Ringhals. Such assumptions regarding the MTC evaluation by noise analysis are actually verified in (Demazière *et al*, 2002), where the MTC is estimated by noise analysis based on the use of gamma-thermometers and compared to its actual value.
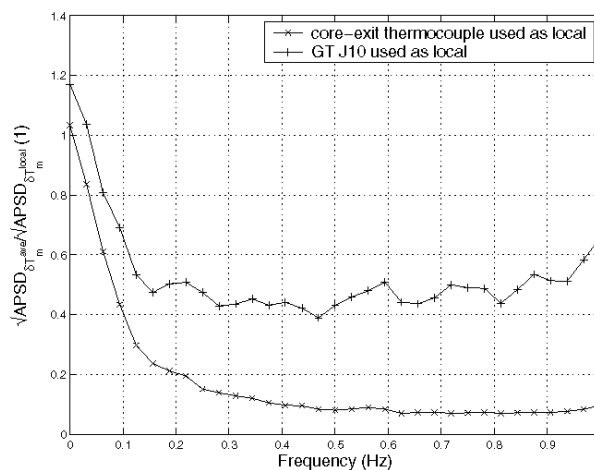
Fig. 5. Comparison between the noise levels recorded by the core-exit thermocouple, a GT, and the planar average of all the GTs

This preliminary evaluation shows that using the concept of the average temperature, and determining it experimentally with in-core gamma thermometers has a very good potential to become an accurate way of determining the MTC with noise methods. This also shows that the use of in-core gamma thermometers is a very versatile tool for both routine core surveillance as well as for determining the MTC and core flow.

4. ACKNOWLEDGEMENTS

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PAPER III

PERGAMON

Theoretical investigation of the MTC noise estimate in 1-D homogeneous systems

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Abstract

In this paper, the accuracy of the noise-based determination of the moderator temperature coefficient (MTC) is investigated theoretically and quantitatively. It is known from earlier work that the noise method systematically underestimates the MTC. In this paper, it is found that the main reason for the underestimation lies with the radial incoherence of the temperature fluctuations. The deviation of the reactor response from point-kinetics is another possible reason, but it was found to play a quite insignificant role. The theory of neutron noise, induced by spatially random perturbations is elaborated and by its help the inaccuracy (bias) of the noise based MTC estimation was quantitatively investigated. It was found that a relatively short correlation length of the temperature fluctuations, which is in agreement with experimental evidence, can explain the observed underestimation of the MTC by the noise method. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

The moderator temperature coefficient (MTC) is an important safety parameter in pressurised water reactors (PWRs). For safety reasons, it is kept negative at all times on a design basis. Due to operational circumstances (decrease of boron content in the coolant during the cycle) its absolute value increases during the cycle, leading to the well-known increase of noise level in PWRs with increasing burnup. At the beginning of the cycle (BOC) the MTC is close to zero, i.e. slightly negative,[1] whereas towards the end of the cycle (EOC) it becomes deeply negative. Basically, it has to be measured twice during each fuel cycle: at BOC at hot zero power, to verify

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[1] In cores with high burnup fuel, it is sometimes allowed to be slightly positive at the beginning of the cycle.

Nomenclature

APSD   auto-power spectral density
BOC   beginning of cycle
CCF   cross-correlation function
CPSD   cross-power spectral density
EOC   end of cycle
MTC   moderator temperature coefficient
MOC   middle of cycle
PWR   pressurised water reactor
RMS   root-mean-square

that the MTC is negative, and at near to EOC at full power. The purpose of the latter is to check that the magnitude of the MTC is lower than some prescribed value, in order to prevent a large positive reactivity excursion in case of a sudden unexpected cooldown event.

Whereas it is generally assumed that the traditional measurement of the MTC at the BOC is accurate (due to the advantageous circumstances associated with zero power), at the EOC and full power the MTC cannot be determined with the same ease and level of accuracy. The traditional techniques used such as the boron dilution method, the power change method, the depletion method, or the rod swap method, have the disadvantage that they interfere with reactor operation, are time consuming and costly, and must be supplemented by calculational corrections because the operations made induce change of various parameters other than the moderator temperature change alone.

For these reasons it has been suggested quite some time ago that noise analysis techniques, that are inherently non-intrusive and fast, should be used for determining the MTC (Thie, 1977; Türkcan, 1982). This technique is based on analysis of the neutron noise and temperature noise. An in-core neutron detector and a core-exit thermocouple located within the same channel or in two neighbouring channels are used for that purpose. The MTC is extracted from the auto- and cross-spectra of these signals (APSD and CPSD) by assuming that the temperature fluctuations are homogeneous in space and that the flux response is point-kinetic. This method does not require any perturbation of the system and is therefore very well suited to an on-line and accurate MTC determination. Monitoring the MTC on-line throughout the whole cycle would allow power utilities to use high burnup fuel, for which the MTC might be slightly positive at BOC.

Nevertheless, all attempts made so far to determine the MTC by noise analysis revealed that such a measurement technique underestimates the MTC by a factor of two to five (Thie, 1977; Türkcan, 1982; Thomas et al., 1991; Pór et al., 1985; Herr and Thomas, 1991; Thomas and Clem, 1991; García Cuesta and Blázquez, 1995; Oguma et al., 1995a,b; Upadhyaya et al., 1988; Kostic et al., 1988; Sweeney and Upadhyaya, 1983; Sweeney, 1984; Shieh et al., 1987; Aguilar and Pór, 1987; Kostic et al., 1991; Pór and Jozsa, 1995; Laggiard and Runkel, 1997; Kostic, 1997; Laggiard

and Runkel, 1999). Remarkably enough, at the same time it was found that if the noise measurement is calibrated empirically to the true MTC value early in the cycle, then the calibration remains valid for the rest of the cycle, and most often even during forthcoming cycles (Oguma et al., 1995a,b). This indicates that the basic assumption that the cross-correlation is deterministically related to the MTC is correct, but also that there is some basic error in the formula (also referred to as "estimator") used for the extraction of the MTC from the cross- and auto-spectra of the measured signals.

It is natural to conclude that any such error lies in the simplification of the physical model behind the estimator formula as compared to the real situation. The error (bias) induced by many of these simplifications was investigated in the past. These include the deviation of the reactor response from point-kinetics, the effect of the axial separation distance between the in-core neutron detector and the core-exit thermocouple, the possible generation of temperature fluctuations at all axial elevations during coolant flow, the fact that several other noise sources may coexist at the same time, and finally the presence of the Doppler effect (Thomas et al., 1991; Laggiard and Runkel, 1997; Glöckler, 1989; Laggiard et al., 1997; Laggiard and Runkel, 1999; Antonopoulos-Domis and Housiadas, 1999; Housiadas and Antonopoulos-Domis, 1999). It was among others suggested that the reactivity effect should be determined from integral flux measurements (Antonopoulos-Domis and Housiadas, 1999). These corrections are nevertheless usually small and cannot explain solely the strong deviation of the MTC noise estimate from the true value.

In this paper one basic aspect will be investigated that has not received much attention so far (Demazière, 2000). This is the radially non-homogeneous (incoherent) character of the temperature fluctuations. Is is usually assumed that the temperature, measured by one single thermocouple, is characteristic for the temperature fluctuations in the whole core. This is equivalent to assume a spatially constant (although temporally random) temperature field in the core. This may be a good assumption axially, since as long as the temperature fluctuations propagate along the channel basically unchanged, at low frequencies relevant to the MTC measurement the wavelength of the fluctuations is much longer than the core height. For this reason we shall also neglect the axial variation in our investigations below. However, the assumption of radially coherent temperature fluctuations is rather poor. The temperature fluctuations are radially quite loosely coupled, with a correlation distance of the range of the fuel assemblies (at any rate much smaller than the core diameter) (Demazière, 2000; Demazière et al., 2000). As a consequence, the reactivity worth of the perturbation will be much smaller than if the perturbation was coherent.

One can compare the situation with trying to calibrate the worth of a bank of control rods such that the individual rods move up and down uncorrelated, and one measures the induced neutron noise and the axial displacement of *one* rod only. Clearly, the evaluation of the rod bank worth, based on the assumption that all rods move coherently, will underestimate the rod bank worth. On the other hand, if all rods move simultaneously, the estimation of the rod bank worth will be accurate.

It is clear from the above that the measure of the underestimation of the MTC, i.e. the searched calibration factor of the previous measurements, will depend on the

spatial (radial) correlation structure of the temperature fluctuations (and, in addition, on the other factors mentioned above such as the deviation from point-kinetic response, but these constitute a much smaller effect). The effect of the correlation properties of the temperature fluctuations has not been investigated so far. To our knowledge, it is only the detailed core simulation model of Glöckler (1989) that contains the effect of radial coupling of the temperature fluctuations, but the relationship between this coupling and the MTC underestimation has not been investigated quantitatively.

In this paper the relationship between the MTC underestimation and the correlation structure of the temperature fluctuations is quantified. A simple model for the spatial correlation structure of the temperature fluctuations, used elsewhere before (Williams, 1973; Pázsit 1986,1994), will be introduced. Then the APSD of the neutron noise, induced by such a perturbation, as well as the CPSD between neutron noise and the temperature fluctuations are derived. These can be used in the noise estimator, employed in the traditional evaluation of the measurement, which is based on the assumption of homogeneous temperature fluctuations and point-kinetic response. By this way the error of the traditional method can be explicitly calculated, since the exact MTC of the core is given by the model and is known. It is seen that the underestimation of the MTC by the traditional method increases monotonically with decreasing correlation length (increasing incoherence) of the temperature fluctuations. It is shown that the bias of the estimation, arising from the deviation from point-kinetics of the response, is negligible compared to the effect due to the spatially heterogeneous noise temperature distribution.

Finally some thoughts were devoted to the dependence of the MTC underestimation on the burnup. Experimental evidence shows that the underestimation is rather insensitive to burnup. In the model used here, a simple check was made by using material parameters corresponding to various parts of the fuel cycle of a given core loading. The results show that the accuracy (underestimation) of the MTC is rather insensitive on core burnup. This is not so surprising since the main factor that determines the accuracy is the correlation length of the temperature fluctuations, which is independent of the burnup.

2. Theory of neutron noise induced by spatially random perturbations

In the model presented hereafter, a 1-D system is considered, with the spatial coordinate representing the radial position in the core. Consequently, only the radially inhomogeneous character of the temperature fluctuations is investigated, i.e. the axial dimension is completely disregarded. Nevertheless, in the general derivation of the noise induced by spatially random perturbation, and in the formal analysis of the estimators used in noise measurements, a 3-D notation will be kept for sake of generality.

Inlet temperature fluctuations are thus given as noise sources. In principle, in a PWR, these fluctuations travel upwards and are affected by the coolant temperature fluctuations generated inside the core itself. In this model, the axial dependence of

the temperature noise can be ignored since the radial dependence of both the temperature and the flux noise is known to give stronger effects than the axial one.

2.1. Specification of the noise sources

A one-group diffusion theory approach is used in this study, hence only fluctuations of the absorption cross-section are investigated. In principle, temperature fluctuations affect very much the removal cross-section, and to a lesser extent the absorption cross-section. However, the noise induced by the removal cross-section fluctuations can only be accounted for in a two-group representation. On the other hand, a shift of the thermal spectrum of the moderator and thus that of the thermal neutrons will lead to increased absorption in the fuel. This phenomenon can be modelled in a one-group model and this is what we shall use here.

It is thus assumed that the temperature fluctuations induce fluctuations of the absorption cross-section, so that the fluctuations in the absorption cross-section can be directly related to temperature noise through a space- and time-independent coefficient as given by:

$$\delta T_m(\mathbf{r}, t) = K \times \delta \Sigma_a(\mathbf{r}, t) \tag{1}$$

It is supposed that $\delta \Sigma_a(\mathbf{r}, t)$ is stationary and ergodic in time and has a zero expected value, i.e.

$$\left\langle \delta \Sigma_a(\mathbf{r}, t) \right\rangle = 0 \quad \forall \quad \mathbf{r}, t \tag{2}$$

where, due to temporal ergodicity, the ensemble average can be substituted by time averages, and therefore the Wiener–Khinchin theorem can be used in order to evaluate the spectral characteristics of the neutron and temperature noise.

As mentioned previously, the space-dependence of the temperature noise is not known in a deterministic way. Rather, the space–time behaviour can only be defined in a statistical sense through the temporal and spatial cross-correlation function of temperature fluctuations. For this correlation function, we shall assume the simplest non-trivial model which is described with a few parameters. This model was introduced for cross-section fluctuations by Williams (1973) and then was developed further to describe spatial density correlations of two-phase flow (Pázsit, 1986), and later it was also applied to fusion plasma transport (Pázsit, 1994). It is assumed that the correlations can be factorised into a temporal component and a spatial component. The temporal part is given by $\delta(\tau)$ (white noise in the frequency range of interest for the MTC investigations, i.e. typically from 0.1 to 1 Hz). The spatial part, given by $R(\mathbf{r}, \mathbf{r}')$, is further factorised into a fast decaying function of the distance between the two points, representing the decay of correlations, and a much slower varying shape (amplitude) function $\sigma^2(\hat{\mathbf{r}})$ of the centre $\hat{\mathbf{r}}$ of the two spatial coordinates, representing the space-dependent strength of the noise source (can be related to the RMS or the variance of the noise source). In formula, one has:

$$CCF_{\delta\Sigma_a}(\mathbf{r}, \mathbf{r}', \tau) = \left\langle \delta\Sigma_a(\mathbf{r}, t)\delta\Sigma_a(\mathbf{r}', t + \tau)\right\rangle = \delta(\tau)R(\mathbf{r}, \mathbf{r}') = \delta(\tau)\sigma^2(\hat{\mathbf{r}})e^{-\frac{|\mathbf{r}-\mathbf{r}'|}{l}} \tag{3}$$

with:

$$\hat{\mathbf{r}} \equiv \frac{\mathbf{r} + \mathbf{r}'}{2} \tag{4}$$

Here l is called the correlation length of the temperature fluctuations and is assumed to be space-independent, i.e. the decay of correlations is uniform throughout the core. It can be noted that at the level of the correlation function, the assumption of spatially constant (homogeneous or coherent) temperature fluctuations is equivalent to an infinite correlation length and a constant intensity $\sigma^2(\hat{\mathbf{r}})$, i.e. $R(\mathbf{r}, \mathbf{r}') = $ const. For the space-dependent variance $\sigma^2(\hat{\mathbf{r}})$, three basic types will be used, all changing in a much longer scale than the exponential part. In particular, for long correlation lengths, only the case of constant $\sigma^2(\hat{\mathbf{r}})$ is physically realistic.

The above form of the correlation structure is of course very largely simplified. The assumed factorisation into two components as well as the assumption of spatially constant correlation length do not hold in practice. Nevertheless, the model does represent a deviation from the assumption of spatially constant temperature fluctuations and introduces the spatially decaying correlations. Hence, it is suitable for a conceptual study even in quantitative terms.

Due to the temporally white noise behaviour of the noise sources, the power spectrum of the perturbation is constant in frequency. Consequently, the Fourier transform of Eq. (3) only retains the spatial part and defines the cross-power spectral density (CPSD) of the temperature fluctuations:

$$\text{CPSD}_{\delta\Sigma_a}(\mathbf{r}, \mathbf{r}', \omega) = R(\mathbf{r}, \mathbf{r}') = \sigma^2(\hat{\mathbf{r}})e^{\frac{|\mathbf{r}-\mathbf{r}'|}{l}} \tag{5}$$

The amplitude function $\sigma^2(\hat{\mathbf{r}})$ and the correlation length l allow defining entirely the spatial structure of the temperature noise. Several noise sources, i.e. different sets of these two parameters, are investigated in the following (see Section 4). Both short and long correlation lengths (compared to the core size) are studied, where the most interesting case from theoretical and practical viewpoints is the one corresponding to short correlation lengths.

2.2. Calculation of the space-dependent neutron noise

In one-group diffusion theory, the time- and space-dependent flux is given by the following equations:

$$\frac{1}{v_0}\frac{\partial\phi(\mathbf{r}, t)}{\partial t} = D_0\nabla^2\phi(\mathbf{r}, t) + \nu\Sigma_{f,0}(1 - \beta)\phi(\mathbf{r}, t) - \Sigma_a(\mathbf{r}, t)\phi(\mathbf{r}, t) + \lambda_0 C(\mathbf{r}, t) \tag{6}$$

$$\frac{\partial C(\mathbf{r}, t)}{\partial t} = \nu \Sigma_{f,0} \beta \phi(\mathbf{r}, t) - \lambda_0 C(\mathbf{r}, t) \tag{7}$$

where

$$\Sigma_a(\mathbf{r}, t) = \Sigma_{a,0} + \delta \Sigma_a(\mathbf{r}, t) \tag{8}$$

$$\phi(\mathbf{r}, t) = \phi_0(\mathbf{r}) + \delta \phi(\mathbf{r}, t) \tag{9}$$

$$C(\mathbf{r}, t) = C_0 + \delta C(\mathbf{r}, t) \tag{10}$$

All the symbols have their usual meaning and the subscript 0 represents the static case. If one neglects the second-order terms, subtracts the static equations from Eqs. (6) and (7), and eliminates the precursor density through a temporal Fourier transform, one obtains the following equation:

$$\nabla^2 \delta \phi(\mathbf{r}, \omega) + B^2(\omega) \delta \phi(\mathbf{r}, \omega) = \frac{\delta \Sigma_a(\mathbf{r}, \omega) \phi_0(\mathbf{r})}{D_0} \tag{11}$$

where

$$B^2(\omega) = B_0^2 \left(1 - \frac{1}{\rho_\infty G_0(\omega)} \right) \tag{12}$$

and

$$G_0(\omega) = \frac{1}{i\omega \left(\Lambda + \dfrac{\beta}{i\omega + \lambda} \right)} \tag{13}$$

The solution of Eq. (11) can be determined through the use of the Green's function technique:

$$\delta \phi(\mathbf{r}, \omega) = \frac{1}{D_0} \int G(\mathbf{r}, \mathbf{r}', \omega) \delta \Sigma_a(\mathbf{r}', \omega) \phi_0(\mathbf{r}') d\mathbf{r}' \tag{14}$$

where the Green's function is the solution of the following equation:

$$\nabla_{\mathbf{r}}^2 G(\mathbf{r}, \mathbf{r}', \omega) + B^2(\omega) G(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}') \tag{15}$$

More precisely, since only the statistical properties of the temperature fluctuations are known, the APSD and (spatial) CPSD of the neutron noise are calculated by using the Wiener–Khinchin theorem, which connects the Fourier transform of a stochastic process with its power spectrum. One obtains the following expressions:

$$\text{APSD}_{\delta\phi}(\mathbf{r}, \omega) = \frac{1}{D_0^2} \int\int G^*(\mathbf{r}, \mathbf{r}', \omega)G(\mathbf{r}, \mathbf{r}'', \omega)\text{CPSD}_{\delta\Sigma_a}(\mathbf{r}', \mathbf{r}'', \omega)\phi_0(\mathbf{r}')\phi_0(\mathbf{r}'')\mathrm{d}\mathbf{r}'\mathrm{d}\mathbf{r}''$$

(16)

$$\text{CPSD}_{\delta\phi}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{1}{D_0^2} \int\int G^*(\mathbf{r}_1, \mathbf{r}', \omega)G(\mathbf{r}_2, \mathbf{r}'', \omega)\text{CPSD}_{\delta\Sigma_a}(\mathbf{r}', \mathbf{r}'', \omega)\phi_0(\mathbf{r}')\phi_0(\mathbf{r}'')\mathrm{d}\mathbf{r}'\mathrm{d}\mathbf{r}''$$

(17)

Eqs. (16) and (17), together with Eq. (5), constitute the exact solution of the problem. They can be now used to check the validity of point-kinetic behaviour of the flux response as well as to check the accuracy of the traditional MTC evaluation formula. In this latter both the deviation from point-kinetics as well as the bias introduced by the assumption of spatially homogeneous perturbation are included.

2.3. Comparison with the point-kinetic approximation

To investigate the deviation of the noise auto- and cross-spectra from their point-kinetic approximation, we recall that according to the latter the neutron noise in the frequency domain is given as

$$\delta\phi^{\text{pk}}(\mathbf{r}, \omega) = \delta P(\omega)\phi_0(\mathbf{r})$$

(18)

As is also known, the fluctuation of the amplitude factor, $\delta P(\omega)$ is given as

$$\delta P(\omega) = \delta\rho(\omega)G_0(\omega)$$

(19)

Here $\delta\rho(\omega)$ is the Fourier transform of the reactivity effect $\delta\rho(t)$, given by the first-order perturbation formula as:

$$\delta\rho(t) = \frac{-\int \delta\Sigma_a(\mathbf{r}, t)\phi_0^2(\mathbf{r})\mathrm{d}\mathbf{r}}{\nu\Sigma_{f,0} \int \phi_0^2(\mathbf{r})\mathrm{d}\mathbf{r}}$$

(20)

which gives in the frequency domain:

$$\delta\rho(\omega) = \frac{-\int \delta\Sigma_a(\mathbf{r}, \omega)\phi_0^2(\mathbf{r})\mathrm{d}\mathbf{r}}{\nu\Sigma_{f,0} \int \phi_0^2(\mathbf{r})\mathrm{d}\mathbf{r}}$$

(21)

With this, one obtains

$$\delta\phi^{\text{pk}}(\mathbf{r}, \omega) = \phi_0(\mathbf{r})\delta\rho(\omega)G_0(\omega)$$

(22)

As in the full space-dependent case, we need to turn to the auto- and cross-spectra of the neutron noise, expressed as a function of the APSD of the reactivity, which in

turn is expressed through the CPSD of the cross-section fluctuations. One readily obtains

$$\text{APSD}_{\delta\phi}^{\text{pk}}(\mathbf{r}, \omega) = \left|G_0(\omega)\right|^2 \phi_0^2(\mathbf{r}) \text{APSD}_{\delta\rho}(\omega) \tag{23}$$

$$\text{CPSD}_{\delta\phi}^{\text{pk}}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \left|G_0(\omega)\right|^2 \phi_0(\mathbf{r}_1) \phi_0(\mathbf{r}_2) \text{APSD}_{\delta\rho}(\omega) \tag{24}$$

where

$$\text{APSD}_{\delta\rho}(\omega) = \frac{\int\int \text{CPSD}_{\delta\Sigma_a}(\mathbf{r}, \mathbf{r}', \omega)\phi_0^2(\mathbf{r})\phi_0^2(\mathbf{r}')\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}'}{\left(\nu\Sigma_{f,0} \int \phi_0^2(\mathbf{r})\mathrm{d}\mathbf{r}\right)^2} \tag{25}$$

A comparison of the full space-dependent solutions, i.e. the set of Eqs. (16) and (17) together with Eq. (5) ("exact solution"), to the point-kinetic approximations, Eqs. (23) and (24) ("0-D approximation"), will display the error induced by the assumption of point- kinetic behaviour. This will be performed quantitatively in the next section. A quite detailed investigation of this formula was also given in (Demazière and Pázsit, 2000).

3. Principles of the MTC noise estimate

By definition, the MTC is the reactivity variation due to a change of the inlet temperature of the coolant, divided by the average temperature change [see the newest American Nuclear Standard (ANS), 1997]:

$$\delta\rho(t) = \text{MTC} \times \delta T_{\text{m}}^{\text{ave}}(t) \tag{26}$$

This definition is only useful as long as the temperature change is constant in space. When the temperature change is relatively homogeneous through the core, the axial variation can be handled by using the temperature average according to the formula

$$\delta T_{\text{m}}^{\text{ave}}(t) = \frac{\delta T_{\text{in}}(t) + \delta T_{\text{out}}(t)}{2} \tag{27}$$

where the subscripts in and out stand for the inlet and the outlet of the core, respectively. As mentioned before, when using noise analysis, at the frequency used in the measurement the axial variation of the temperature fluctuations can be neglected, and therefore it suffices to use one axial position (the core outlet) in the measurement. Correspondingly, we have neglected the axial variable in the present model and only will consider the radial variation.

The radial variation of the temperature constitutes a much more serious concern, and this is the main subject of the present paper. The Standard (ANS 1997) only says that if the temperature variation is not distributed homogeneously throughout the core, an average that reflects this distribution must be used. There are two problems associated with this question. One is to decide what weighting function should be used to calculate this average. Second, the radial profile of the temperature fluctuations (or their spatial correlations) are not utilized in a measurement since the temperature fluctuations as measured in one single point are usually used.

The Standard states that the weighting function does not play a significant role as long as the same definition is used when comparisons are made between calculations and measurements. Although the Standard recommends to use a volume-average (i.e. a weighting function equal to unity), this definition appears to be inappropriate as will be shown shortly.

Assuming for a while that the temperature fluctuations are constant in space, a very simple algebra yields from Eqs. (26), (1) and (21) that the MTC is given by

$$\text{MTC} = -\frac{1}{K\nu\Sigma_{\text{f},0}} \tag{28}$$

By definition, this is the exact MTC of the system, irrespective of the space dependence of the temperature fluctuations. The MTC given by Eq. (28) will thus be called in the following the "reference" MTC with which the noise estimate will be compared. So far, nothing was said about how the MTC is determined experimentally.

If the temperature fluctuations are not homogeneous in space, one needs to define the average temperature variation whose usage in (26) leads to the same MTC. It is easy to confirm that if the average temperature fluctuation in the core is defined as

$$\delta T_{\text{m}}^{\text{ave}}(t) = \frac{\int \delta T_{\text{m}}(\mathbf{r}, t)\phi_0^2(\mathbf{r})d\mathbf{r}}{\int \phi_0^2(\mathbf{r})d\mathbf{r}} \tag{29}$$

then, Eqs. (26), (1) and (21) will still lead to the exact value (28). This shows that the average temperature change must be calculated by using the square of the static flux as a weighting function.

Conceptually, the MTC given by Eq. (28) represents the actual value of the reactivity coefficient in our model. In practice, one should use noise estimators with measured noise signals as input data to determine this reference value experimentally. In case of spatially homogeneous temperature perturbation, Thie (1977), Türkcan (1982), and Pór et al. (1985) showed that the MTC can be inferred from the statistical properties of the reactivity noise and the moderator temperature noise. In such a case, the reactor response is necessarily point-kinetic and therefore the correct MTC value is given by what will be called later the H_1 estimator. It has to be emphasized that this estimator, which is given by Eq. (34) in the following, uses directly the flux noise instead of the reactivity noise (through the use of the zero-power reactor transfer function), but this substitution is valid as long as the reactor

behaves in a point-kinetic way, i.e. when the temperature noise is distributed homogeneously throughout the core. In case of heterogeneous temperature distribution, the reactor response is not any longer point-kinetic, thus the reactivity noise cannot be replaced by the flux noise. Further, the spatial heterogeneity of the temperature noise still need to be accounted for in order to fulfil Eqs. (26) and (29).

Consequently, we define a modified H_1 estimator that yields the correct MTC value given by Eq. (28):

$$H_1 = \frac{\mathrm{CCF}_{\delta\rho, \delta T_{\mathrm{m}}^{\mathrm{ave}}}(\tau)}{\mathrm{ACF}_{\delta T_{\mathrm{m}}^{\mathrm{ave}}}(\tau)} \tag{30}$$

or in the frequency domain:

$$H_1 = \frac{\mathrm{CPSD}_{\delta\rho, \delta T_{\mathrm{m}}^{\mathrm{ave}}}(\omega)}{\mathrm{APSD}_{\delta T_{\mathrm{m}}^{\mathrm{ave}}}(\omega)}, \tag{31}$$

with the definition of the average temperature as in (29). When the temperature noise is spatially homogeneous, this modified estimator coincides with the one introduced by Pór et al. (1985). Otherwise, the reactor response will deviate from point-kinetics, and, more important, the local temperature fluctuation will also deviate from the average one. Hence, in such cases, the reactivity noise and the average temperature noise must be used to obtain the expected MTC value, as Eqs. (30) and (31) show. Although both the numerator and the denominator of (31) depend on frequency, the dependences cancel in the expression and the result is a frequency-independent constant, equal to the MTC. Other noise estimators have been proposed to measure the MTC (Herr and Thomas, 1991; García Cuesta and Blázquez, 1995). But in case of coherent reactivity and moderator temperature noise, i.e. when the reactivity noise is solely induced by the temperature noise, these estimators give the same result. Since in our model one single type of noise source is considered and therefore the reactivity noise is fully induced by the moderator temperature noise, for the sake of simplicity we shall only discuss the performance of the most frequently used estimator, the H_1 estimator.

The problem with the definition (31) is that neither the reactivity, nor the average temperature is measured directly. Therefore, in the first step, it is assumed that the reactor response is point-kinetic, and thus the reactivity noise can be inferred from the flux noise. Using (22) in (31) will lead to the biased estimator $\tilde{H}_1^{\mathrm{biased}}(\mathbf{r}, \omega)$ as

$$\tilde{H}_1^{\mathrm{biased}}(\mathbf{r}, \omega) = \frac{1}{G_0(\omega)\phi_0(\mathbf{r})} \frac{\mathrm{CPSD}_{\delta\phi, \delta T_{\mathrm{m}}^{\mathrm{ave}}}(\mathbf{r}, \omega)}{\mathrm{APSD}_{\delta T_{\mathrm{m}}^{\mathrm{ave}}}(\omega)} \tag{32}$$

Because of the approximation introduced, this expression will only be approximately equal to the MTC, and the estimator becomes both space- and frequency-dependent. As usual in the noise measurement, its value in the frequency range 0.1–1 Hz is used to estimate the MTC.

In our analytical model with the specified temperature and cross-section fluctuations, we can derive an explicit expression for (32) by using Eqs. (1), (16) and (17). The result is

$$\tilde{H}_1^{\text{biased}}(\mathbf{r}, \omega) = \frac{\int \phi_0^2(\mathbf{r})d\mathbf{r}}{G_0(\omega)\phi_0(\mathbf{r})KD_0} \frac{\int\int G(\mathbf{r}, \mathbf{r}', \omega)\text{CPSD}_{\delta\Sigma_a}(\mathbf{r}', \mathbf{r}'', \omega)\phi_0^2(\mathbf{r}')\phi_0^2(\mathbf{r}'')d\mathbf{r}'d\mathbf{r}''}{\int\int \text{CPSD}_{\delta\Sigma_a}(\mathbf{r}', \mathbf{r}'', \omega)\phi_0^2(\mathbf{r}')\phi_0^2(\mathbf{r}'')d\mathbf{r}'d\mathbf{r}''}$$

$$(33)$$

Employing Eq. (5) in the above, the biased estimator can be quantitatively evaluated and compared to the exact value. This will be performed in the next section. The bias of this estimator will show the error induced by the assumption of the point-kinetic behaviour of the system. The magnitude of this bias is equal to the deviation between the APSD and CPSD of the neutron noise calculated exactly and in the point-kinetic approximation, respectively. This bias is not significant as it will be seen.

However the above estimator still cannot be measured in practice, since only the moderator temperature noise $\delta T_m(\mathbf{r}, t)$ in one radial point is measured, not the average temperature fluctuations throughout the core. Therefore the following estimator is used in measurements:

$$H_1^{\text{biased}}(\mathbf{r}, \omega) = \frac{1}{G_0(\omega)\phi_0(\mathbf{r})} \frac{\text{CPSD}_{\delta\phi, \delta T_m}(\mathbf{r}, \omega)}{\text{APSD}_{\delta T_m}(\mathbf{r}, \omega)}$$

$$(34)$$

For the same reasons as before, this estimator is also space- and frequency-dependent. This estimator will now be biased both due to the assumption of point-kinetic response, as well as the assumption of homogeneous temperature fluctuations. The latter can also be formulated that using the local temperature instead of the average one introduces a bias into the estimator (34).

This estimator can also be calculated analytically from our model as follows:

$$H_1^{\text{biased}}(\mathbf{r}, \omega) = \frac{1}{G_0(\omega)\phi_0(\mathbf{r})KD_0} \frac{\int G(\mathbf{r}, \mathbf{r}', \omega)\text{CPSD}_{\delta\Sigma_a}(\mathbf{r}', \mathbf{r}, \omega)\phi_0(\mathbf{r}')d\mathbf{r}'}{\text{APSD}_{\delta\Sigma_a}(\mathbf{r}, \omega)}$$

$$(35)$$

Evaluating the above expression and comparing it with the exact value makes it possible to quantify the deviation of the estimated value from the reference MTC. This too will be performed in the next section.

4. Quantitative results and discussion

In the quantitative work, the previous model is applied to a realistic commercial PWR, namely Ringhals-4 (Sweden). The material constants are obtained from the Studsvik Scandpower SIMULATE-3 code (Umbarger and DiGiovine, 1992). Nevertheless, even if the material constants are given in a two-group point-kinetic

formulation, they represent a 3-D system. Consequently, some modifications are necessary to apply them to our 1-group 1-D model. The energy condensation and the 3-D to 1-D spatial transformation is carried out such that the actual migration area and the diffusion coefficient are preserved. Finally, the criticality is maintained by modifying the $\nu\Sigma_{f,0}$ cross-section.

4.1. Parameters of the model

The material constants are representative of the fuel cycle 15 in Ringhals-4. To have some feeling of the influence of the burnup on the accuracy of the MTC determination, three sets of material data were used: one at the beginning of the cycle, one at the burnup of 53.438 GWd/tHM (MOC) and one at 9.378 GWd/tHM (corresponding to the point where a noise measurement was performed; for simplicity this will be called EOC data), respectively (Aira, 1999). These data are shown in Table 1. In the following, a denotes the core radius (core half-width), which is chosen to be equal to 150 cm.

Regarding the spatial variation of the noise variance, i.e. noise strength, described by Eq. (5), several forms have been investigated, and for each of them three different correlation lengths have been examined ($l = a, l = a/10$, and $l = a/100$). The three noise variance functions that were used are as follows:

$$\sigma_h(\hat{x}) = 1 \quad \text{(``homogeneous'' case);} \tag{36a}$$

$$\sigma_c(\hat{x}) = \cos(B_0\hat{x}) \text{ (``central'' case);} \tag{36b}$$

$$\sigma_p(\hat{x}) = \cfrac{1}{1 - \left(\cfrac{\hat{x}}{a + \delta a}\right)^2} \quad \text{where} \quad \delta a = a/5 \quad \text{(``peripheral'' case).} \tag{36c}$$

Case (36a), where h stands for homogeneous, represents the simplest possible choice and is the first guess if nothing is known in advance. With an infinite correlation length, it coincides with the point-kinetic solution since the temperature fluctuation is homogeneous. This specific case is used to test that the exact solution and the point-kinetic approximation of the neutron noise are the same, thus leading to an accurate MTC noise estimation. Case (36b), where c stands for central, assumes that the strength of the temperature noise σ^2, is proportional to the power, and then the amplitude function, expressed in form of a variance, is proportional to the

Table 1
Parameters of the 1-group 1-D model

Core burnup	β (pcm)	Λ (μs)	λ (ms)	$\Sigma_a, 0$ (cm^{-1})	$\nu\Sigma_{f,0}$ (cm^{-1})	D_0 (cm)	ρ_∞ ($)
BOC	590	19.3	84.8	2.315×10^{-2}	2.330×10^{-2}	1.331	1.062
MOC	551	20.3	34.9	2.315×10^{-2}	2.330×10^{-2}	1.341	1.145
EOC	529	21.6	85.2	2.302×10^{-2}	2.316×10^{-2}	1.331	1.191

square of the static flux. Finally, case (36c), where p stands for peripheral, is based on experimental evidence which shows that, contrary to the expectations, there are observed cases when the temperature noise is somewhat larger close to the core boundary than at the core centre (Karlsson, 2000). In cases (36b) and (36c), only the short correlation lengths are physically relevant, i.e. $l = a/10$ and $l = a/100$. Nevertheless, for the sake of completeness, a correlation length equal to the core half width, i.e. $l = a$, is also investigated in these cases. These three different shape (amplitude) functions σ^2 are shown in Fig. 1.

4.2. Results

The results of the calculations are shown in the following figs. (Figs. 2–7). Only the calculations performed at a frequency of 1 Hz are presented, since the MTC in the noise analysis method is usually evaluated in the frequency range 0.1 to 1 Hz. Further, only the results corresponding to EOC are displayed. This is because the calculations showed that the relative error in the neutron flux noise due to the point-kinetic approximation, as well as that of the MTC noise estimates (the ratio between the noise estimators and the actual value of the MTC) seem to be very weakly dependent of the core burnup. This supports the fact that one single calibration factor can be used throughout the whole fuel cycle, and maybe even throughout several cycles, as long as the same pair of detectors (in-core neutron detector and core-exit thermocouple for the traditional H_1 estimator) are used, and the corresponding instrumented bundles have similar characteristics, from cycle to cycle

Fig. 1. Shape of the σ functions.

Fig. 2. APSD of the neutron noise for $\sigma_h(\hat{x})$ at 1 Hz and EOC (left column) and the relative error of the point-kinetic approximation (right column).

Fig. 3. MTC comparisons for $\sigma_h(\hat{x})$ at 1 Hz and EOC. Ratio of the traditional noise estimated MTC to the true MTC (left column) and ratio of the newly proposed estimator $\tilde{H}_1$ to the true MTC (right column).
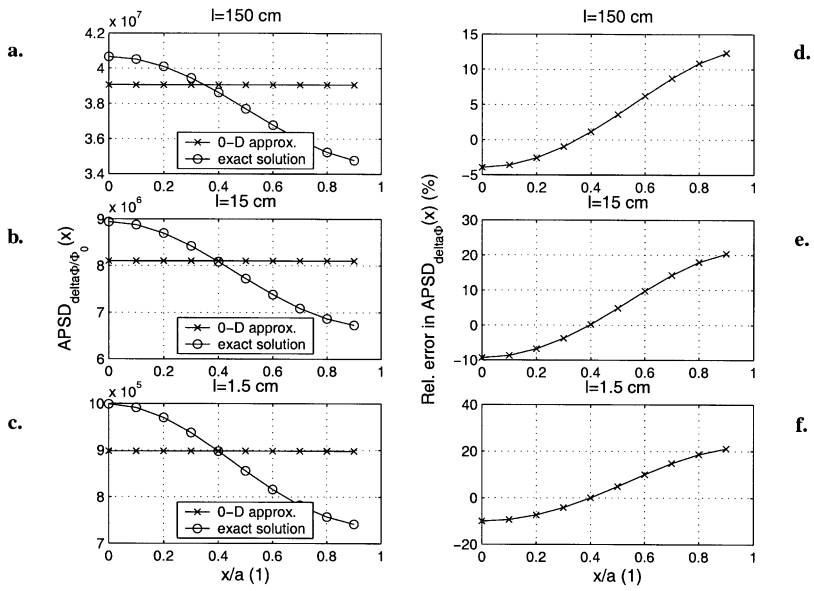
Fig. 4. APSD of the neutron noise for $\sigma_c(\hat{x})$ at 1 Hz and EOC (left column) and the relative error of the point-kinetic approximation (right column).
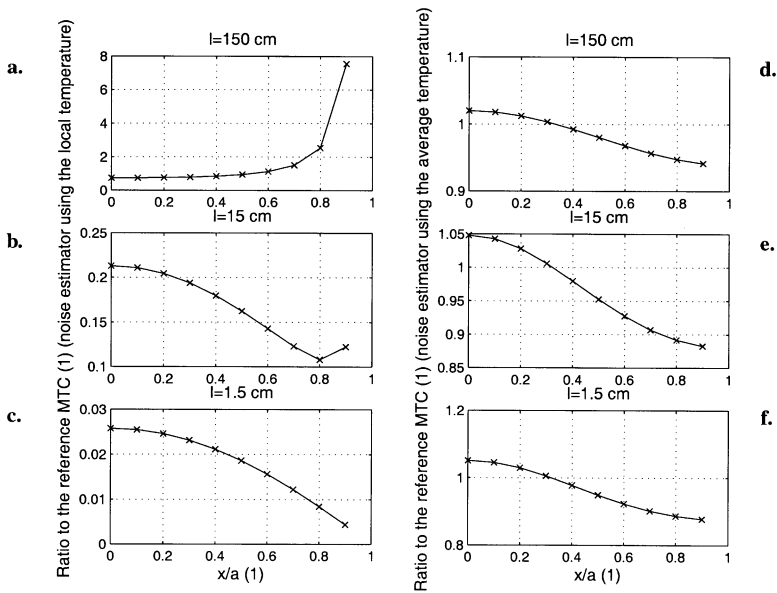
Fig. 5. MTC comparisons for $\sigma_c(\hat{x})$ at 1 Hz and EOC. Ratio of the traditional noise estimated MTC to the true MTC (left column) and ratio of the newly proposed estimator $\tilde{H}_1$ to the true MTC (right column).
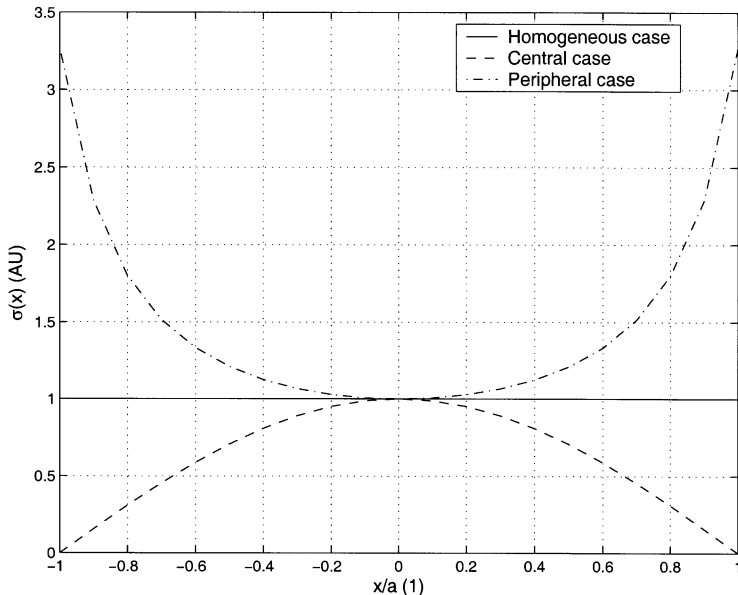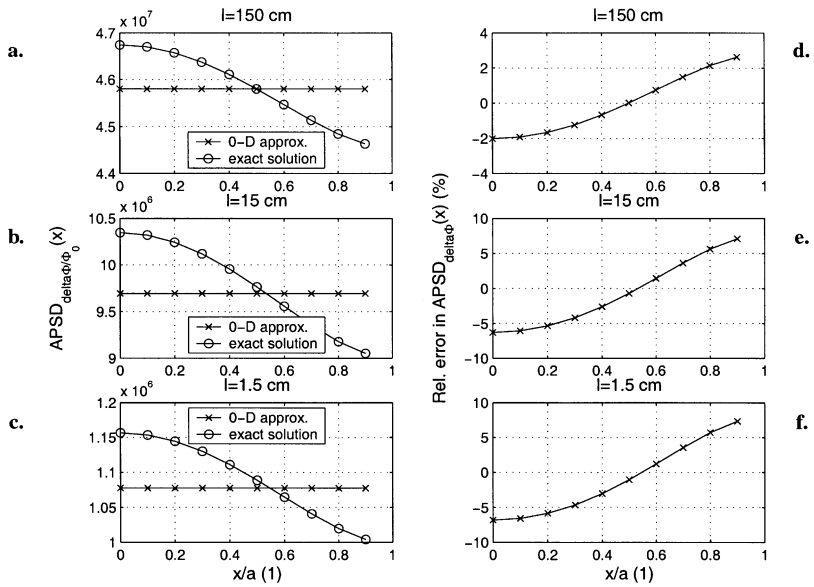
Fig. 6. APSD of the neutron noise for $\sigma_p(\hat{x})$ at 1 Hz and EOC (left column) and the relative error of the point-kinetic approximation (right column).
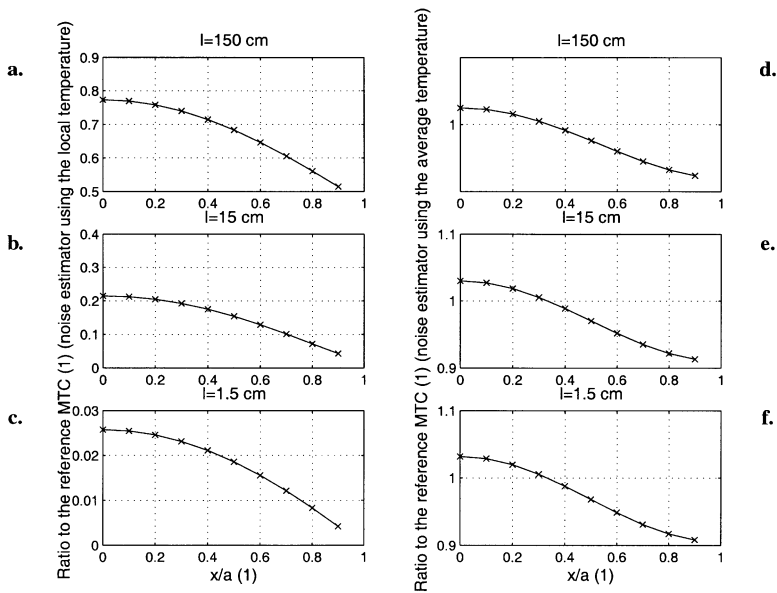

Fig. 7. MTC comparisons for $\sigma_p(\hat{x})$ at 1 Hz and EOC. Ratio of the traditional noise estimated MTC to the true MTC (left column) and ratio of the newly proposed estimator $\tilde{H}_1$ to the true MTC (right column).
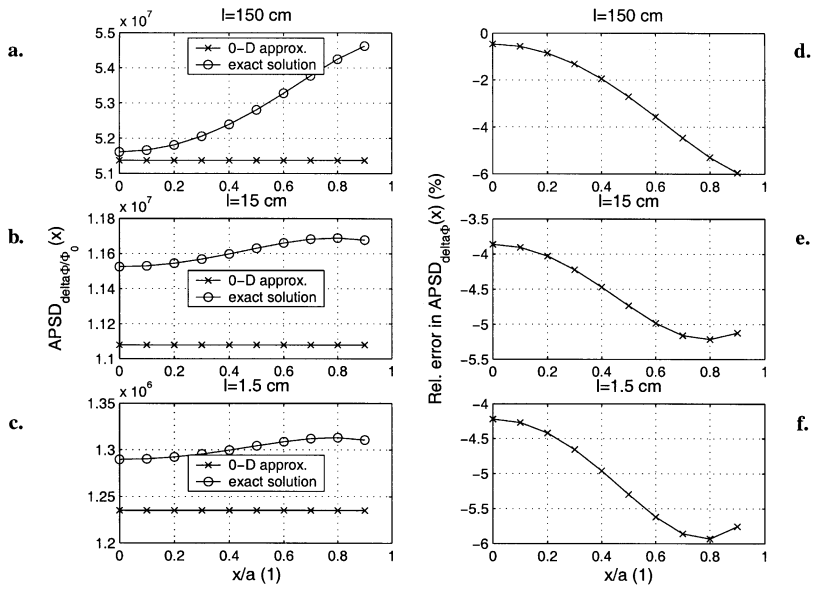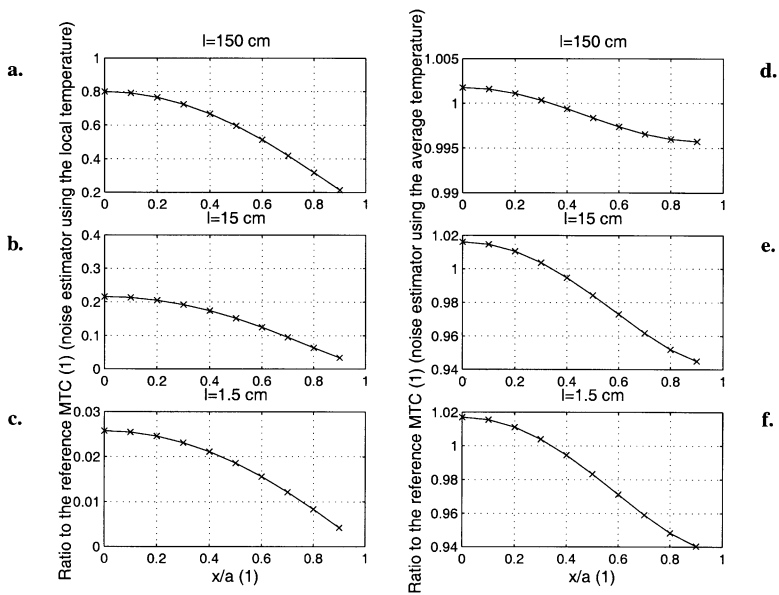
(Oguma et al., 1995a,b). As was mentioned in the Introduction, the confirmation of this fact in the present model is not surprising since the main parameter which determines the MTC underestimation is the correlation length of the temperature fluctuations, which is independent of the burnup.

Regarding the first case, Case (36a) ["homogeneous" variance, see Eq. (36)], Fig. 2 shows the results of the calculations of the APSD of the neutron noise in the point-kinetic approximation ("0-D approx.") and the exact APSD calculations ("exact solution") as a function of the position in the core. The relative difference between the two solutions is also given. Due to the symmetry of the system, only the results for $x \in [0, a]$ are presented. In this simple, symmetric model with constant correlation length, the point-kinetic solution underestimates the exact solution at the core centre, whereas the opposite behaviour prevails towards the core boundary. The relative difference between the two solutions remains nevertheless quite moderate. It can be seen also that the deviation from point-kinetics increases monotonically with decreasing correlation length.

The calculations concerning the cross-spectrum of the noise show a similar tendency. They will however not be shown here. We refer instead to the extensive material published in Demazière and Pázsit (2000).

The results of the MTC calculations regarding Case (36a) of Eq. (36) are shown in Fig. 3. The comparison between the results given by the H_1^{biased} estimator [Eq. (35), which is the formula used in the traditional noise method] and the actual (true) MTC is given in the left column. The comparison between the $\tilde{H}_1^{\text{biased}}$ estimator [Eq. (33), which is the one suggested in the present paper and which is based on the use of the average temperature] and the actual MTC is plotted in the right column. From the left column, one can notice that the usual way of determining the MTC by noise analysis systematically underestimates the actual value of the MTC, and that this deviation increases drastically with decreasing correlation length. Moreover, the deviation is also strongly space-dependent. On the contrary, the MTC estimator relying on the use of the average temperature fluctuation shows only small deviation from its actual value. It has to be emphasised also that the difference between the point-kinetic solution and the exact solution regarding the calculation of the flux noise is almost the same for $l = a/10$ and $l = a/100$. So is the MTC estimator using the average temperature noise, whereas the usual MTC estimator shows large difference between these two correlation lengths.

Figs. 4 and 5 represent Case (36b) of Eq. (36), i.e. the "central" case. The deviation from point-kinetics is somewhat larger than for the previous case, but the same tendencies can be noticed: the point-kinetic solution underestimates the exact one at the core centre, overestimates it at the boundary, and the difference between $l = a/10$ and $l = a/100$ is only noticeable for the classical MTC estimator. Nevertheless, one major difference with Case (36a) can be seen in the comparison of the usual MTC estimator to its actual value for long correlation length $l = a$ (Fig. 5a), where the noise method seems to overestimate the MTC close to the boundary. Nevertheless, as mentioned previously, this case is physically unrealistic.

Case (36c), i.e. the "peripheral" case, is presented in Figs. 6 and 7. Here the point-kinetic solution systematically underestimates the exact solution, but this deviation

is smaller towards the core centre. As mentioned earlier, the difference between short and medium correlation lengths, i.e. $l = a/100$ and $l = a/10$, respectively, is only noticeable when evaluating the MTC with the usual noise estimator.

It is thus demonstrated that in the traditional noise method the MTC is systematically and significantly underestimated. This underestimation depends primarily on the correlation length of the temperature fluctuations, whereas the spatial strength distribution $\sigma(\hat{x})$ as well as the burnup play a much smaller, practically negligible role.

An additional point is that, as is seen in Figs. 3, 5, and 7, in case of inhomogeneous temperature fluctuations, the MTC estimate becomes strongly space-dependent. The physical explanation is that the reactivity effect of a local perturbation is closely related to the importance (weight) of the position, and this weight decreases monotonically from the centre of the core towards the periphery. This is another phenomenon which was observed experimentally (Demazière et al., 2000; Demazière, 2000). For this reason the underestimation of the MTC, according to the data reported in Figs. 3–7, covers a very wide area, between a factor 2 to about 50. To restrict this arbitrariness, we shall only consider the results in the central positions, i.e. in the vicinity of the core centre. This choice is justified by the fact that in the experiments, only such measurement positions are used because only those yield acceptable results (Demazière et al., 1999, 2000; Demazière, 2000). Thus as Figs. 3b, 5b, and 7b show, for a central perturbation, and a correlation length of 15 cm, the MTC is underestimated with a factor of 5. Both this latter factor, and the correlation length cited, agree well with measurements performed in Ringhals (Demazière, 2000; Demazière et al., 2000). This correlation length is about the size of a PWR fuel assembly, and in preliminary measurements using three strings of gamma-thermometers, a correlation length in this range was found. Measurements using 12 radial gamma-thermometer positions, yielding a much better spatial resolution, will be performed soon. From those measurements a better estimate of the correlation length will be obtained.

4.3. Analysis of the MTC underestimation

Our calculations show that using the traditional H_1 estimator (where only the local temperature noise is measured) will lead to a significantly underestimated value of the MTC with short correlation lengths. It has to be emphasised also that this H_1 estimator is strongly space-dependent. Moreover, for a correlation length close to the size of a PWR assembly and a central position, the bias between the MTC estimator and its actual value is in the same range as the one usually measured in noise analysis, i.e. a factor of about 5.

The first effect that could explain why noise analysis underestimates the actual value of the MTC is the deviation from point-kinetics of the reactor response. This effect is not very significant since the $\tilde{H}_1^{\text{biased}}$ estimator only deviates slightly from the actual MTC value, as can be seen in Figs. 3d–f, 5d–f, and 7d–f. Likewise, the deviation from point-kinetics of the neutron noise itself is appreciable, but quite moderate at plateau frequencies (see Figs. 2,4, and 6), and therefore does not allow explaining solely the strong MTC underestimation. The reason for the small deviation

of the $\tilde{H}_1^{\text{biased}}$ estimator from the expected MTC value lies with the fact that this estimator is able to take the spatial heterogeneous structure of the temperature fluctuation into account [see Eq. (29)], and thereby being free from some discrepancies that will be analysed below.

The relationship between the exact (space-dependent) neutron noise and the point-kinetic approximation can be found by expanding the flux noise, solution of Eq. (11), with respect to spatial static eigenfunctions $\phi_k(\mathbf{r})$:

$$\delta\phi(\mathbf{r}, \omega) = \sum_k A_k(\omega)\phi_k(\mathbf{r}) \tag{37}$$

or similarly by expanding the Green's function with respect to the $\phi_k(\mathbf{r})$ functions:

$$G(\mathbf{r}, \mathbf{r}', \omega) = \sum_k B_k(\omega)\phi_k(\mathbf{r})\phi_k(\mathbf{r}') \tag{38}$$

The $A_k(\omega)$ coefficients are given by

$$A_k(\omega) = \frac{\int \delta\Sigma_a(\mathbf{r}, \omega)\phi_0(\mathbf{r})\phi_k(\mathbf{r})\mathrm{d}\mathbf{r}}{D_0\left[B^2(\omega) - B_k^2\right]\int \phi_k^2(\mathbf{r})\mathrm{d}\mathbf{r}} \tag{39}$$

the $B_k(\omega)$ coefficients by

$$B_k(\omega) = \frac{1}{\left[B^2(\omega) - B_k^2\right]\int \phi_k^2(\mathbf{r})\mathrm{d}\mathbf{r}} \tag{40}$$

whereas the $\phi_k(\mathbf{r})$ eigenfunctions satisfy the following equation:

$$\nabla^2\phi_k(\mathbf{r}) + B_k^2\phi_k(\mathbf{r}) = 0 \tag{41}$$

The $A_0(\omega)$ [or the $B_0(\omega)$] coefficient gives the point-kinetic contribution to the flux-noise, so that:

$$A_0(\omega) = \delta P(\omega) = -\frac{G_0(\omega)\int \delta\Sigma_a(\mathbf{r}, \omega)\phi_0^2(\mathbf{r})\mathrm{d}\mathbf{r}}{\nu\Sigma_{f,0}\int \phi_0^2(\mathbf{r})\mathrm{d}\mathbf{r}} \tag{42}$$

or

$$B_0(\omega) = -\frac{DG_0(\omega)}{\nu\Sigma_f\int \phi_0^2(\mathbf{r})\mathrm{d}\mathbf{r}} \tag{43}$$

By using the Wiener–Khinchin theorem, the deviation from point-kinetics of the reactor response can be assessed:

$$\text{APSD}_{\delta\phi}(\mathbf{r}, \omega) = \text{APSD}_{\delta\phi}^{\text{pk}}(\mathbf{r}, \omega) + \sum_{k+1>0} A_k(\omega)A_l^*(\omega)\phi_k(\mathbf{r})\phi_l(\mathbf{r}) \tag{44}$$

$$\text{CPSD}_{\delta\phi}(\mathbf{r}, \mathbf{r}', \omega) = \text{CPSD}_{\delta\phi}^{\text{pk}}(\mathbf{r}, \mathbf{r}', \omega) + \sum_{k+l>0} A_k(\omega)A_l^*(\omega)\phi_k(\mathbf{r})\phi_l(\mathbf{r}') \tag{45}$$

with

$$\text{APSD}_{\delta\phi}^{\text{pk}}(\mathbf{r}, \omega) = \left|A_0(\omega)\right|^2 \phi_0^2(\mathbf{r}) \tag{46}$$

$$\text{CPSD}_{\delta\phi}^{\text{pk}}(\mathbf{r}, \mathbf{r}', \omega) = \left|A_0(\omega)\right|^2 \phi_0(\mathbf{r})\phi_0(\mathbf{r}') \tag{47}$$

From these expressions, it is easy to see that only the case of a homogeneous structure of the temperature noise, i.e. $\delta\Sigma_a(\mathbf{r}, \omega) = \delta\Sigma_a(\omega), \forall \mathbf{r}$, gives a point-kinetic response of the reactor, since the orthogonality of the eigenfunctions $\phi_k(\mathbf{r})$ implies $A_k(\omega) = 0, k \geqslant 1$. Otherwise, the flux noise deviates from point-kinetics, as shown by Eqs. (44) and (45). The deviation increases with the deviation of the perturbation from homogeneous, i.e. with decreasing correlation lengths. As mentioned earlier, this effect is not significant.

The second effect that could explain the MTC underestimation by noise analysis lies with the underestimation of the reactivity effect in case of heterogeneous noise sources. With spatially inhomogeneous temperature fluctuations, the reactivity effect will be smaller than it is assumed in the traditional method, since the ratio between the true reactivity effect and the assumed one (infinite correlation, i.e. homogeneous temperature noise) will be smaller than unity, as shown below. This fact would lead to an underestimation of the MTC even if the reactor response was point-kinetic.

This ratio, as the function of the correlation length, can be easily calculated in the present model. Using Eq. (25), which assumes a point-kinetic behaviour of the reactor, together with Eq. (5) with a finite correlation length and with $l = \infty$, one obtains

$$\frac{\text{APSD}_{\delta\rho}^{\text{true}}}{\text{APSD}_{\delta\rho}^{\text{assumed}}}(l) = \frac{\int\int e^{\frac{-|\mathbf{r}-\mathbf{r}'|}{l}}\phi_0^2(\mathbf{r})\phi_0^2(\mathbf{r}')\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}'}{\int\int \phi_0^2(\mathbf{r})\phi_0^2(\mathbf{r}')\mathrm{d}\mathbf{r}\mathrm{d}\mathbf{r}'} \tag{48}$$

in case of a constant noise strength, i.e. $\sigma_h^2(\hat{\mathbf{r}}) = 1$. The variation of this ratio with the correlation length is depicted in Fig. 8. It is seen that for a correlation length equal to the core half-width, i.e. $l = a$, the overestimation of the reactivity effect by the idealised model is only about 30%. However, with decreasing correlation length, the true reactivity effect becomes a smaller and smaller fraction of the one assumed in the traditional MTC formula.

Nevertheless, the above described reactivity underestimation is space-independent. Since, as explained above, the deviation from point-kinetics is space-dependent but weak, another strongly space-dependent effect takes place in the MTC underestimation.
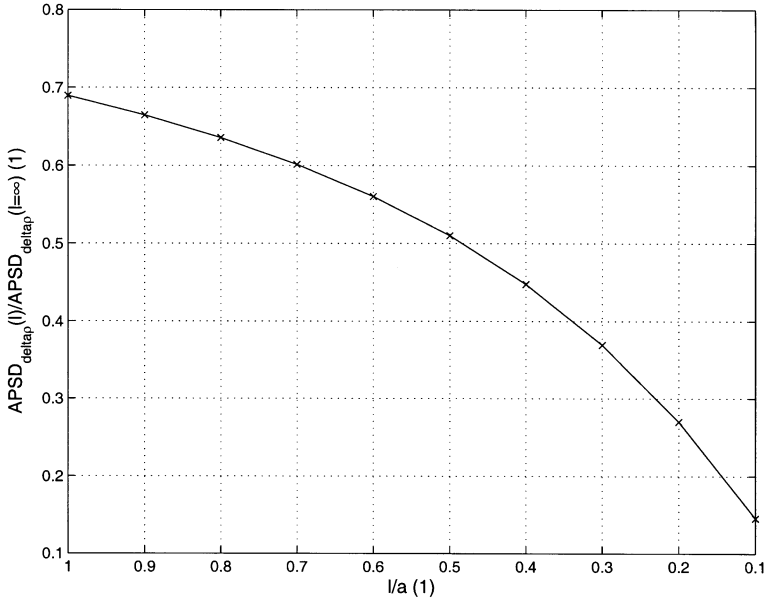
Fig. 8. Variation of the ratio of the APSDs of the reactivity noise for inhomogeneous and homogeneous temperature fluctuations, $\mathrm{APSD}_{\delta\rho}^{\mathrm{true}}/\mathrm{APSD}_{\delta\rho}^{\mathrm{assumed}}$, for $\sigma_{\mathrm{h}}(\hat{x})$ at 1 Hz and EOC, with decreasing correlation length.

This last effect is due to the underestimation of the cross-correlation between the neutron noise and the temperature noise when temperature fluctuations are non-homogeneous. As before, assuming a point-kinetic behaviour of the reactor, one can calculate the ratio between the true CPSD and the assumed one (infinite correlation length, i.e homogeneous temperature noise).

In case of point-kinetic response, the CPSD between neutron noise and temperature noise (local measurement by using the H_1^{biased} estimator) can be written, by virtue of the Wiener–Khinchin theorem:

$$\mathrm{CPSD}_{\delta\phi^{\mathrm{pk}}/\phi_0,\delta T_{\mathrm{m}}}(\mathbf{r},\omega) = G_0(\omega)\delta\rho(\omega)K\delta\Sigma_{\mathrm{a}}^*(\mathbf{r},\omega) \tag{49}$$

Using Eq. (42) gives:

$$\mathrm{CPSD}_{\delta\phi^{\mathrm{pk}}/\phi_0,\delta T_{\mathrm{m}}}(\mathbf{r},\omega) = -\frac{KG_0(\omega)}{\nu\Sigma_{\mathrm{f},0}}\frac{\int R(\mathbf{r},\mathbf{r}')\phi_0^2(\mathbf{r}')\mathrm{d}\mathbf{r}'}{\int \phi_0^2(\mathbf{r})\mathrm{d}\mathbf{r}} \tag{50}$$

so that, in case of a constant shape (amplitude) function, i.e. $\sigma_{\mathrm{h}}^2(\hat{\mathbf{r}}) = 1$:

$$\frac{\mathrm{CPSD}_{\delta\phi^{\mathrm{pk}}/\phi_0,\delta T_{\mathrm{m}}}^{\mathrm{true}}(\mathbf{r},\omega)}{\mathrm{CPSD}_{\delta\phi^{\mathrm{pk}}/\phi_0,\delta T_{\mathrm{m}}}^{\mathrm{assumed}}(\mathbf{r},\omega)} = \frac{\int \mathrm{e}^{\frac{|\mathbf{r}-\mathbf{r}'|}{l}}\phi_0^2(\mathbf{r}')\mathrm{d}\mathbf{r}'}{\int \phi_0^2(\mathbf{r}')\mathrm{d}\mathbf{r}'} \tag{51}$$
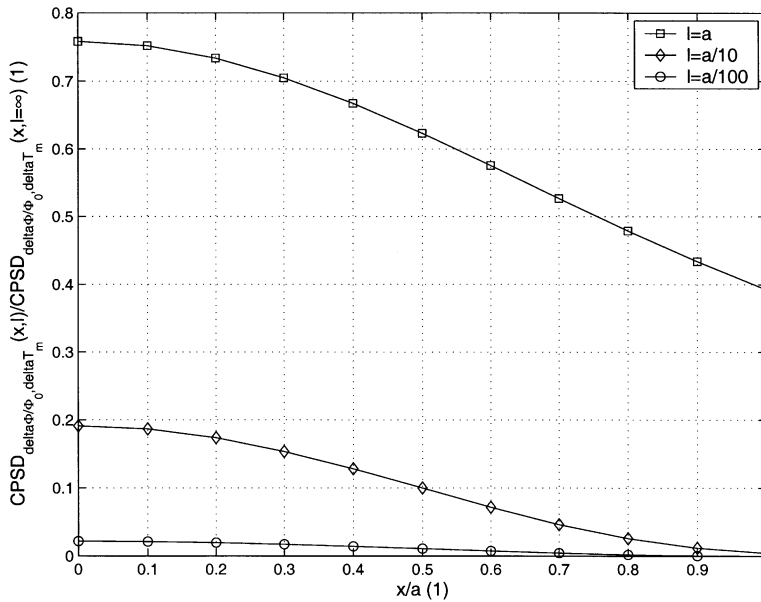
Fig. 9. Variation of the ratio of the CPSDs between neutron (point-kinetic case) and temperature noise for inhomogeneous and homogeneous temperature fluctuations, $\mathrm{CPSD}_{\delta\rho}^{\mathrm{true}}/\mathrm{CPSD}_{\delta\rho}^{\mathrm{assumed}}$, for $\sigma_h(\hat{x})$ at 1 Hz and EOC.

The variation of this ratio with the position, for three different correlation lengths, is depicted in Fig. 9. One notices that this ratio is partly smaller than unity, and partly it is space-dependent.

To summarise, it is the strong space-dependence of the temperature fluctuations that is the reason for the significant underestimation of the MTC by the traditional noise method. One way of alleviating this discrepancy is to use integral methods regarding the temperature noise, i.e. to use the average temperature as defined by Eq. (29). Integral methods have so far been suggested only for the extraction of the reactivity from the flux measurements (see e.g. Antonopoulos-Domis and Housia-das, 1999). These latter play however a much less significant role, as was shown by the numerical analysis presented in the foregoing.

5. Discussion and conclusions

The analysis presented in this paper shows that the main reason of the under-estimation of the MTC in the traditional noise measurement lies in the fact that the spatially incoherent structure of the temperature fluctuations is neglected. There is a monotonic relationship between the correlation structure of the temperature fluc-tuations and the underestimation of the MTC (which can be quantified with a cali-bration factor that has to be used with the noise measurement to obtain the correct result). It was seen that the underestimation of the reference MTC strongly depends

on the correlation length (stronger underestimation with decreasing correlation length monotonically) and the spatial position in the core (larger discrepancy at the core boundary than at the core centre), but does not depend significantly on the spatial shape of the strength (variance) of the temperature noise itself. In addition, the underestimation practically does not depend on the burnup. For a correlation length of the size of a typical PWR bundle and a flux noise estimated in the centre of the core, the ratio between the reference MTC and its noise estimate varies between two and five, which corresponds to what was experimentally noticed previously.

The fact that the measure of the underestimation depends mainly on the correlation structure of temperature fluctuations and the measuring position agrees well with the experimental facts which show that for a given core position, the calibration factor is approximately constant during the cycle and even for several cycles. This is possible if the temperature correlations do not change with burnup or between several cycles, which sounds quite plausible.

The question remains how to utilize the findings of the present work for a more accurate evaluation of the traditional noise measurement. As was seen, the main problem is the use of a local temperature value without knowledge of the temperature correlations. If the correlations were known, the calibration factor could be calculated with a more accurate (two-group nodal) calculation of the Green's function and the flux and by evaluating the formulas (16) and (17) numerically. This can be performed without the need for a simplified form of the correlations as in (5). If the correlation function of the temperature fluctuations is mapped once, it can be used for a longer period and with arbitrary core positions to calculate a calibration function. This method then would work as long as the temperature correlation does not change significantly. In principle, this method bears resemblance with the present way of handling the situation where the calibration factor is determined empirically (comparing the measured value to the true one, obtained from traditional measurements or core calculation) (Analytis, 2000).

Another possibility arises if the temperature fluctuations can be measured in several points in the core during an MTC noise measurement. In that case the average temperature fluctuations, as defined by (29), can be estimated by approximating the integral with a finite sum. After that the slightly biased estimator (32) can be used. This estimator is only biased by the deviation of the reactor response from point-kinetic. As was seen this bias is quite small. In this method the knowledge of the temperature correlations is not necessary. The advantage is that the evaluation is much simpler. The disadvantage is that it does not yield a calibration factor that can be used in later measurements in which the temperature is only measured in single points.

There exist cores in which there are sufficient number of thermocouples available such that the second method be possible to use. One case is the Ringhals R2 reactor, in which there are 108 permanent gamma-thermometers (12 strings with nine gamma-thermometers in each) that can be used as ordinary thermoelements in the frequency range considered (Andersson, 1994; Demazière et al., 2000). With such a large number of temperature values, a good approximation of the average temperature fluctuation can be obtained. This method will be tested in the near future

within a joint research project between the Ringhals Power Plant and the Department of Reactor Physics at Chalmers.

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PAPER IV

PROCEEDINGS
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2-D 2-GROUP NEUTRON NOISE SIMULATOR
AND
ITS APPLICATION TO ANOMALY LOCALISATION

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Keywords: neutron noise, dynamic transfer function, diffusion approximation, finite difference scheme, benchmarking

ABSTRACT

This paper presents a so-called neutron noise simulator, essentially an algorithm to calculate the dynamic transfer function, and its use in a procedure allowing to locate a noise source from the neutron detector readings. The noise simulator relies on the two-group diffusion approximation in 2-D. Benchmarking of this calculator versus analytical solutions showed that the finite difference discretisation scheme used in the simulator was accurate in case of homogeneous cores and a central noise source. The localisation algorithm was found to give correct results as long as one single noise source exists in the core and when the transfer function from the removal cross-section noise to the thermal neutron noise was used. Applying this localisation procedure to the Forsmark-1 BWR (Sweden) when a local instability event occurred (cycle 16) pointed out, via the use of an appropriate set of detectors, a region close to where an unseated fuel element was discovered.

1. INTRODUCTION

It is well known that the neutron noise, i.e. the difference between the time-dependent neutron flux and its time-averaged value, assuming that all the processes are stationary and ergodic in time, allows determining many interesting features of a reactor. The neutron noise can be used either for diagnostic purposes, when an abnormal situation is suspected, or for estimating a dynamical core parameter, whereas the reactor is at steady-state conditions. Many examples can be found in the literature. For the former, it is now common practice to determine any flow blockage by estimating the flow velocity in the corresponding fuel channel. The cross-correlation between two neutron detectors can be used for that purpose. But the neutron noise can be utilized for other diagnostics tasks, such as the estimation of core barrel vibration, or as it will be presented in this paper, the localisation of an unseated fuel element. For the latter category, namely the determination of global dynamical core parameters, the Decay Ratio (DR) in Boiling Water Reactors (BWRs) and the Moderator Temperature Coefficient of reactivity (MTC) in Pressurised Water Reactors (PWRs) are probably the two most significant applications.

In order to unfold the noise sources from the measured neutron noise, the transfer function between the noise source and the induced noise should be calculated. Nevertheless, to our knowledge, there is no commercial code that is able to calculate the transfer function of the neutron noise. Even if time-dependent codes are used to estimate the Decay Ratio for instance, the neutron noise was not given enough attention so far. Compared to the static flux, in certain aspects the calculation of the neutron noise in the frequency domain is even simpler than the estimation of the static flux, as will be seen in the following. The main reason lies with the fact that the first-order neutron noise can be expressed as a source problem, not an eigenvalue problem as for the static flux. Due to the lack of such a code, a neutron noise simulator was developed at the Department of Reactor Physics, Chalmers University of Technology. In this paper, only the neutronic model is described for 2-D homogeneous/heterogeneous systems. But the final goal of this project is the development of a fully coupled thermal-hydraulic/neutronic core model in 3-D.

After a brief description of the neutronic model and the numerical scheme used to perform the calculation of the neutron noise, a benchmark of the numerical noise simulator versus analytical solutions is reported. For noise sources located at the centre of a homogeneous core, the agreement of both the amplitude and the phase of the flux noise was found to be excellent. Finally, the noise simulator was used to localise a noise source, assuming that only the detectors signals were available as input parameters. The localisation algorithm was first used in different test cases and then applied to a realistic core, namely the Forsmark-1 reactor. The detector readings were representative of the local instability event reported previously in (Karlsson, 1999). The noise source was successfully located in the neighbourhood of the unseated fuel element which was found during the core outage following the instability event and assumed to be responsible for the local oscillations.

2. NEUTRON NOISE SIMULATOR

The neutronic model of the noise simulator relies on the two-group diffusion approximation. All the calculations are performed in the frequency domain directly, which is equivalent to define complex cross-sections. The main advantage of using the frequency domain instead of the time domain is twofold. First, it is common practice to use the Fourier transform of the measured signals. Second, because of the Fourier transform, the time derivative in the equations is eliminated. There is consequently no need to properly choose a time discretisation which allows taking into account the phenomena one wants to study, and for which the neutron noise has to be evaluated at each time step. In the frequency domain instead, the calculation needs only to be performed once, assuming that the frequency of interest is known. If not, scanning a frequency range does not appear to be a big burden. Finally, the spatial discretisation is carried out by using a finite difference scheme.

2.1 Neutron noise in the 2-group diffusion approximation

In the two-group diffusion approximation, the time- and space-dependent flux can be expressed as the following:

$$\frac{1}{v_1}\frac{\partial \phi_1}{\partial t}(r, t) \tag{1}$$

$$= D_1(r, t)\nabla^2\phi_1(r, t) + v\Sigma_{f, 2}(r, t)(1 - \beta_{eff})\phi_2(r, t) + \lambda C(r, t)$$

$$+ [v\Sigma_{f, 1}(r, t)(1 - \beta_{eff}) - \Sigma_{a, 1}(r, t) - \Sigma_{rem}(r, t)]\phi_1(r, t)$$

and

$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t}(r, t) = D_2(r, t)\nabla^2\phi_2(r, t) - \Sigma_{a, 2}(r, t)\phi_2(r, t) + \Sigma_{rem}(r, t)\phi_1(r, t) \tag{2}$$

with the precursor density given as:

$$\frac{\partial C}{\partial t}(r, t) = \beta_{eff}[v\Sigma_{f, 1}(r, t)\phi_1(r, t) + v\Sigma_{f, 2}(r, t)\phi_2(r, t)] - \lambda C(r, t) \tag{3}$$

Assuming that all the time-dependent parameters can be expressed as:

$$X(r, t) = X_0(r) + \delta X(r, t) \tag{4}$$

where the index 0 represents the static case, substracting the static case to Eqs. (1)-(3), performing a temporal Fourier transform and neglecting the second-order terms lead to the following matrix formulation:

$$(D_{\Delta\delta\phi}(r)\nabla^2 + D_{\delta\phi}(r, \omega))\begin{bmatrix} \delta\phi_1(r, \omega) \\ \delta\phi_2(r, \omega) \end{bmatrix} \tag{5}$$

$$= D_{\delta D}(r)\begin{bmatrix} \delta D_1(r, \omega) \\ \delta D_2(r, \omega) \end{bmatrix} + D_{\delta\Sigma_{rem}}(r)\delta\Sigma_{rem}(r, \omega) + D_{\delta\Sigma_a}(r)\begin{bmatrix} \delta\Sigma_{a, 1}(r, \omega) \\ \delta\Sigma_{a, 2}(r, \omega) \end{bmatrix}$$

$$+ D_{\delta v\Sigma_f}(r, \omega)\begin{bmatrix} \delta v\Sigma_{f, 1}(r, \omega) \\ \delta v\Sigma_{f, 2}(r, \omega) \end{bmatrix}$$

where the different matrices are given as:

$$D_{\Delta\delta\phi}(r) = \begin{bmatrix} D_1(r) & 0 \\ 0 & D_2(r) \end{bmatrix} \tag{6}$$

$$D_{\delta\phi}(r, \omega) = \begin{bmatrix} -\Sigma_1(r, \omega) & v\Sigma_{f, 2}(r, \omega) \\ \Sigma_{rem, 0}(r) & -\Sigma_{a, 2}(r, \omega) \end{bmatrix} \tag{7}$$

$$D_{\delta D}(r) = \begin{bmatrix} -\nabla^2 \phi_{1,0}(r) & 0 \\ 0 & -\nabla^2 \phi_{2,0}(r) \end{bmatrix} \tag{8}$$

$$D_{\delta \Sigma_{rem}}(r) = \begin{bmatrix} \phi_{1,0}(r) \\ -\phi_{1,0}(r) \end{bmatrix} \tag{9}$$

$$D_{\delta \Sigma_a}(r) = \begin{bmatrix} \phi_{1,0}(r) & 0 \\ 0 & \phi_{2,0}(r) \end{bmatrix} \tag{10}$$

$$D_{\delta \nu \Sigma_f}(r, \omega) = \begin{bmatrix} -\phi_{1,0}(r)\left(1 - \dfrac{i\omega\beta_{eff}}{i\omega + \lambda}\right) & -\phi_{2,0}(r)\left(1 - \dfrac{i\omega\beta_{eff}}{i\omega + \lambda}\right) \\ 0 & 0 \end{bmatrix} \tag{11}$$

and the different coefficients are:

$$\Sigma_1(r, \omega) = \Sigma_{a,1,0}(r) + \frac{i\omega}{v_1} + \Sigma_{rem,0}(r) - \nu\Sigma_{f,1,0}(r)\left(1 - \frac{i\omega\beta_{eff}}{i\omega + \lambda}\right) \tag{12}$$

$$\nu\Sigma_{f,2}(r, \omega) = \nu\Sigma_{f,2,0}(r)\left(1 - \frac{i\omega\beta_{eff}}{i\omega + \lambda}\right) \tag{13}$$

$$\Sigma_{a,2}(r, \omega) = \Sigma_{a,2,0}(r) + \frac{i\omega}{v_2} \tag{14}$$

From Eq. (5), it is obvious that the right-hand-side represents the neutron noise source. Compared to the calculation of the static neutron flux (which is an eigenvalue problem), the estimation of the neutron noise is a simpler task since there is no need to iterate on the fission term. The only difficulty lies with the fact that Eq. (5) uses complex cross-sections, as the ones defined by Eqs. (12)-(14). These cross-sections are frequency-dependent. This means that a new set of calculation has to be performed for each frequency that one might consider.

2.2 2-D spatial discretisation

Due to the ∇^2 operator in Eq. (5), a spatial discretisation scheme has to be chosen. The finite difference scheme was retained for its simplicity and its efficiency. As will be shown in §3., this scheme is satisfactory for homogeneous systems. Nevertheless, it is known that the number of nodes needs to be increased significantly in heterogeneous systems, if one wants to obtain an acceptable level of accuracy. Therefore, other more

powerful discretisation schemes, such as nodal methods or finite elements, will be considered at a later stage.

The starting point of the discretisation procedure is the integration of Eq. (5) on an elementary volume. The unknowns are thus expressed by the following generic formulation:

$$\delta X_{I,J}(\omega) = \frac{1}{\Delta x \cdot \Delta y} \int\limits_{(I,J)} \delta X(r, \omega) dr \qquad (15)$$

whereas the elements of the matrices satisfy the following relationship:

$$m_{I,J}(\omega)\delta X_{I,J}(\omega) = \frac{1}{\Delta x \cdot \Delta y} \int\limits_{(I,J)} m(r, \omega)\delta X(r, \omega) dr \qquad (16)$$

$$\Leftrightarrow m_{I,J}(\omega) = \frac{\int\limits_{(I,J)} m(r, \omega)\delta X(r, \omega) dr}{\int\limits_{(I,J)} \delta X(r, \omega) dr}$$

This way of averaging is consistent with the two-group constants provided by any static core calculator, so that the actual reaction rates are preserved.

If one represents a node I,J by the system of axes and numbering as shown in Fig. 1, the spatial discretisation of the neutron noise can be carried out according to the "box-scheme" (Nakamura, 1977) that allows writing:

$$\frac{1}{\Delta x \cdot \Delta y} \int\limits_{(I,J)} D_{\Delta\delta\phi}\nabla^2\delta\phi(r, \omega) dr \qquad (17)$$

$$= -\frac{[\delta J_{I,J}^x(\omega) - \delta J_{I-1,J}^x(\omega)]}{\Delta x} - \frac{[\delta J_{I,J}^y(\omega) - \delta J_{I,J-1}^y(\omega)]}{\Delta y}$$

For each direction (either the x or y direction), two expressions for the current noise $\delta J(\omega)$ can be written by considering the node I,J and its neighbours (either the $I+1,J$ or $I,J+1$ node respectively). Equating these two expressions allows eliminating the flux noise at the boundary of the two nodes, so that the current noise can be expressed as directly depending on the node-average flux noise in the node I,J and the node-average flux noise in its neighbouring nodes. In a given energy-group g, one obtains:
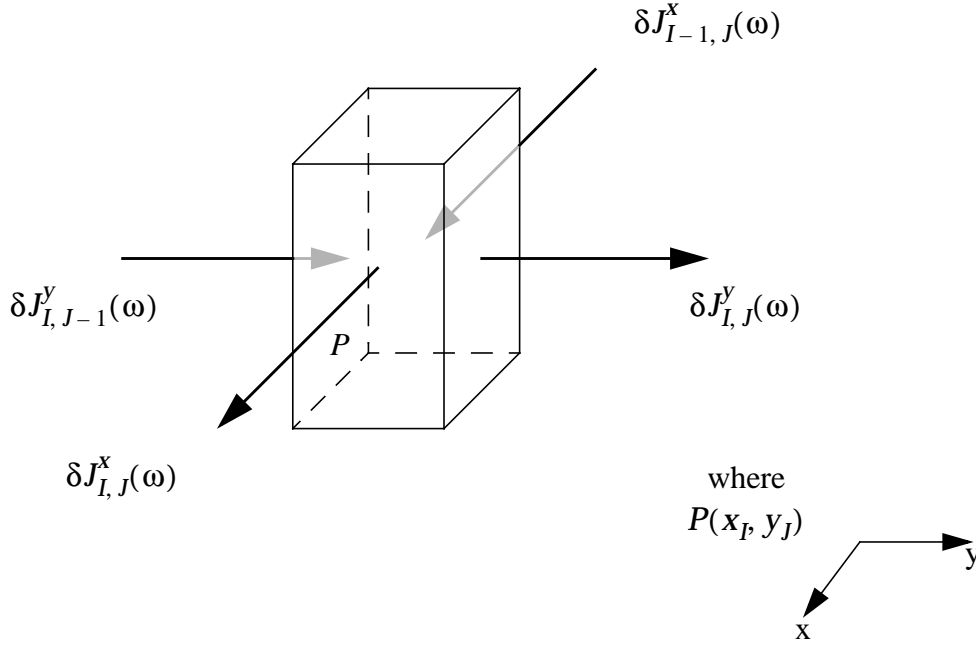
Fig. 1 Principles and convention used in the discretisation scheme (2-D case).

$$\delta J^x_{g, I, J}(\omega) - \delta J^x_{g, I-1, J}(\omega) \tag{18}$$
$$= a^x_{g, I, J}\delta\phi_{g, I, J}(\omega) + b^x_{g, I, J}\delta\phi_{g, I+1, J}(\omega) + c^x_{g, I, J}\delta\phi_{g, I-1, J}(\omega)$$

$$\delta J^y_{g, I, J}(\omega) - \delta J^y_{g, I, J-1}(\omega) \tag{19}$$
$$= a^y_{g, I, J}\delta\phi_{g, I, J}(\omega) + b^y_{g, I, J}\delta\phi_{g, I, J+1}(\omega) + c^y_{g, I, J}\delta\phi_{g, I, J-1}(\omega)$$

The different coefficients $a^x_{g, I, J}$, $a^y_{g, I, J}$, $b^x_{g, I, J}$, $b^y_{g, I, J}$, $c^x_{g, I, J}$, and $c^y_{g, I, J}$ are summarised in Table 1 and Table 2 for the x and y directions respectively.

Table 1 Coupling coefficients in the x direction.

	$a^x_{g, I, J}$	$b^x_{g, I, J}$	$c^x_{g, I, J}$
if the node I-1 does not exist	$\dfrac{2D_{g, I, J}D_{g, I+1, J}}{\Delta x(D_{g, I, J} + D_{g, I+1, J})}$ $+ \dfrac{2D_{g, I, J}}{\Delta x}$	$-\dfrac{2D_{g, I, J}D_{g, I+1, J}}{\Delta x(D_{g, I, J} + D_{g, I+1, J})}$	0
if the nodes I-1 and I+1 both exist	$\dfrac{2D_{g, I, J}D_{g, I+1, J}}{\Delta x(D_{g, I, J} + D_{g, I+1, J})}$ $+ \dfrac{2D_{g, I, J}D_{g, I-1, J}}{\Delta x(D_{g, I, J} + D_{g, I-1, J})}$	$-\dfrac{2D_{g, I, J}D_{g, I+1, J}}{\Delta x(D_{g, I, J} + D_{g, I+1, J})}$	$-\dfrac{2D_{g, I, J}D_{g, I-1, J}}{\Delta x(D_{g, I, J} + D_{g, I-1, J})}$

(6)

Table 1   Coupling coefficients in the *x* direction.

	$a^x_{g,I,J}$	$b^x_{g,I,J}$	$c^x_{g,I,J}$
if the node *I+1* does not exist	$\dfrac{2D_{g,I,J}}{\Delta x} + \dfrac{2D_{g,I,J}D_{g,I-1,J}}{\Delta x(D_{g,I,J}+D_{g,I-1,J})}$	0	$-\dfrac{2D_{g,I,J}D_{g,I-1,J}}{\Delta x(D_{g,I,J}+D_{g,I-1,J})}$

Table 2   Coupling coefficients in the *y* direction.

	$a^y_{g,I,J}$	$b^y_{g,I,J}$	$c^y_{g,I,J}$
if the node *J-1* does not exist	$\dfrac{2D_{g,I,J}D_{g,I,J+1}}{\Delta y(D_{g,I,J}+D_{g,I,J+1})} + \dfrac{2D_{g,I,J}}{\Delta y}$	$-\dfrac{2D_{g,I,J}D_{g,I,J+1}}{\Delta y(D_{g,I,J}+D_{g,I,J+1})}$	0
if the nodes *J-1* and *J+1* both exist	$\dfrac{2D_{g,I,J}D_{g,I,J+1}}{\Delta y(D_{g,I,J}+D_{g,I,J+1})} + \dfrac{2D_{g,I,J}D_{g,I,J-1}}{\Delta y(D_{g,I,J}+D_{g,I,J-1})}$	$-\dfrac{2D_{g,I,J}D_{g,I,J+1}}{\Delta y(D_{g,I,J}+D_{g,I,J+1})}$	$-\dfrac{2D_{g,I,J}D_{g,I,J-1}}{\Delta y(D_{g,I,J}+D_{g,I,J-1})}$
if the node *J+1* does not exist	$\dfrac{2D_{g,I,J}}{\Delta y} + \dfrac{2D_{g,I,J}D_{g,I,J-1}}{\Delta y(D_{g,I,J}+D_{g,I,J-1})}$	0	$-\dfrac{2D_{g,I,J}D_{g,I,J-1}}{\Delta y(D_{g,I,J}+D_{g,I,J-1})}$

By using Eqs. (15)-(19), the discretised system of equations that has to be solved can be derived from Eq. (5) as follows:

$$
\begin{aligned}
D^{discr}_{\delta\phi}(\omega)\delta\phi^{discr}(\omega) \\
= D^{discr}_{\delta D}\delta D^{discr}(\omega) + D^{discr}_{\delta\Sigma_{rem}}\delta\Sigma^{discr}_{rem}(\omega) + D^{discr}_{\delta\Sigma_{a}}\delta\Sigma^{discr}_{a}(\omega) \\
+ D^{discr}_{\delta\nu\Sigma_{f}}(\omega)\delta\nu\Sigma_{f}(\omega)
\end{aligned}
\tag{20}
$$

Whereas the expression of each term on the right-hand-side of Eq. (20) is relatively straightforward, the left-hand-side needs to be clarified a little bit further. If one considers a given node *I,J*, one has:

$$[D_{\delta\phi}^{discr}(\omega)\delta\phi^{discr}(\omega)]_{(I,J)} \tag{21}$$

$$= \begin{bmatrix} -\Sigma_{1,I,J}(\omega)-\dfrac{a_{1,I,J}^x}{\Delta x}-\dfrac{a_{1,I,J}^y}{\Delta y} & \nu\Sigma_{f,2,I,J}(\omega) \\[2ex] \Sigma_{rem,0,I,J} & -\Sigma_{a,2,I,J}(\omega)-\dfrac{a_{2,I,J}^x}{\Delta x}-\dfrac{a_{2,I,J}^y}{\Delta y} \end{bmatrix} \times \begin{bmatrix} \delta\phi_{1,I,J}(\omega) \\[2ex] \delta\phi_{2,I,J}(\omega) \end{bmatrix}$$

$$+ \begin{bmatrix} -\dfrac{b_{1,I,J}^x}{\Delta x} & 0 \\[2ex] 0 & -\dfrac{b_{2,I,J}^x}{\Delta x} \end{bmatrix} \times \begin{bmatrix} \delta\phi_{1,I+1,J}(\omega) \\[2ex] \delta\phi_{2,I+1,J}(\omega) \end{bmatrix} + \begin{bmatrix} -\dfrac{b_{1,I,J}^y}{\Delta x} & 0 \\[2ex] 0 & -\dfrac{b_{2,I,J}^y}{\Delta x} \end{bmatrix} \times \begin{bmatrix} \delta\phi_{1,I,J+1}(\omega) \\[2ex] \delta\phi_{2,I,J+1}(\omega) \end{bmatrix}$$

$$+ \begin{bmatrix} -\dfrac{c_{1,I,J}^x}{\Delta x} & 0 \\[2ex] 0 & -\dfrac{c_{2,I,J}^x}{\Delta x} \end{bmatrix} \times \begin{bmatrix} \delta\phi_{1,I-1,J}(\omega) \\[2ex] \delta\phi_{2,I-1,J}(\omega) \end{bmatrix} + \begin{bmatrix} -\dfrac{c_{1,I,J}^y}{\Delta x} & 0 \\[2ex] 0 & -\dfrac{c_{2,I,J}^y}{\Delta x} \end{bmatrix} \times \begin{bmatrix} \delta\phi_{1,I,J-1}(\omega) \\[2ex] \delta\phi_{2,I,J-1}(\omega) \end{bmatrix}$$

The $D_{\delta\phi}^{discr}(\omega)$ matrix is obviously sparse. If one has a 2-D core with N nodes, one has $2N$ unknowns and the matrix is of a $2N$x$2N$ size. This matrix can be inverted so that the flux noise can be directly expressed as (source problem):

$$\delta\phi^{discr}(\omega) \tag{22}$$
$$= [D_{\delta\phi}^{discr}(\omega)]^{-1} D_{\delta D}^{discr}\delta D^{discr}(\omega) + [D_{\delta\phi}^{discr}(\omega)]^{-1} D_{\delta\Sigma_{rem}}^{discr}\delta\Sigma_{rem}^{discr}(\omega)$$
$$+ [D_{\delta\phi}^{discr}(\omega)]^{-1} D_{\delta\Sigma_a}^{discr}\delta\Sigma_a^{discr}(\omega) + [D_{\delta\phi}^{discr}(\omega)]^{-1} D_{\delta\nu\Sigma_f}^{discr}(\omega)\delta\nu\Sigma_f(\omega)$$

2.3 Data required

The only data required by the noise simulator are the static data, i.e. the material constants and the point-kinetic parameters of the core. These data can be easily retrieved from any static core simulator. Nevertheless, there is one particular aspect that is worth mentioning and that could lead to differences in the calculation of the flux noise. Namely, the spatial discretisation scheme used to generate the 2-D material constants needs to be in agreement with the one used in the noise simulator.

More specifically, the first necessary step in the estimation of the noise should be the calculation of the static flux and the eigenvalue with the finite difference scheme, i.e. a scheme which is identical with the one used in the noise determination. It is not granted

that the flux and eigenvalue given by the finite difference scheme will correspond to the one given by the static core simulator. In such a case, using the static data directly from the static core simulator would be equivalent to make the system non-critical.

Nevertheless, recalculating the static flux and the corresponding eigenvalue with a finite difference scheme is identical to neglect the main advantage of any commercial core simulator, i.e. its accuracy. This is why another approach was preferred in this study. This approach is simpler since no calculation of the static flux and eigenvalue is required. The static flux is in fact directly used to adjust the static cross-sections so that the balance equations are fulfilled in each node with the finite difference scheme. This is completely equivalent to make the system critical with the most accurate set of fluxes available and with a scheme compatible to the one used in the noise estimation.

The balance equations that need to be fulfilled are given as:

$$[D_1(r)\Delta\phi_1(r)]_{(I, J)} + \frac{\nu\Sigma_{f, 1, I, J}}{k_{eff}}\phi_{1, I, J} + \frac{\nu\Sigma_{f, 2(1), I, J}}{k_{eff}}\phi_{2, I, J} \qquad (23)$$

$$-\Sigma_{a, 1, I, J}\phi_{1, I, J} - \Sigma_{rem, I, J}\phi_{1, I, J} = 0$$

$$[D_2(r)\Delta\phi_2(r)]_{(I, J)} + \Sigma_{rem, I, J}\phi_{1, I, J} - \Sigma_{a, 2, I, J}\phi_{2, I, J} = 0 \qquad (24)$$

with the leakage terms estimated according to the finite difference scheme:

$$[D_1(r)\Delta\phi_1(r)]_{(I, J)} = \left(\frac{a^x_{1, I, J}}{\Delta x} + \frac{a^y_{1, I, J}}{\Delta y}\right)\phi_{1, I, J} \qquad (25)$$

$$+ \frac{b^x_{1, I, J}}{\Delta x}\phi_{1, I+1, J} + \frac{b^y_{1, I, J}}{\Delta y}\phi_{1, I, J+1} + \frac{c^x_{1, I, J}}{\Delta x}\phi_{1, I-1, J} + \frac{c^y_{1, I, J}}{\Delta y}\phi_{1, I, J-1}$$

$$[D_2(r)\Delta\phi_2(r)]_{(I, J)} = \left(\frac{a^x_{2, I, J}}{\Delta x} + \frac{a^y_{2, I, J}}{\Delta y}\right)\phi_{2, I, J} \qquad (26)$$

$$+ \frac{b^x_{2, I, J}}{\Delta x}\phi_{2, I+1, J} + \frac{b^y_{2, I, J}}{\Delta y}\phi_{2, I, J+1} + \frac{c^x_{2, I, J}}{\Delta x}\phi_{2, I-1, J} + \frac{c^y_{2, I, J}}{\Delta y}\phi_{2, I, J-1}$$

The following procedure has been applied for the adjustment of the cross-sections used in this study. First, the thermal absorption cross-section was modified to fulfil Eqs. (24) and (26). If such an adjustment was not possible (negative cross-section), the removal cross-section was modified instead, and the eigenvalue modified so that Eqs. (23) and (25) could be fulfilled. If this too was impossible, the fast absorption cross-section could also be modified. For the reflector nodes, an adjustment of the absorption cross-sections (both fast and thermal) is first carried out. In case of negative results, the removal cross-section is modified. But due to the coupling between the fast and thermal groups and the relatively

few number of parameters that can be changed, an iterative procedure is required if the removal cross-section is adjusted in the reflector nodes. As a matter of fact, this procedure only affects appreciably the cross-sections in the reflector nodes, and to a lesser extent the cross-sections of the fuel nodes immediately neighbouring the reflector. The main reason lies with the fact that a finite difference scheme does not estimate the static flux accurately in these nodes when only a few nodes are used for the calculation.

3. BENCHMARKING OF THE SIMULATOR

Even if the cross-sections need to be adjusted before using the noise simulator, the modifications of these are almost negligible in the fuel nodes. As pointed out previously, only the fuel nodes directly neighbouring the reflector nodes are appreciably modified. Therefore locating a noise source in the middle of the core and assuming that the core is homogeneous for the estimation of an analytical solution should provide a relatively good reference solution for the numerical scheme far away from the reflector nodes.

3.1 Analytical solution in case of a central noise source

It is assumed in the following that the noise source for the analytical solution is a point source located at the core centre. Three different cases have been considered: a noise source defined in terms of the fluctuation of the fast absorption cross-section, one of the thermal absorption cross-section, and finally one of the removal cross-section. This can be formulated by writing Eq. (5) as follows:

$$(D_{\Delta\delta\phi}(r)\nabla^2 + D_{\delta\phi}(r, \omega)) \begin{bmatrix} \delta\phi_1(r, \omega) \\ \delta\phi_2(r, \omega) \end{bmatrix} = \begin{bmatrix} S_1(r, \omega) \\ S_2(r, \omega) \end{bmatrix} \tag{27}$$

with the following possibilities for the noise source:

$$\begin{bmatrix} S_1(r, \omega) \\ S_2(r, \omega) \end{bmatrix} = \gamma(\omega) \times \begin{bmatrix} \delta(r)\phi_{1, 0}(r) \\ 0 \end{bmatrix} \tag{28}$$

or

$$\begin{bmatrix} S_1(r, \omega) \\ S_2(r, \omega) \end{bmatrix} = \gamma(\omega) \times \begin{bmatrix} 0 \\ \delta(r)\phi_{2, 0}(r) \end{bmatrix} \tag{29}$$

or

$$\begin{bmatrix} S_1(r, \omega) \\ S_2(r, \omega) \end{bmatrix} = \gamma(\omega) \times \begin{bmatrix} \delta(r)\phi_{1, 0}(r) \\ -\delta(r)\phi_{1, 0}(r) \end{bmatrix} \tag{30}$$

(10)

$\gamma(\omega)$ is the noise source strength. This coefficient allows also taking into account the fact that the noise source is homogeneously distributed over one or several nodes in the numerical solution (the nodes representing the core centre), and therefore is not a point-source.

Due to the symmetry of the system, the flux noise is simply given by:

$$\delta\phi_1(r, \omega) = A \times K_0(\lambda r) + B \times I_0(\lambda r) + C \times Y_0(\mu r) + D \times J_0(\mu r) \tag{31}$$

$$\delta\phi_2(r, \omega) \tag{32}$$
$$= A \times S_\lambda \times K_0(\lambda r) + B \times S_\lambda \times I_0(\lambda r) + C \times S_\mu \times Y_0(\mu r) + D \times S_\mu \times J_0(\mu r)$$

where $-\lambda^2$ and μ^2 are the two eigenvalues of the following matrix:

$$\begin{bmatrix} -\Sigma_1(\omega)/D_1 & \nu\Sigma_{f,2}(\omega)/D_1 \\ \Sigma_{rem,0}/D_2 & -\Sigma_{a,2}(\omega)/D_2 \end{bmatrix} \tag{33}$$

and the coupling coefficient S_λ and S_μ are given as follows:

$$S_\lambda = \frac{\Sigma_{rem,0}}{\Sigma_{a,2}(\omega) - D_2\lambda^2} \tag{34}$$

$$S_\mu = \frac{\Sigma_{rem,0}}{\Sigma_{a,2}(\omega) + D_2\mu^2} \tag{35}$$

The coefficients A, B, C, and D are solutions of the system:

$$\begin{bmatrix} K_0(\lambda R) & I_0(\lambda R) & Y_0(\mu R) & J_0(\mu R) \\ S_\lambda \times K_0(\lambda R) & S_\lambda \times I_0(\lambda R) & S_\mu \times Y_0(\mu R) & S_\mu \times J_0(\mu R) \\ -1 & 0 & -[\mu r \times Y_1(\mu r)]_{r \to 0} & 0 \\ -S_\lambda & 0 & -S_\mu \times [\mu r \times Y_1(\mu r)]_{r \to 0} & 0 \end{bmatrix} \times \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \tag{36}$$

$$= \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{2\pi D_1}\displaystyle\int S_1(r, \omega)\,dr \\ \dfrac{1}{2\pi D_2}\displaystyle\int S_2(r, \omega)\,dr \end{bmatrix}$$

(11)

where R is the core radius (fuel + reflector zones).

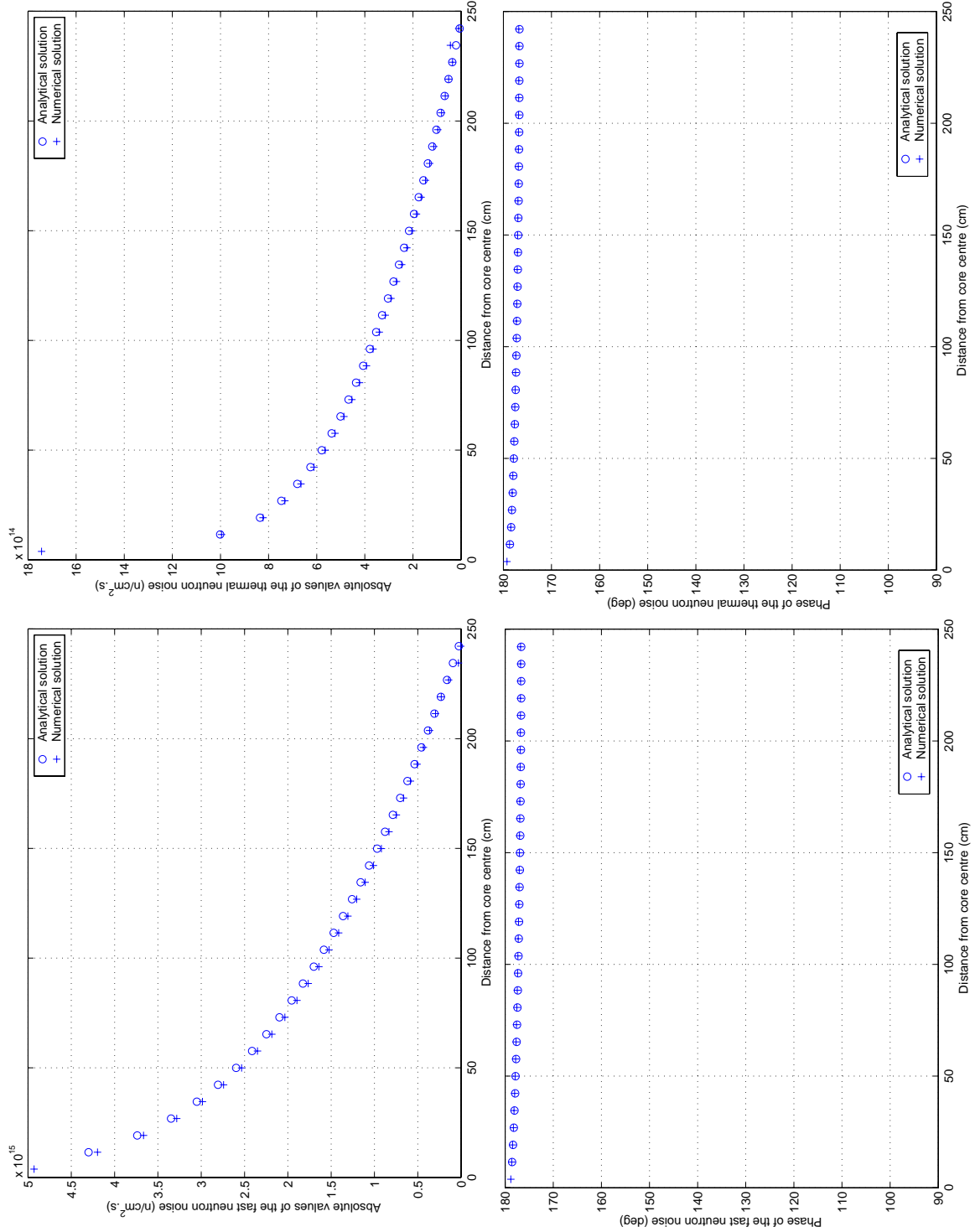## 3.2 Comparison between the numerical and analytical solutions

The following figures (see Figs. 2-4) depict the amplitude of the flux noise and its phase for both the analytical solution and the numerical one. Since the noise simulator calculates a spatially-averaged flux noise over each node, the analytical solution was also averaged on each node, so that both solutions could be directly compared. The first point of the numerical solution (from the core centre) represents the flux noise in the node where the noise source is located. The analytical solution gives obviously a different solution in this node, and therefore the first point of the analytical solution was systematically disregarded.

It can be noticed that the agreement between the analytical and the numerical solutions is excellent. As expected, the discrepancy is a little bit larger at the core boundary than at the core centre due to two main reasons. The first one is simply the presence of the reflector in the numerical simulation, whereas the analytical one does not take any reflector into account. The second effect lies with the fact that close to the reflector nodes, the cross-sections have been adjusted in the noise simulator, so that the system remains critical despite the use of the finite difference scheme. Therefore, while in the analytical case the reactor is homogeneous, the core becomes more and more heterogeneous close to the core boundary in the numerical case. Since the flux noise vanishes at the core boundary, the difference between the analytical and numerical solutions is hardly noticeable for the amplitude of the noise. Even if the discrepancy regarding the phase of the flux noise slightly increases away from the core centre, the accuracy remains very good, as can be seen on the different Figures.

Consequently, the noise simulator seems to reproduce the expected solution rather well. Despite the apparent high level of accuracy, the fact that the core is homogeneous (or more exactly almost homogeneous) has to be strongly emphasized. The finite difference scheme is a very effective (and easy to implement) discretisation scheme as long as the discretised system does not present a strong level of heterogeneity. A realistic core is of course far from being homogeneous. Even if core homogeneity is still an acceptable approximation for PWRs, BWRs are highly heterogeneous systems due to the presence of the control rods (in a PWR, the reactivity adjustment is mainly carried out by the boron concentration). Therefore, the accuracy may deteriorate appreciably when a realistic core is modelled. One way of coping with this could be to increase the number of nodes in the numerical simulation. Nevertheless, a commercial BWR like Forsmark-1 in Sweden has already 800 nodes in the radial direction (676 fuel assemblies + 124 reflector nodes). As pointed out previously, the corresponding matrix representing the transfer function is of a 1600x1600 size. Dividing each node into 4 sub-nodes is still possible, but already appears to be the maximum number of nodes that a 32-bit code could permit. Consequently, an acceptable level of accuracy could only be achieved if a more efficient discretisation scheme than the finite difference one is used. Nodal methods or finite elements are planned to be considered in the near future at our Department.
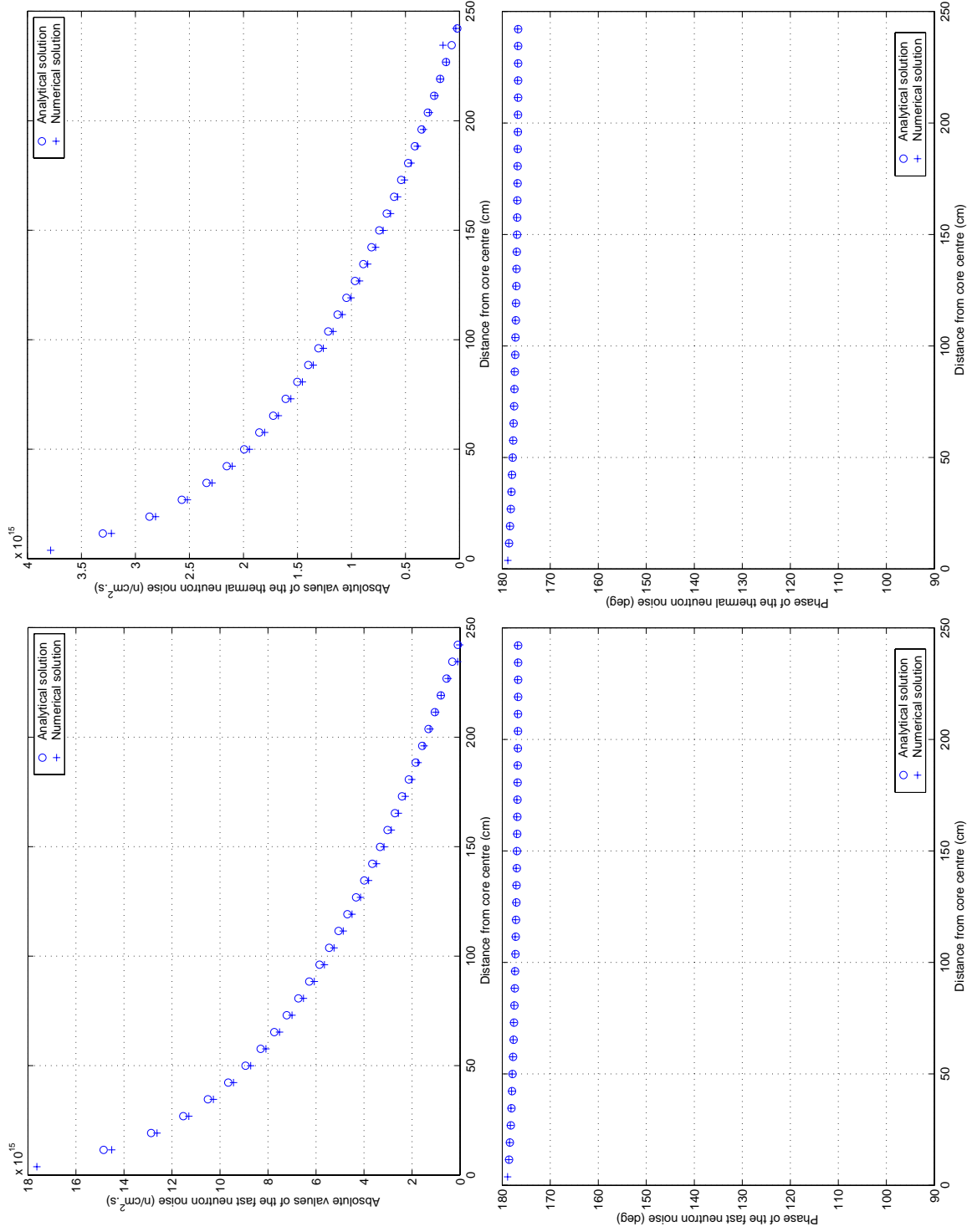
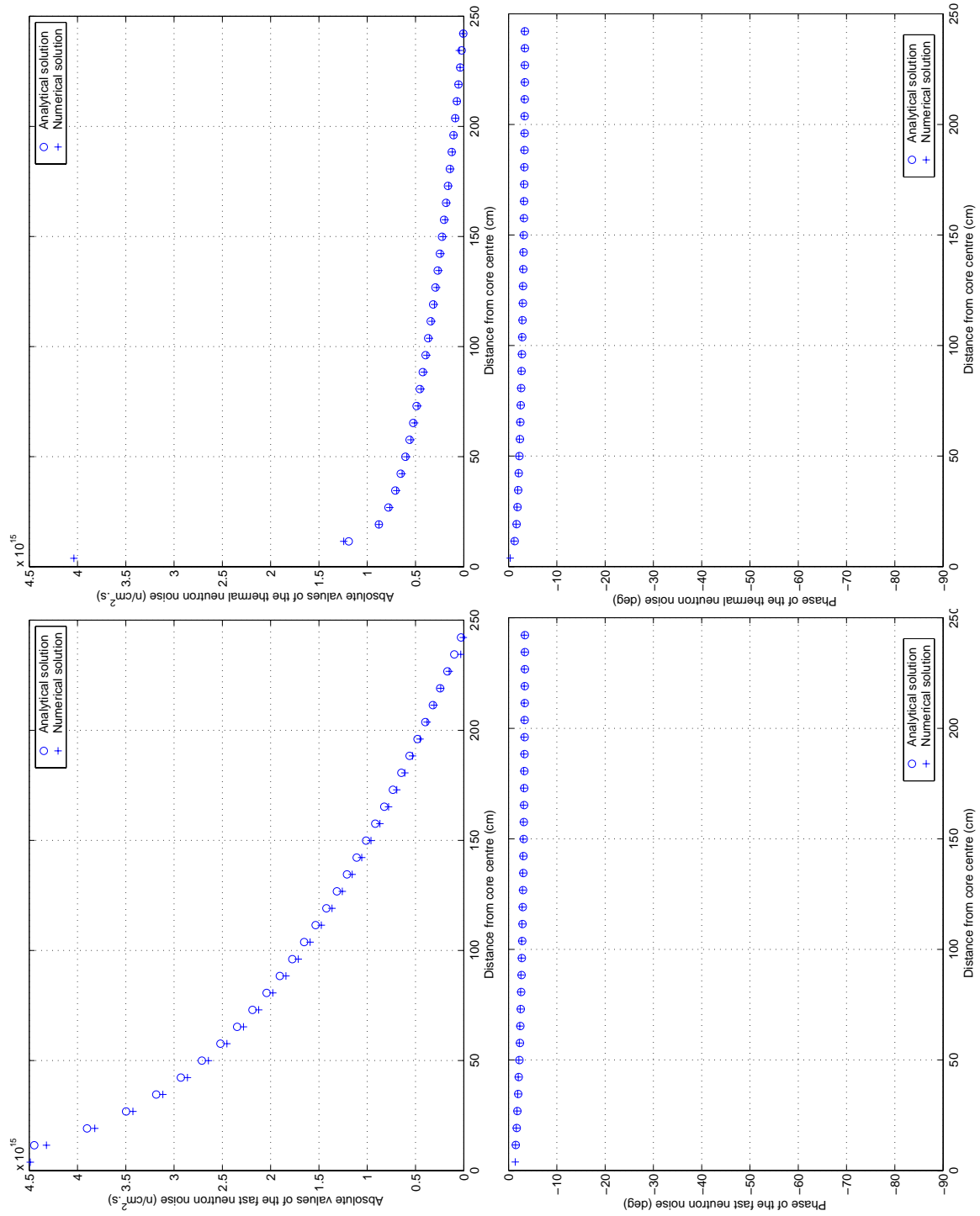**Fig. 2** Comparison between the analytical and numerical solutions, for the case of a fast absorption cross-section noise source.

(13)

Fig. 3 Comparison between the analytical and numerical solutions, for the case of a thermal absorption cross-section noise source.

Fig. 4 Comparison between the analytical and numerical solutions, for the case of a removal cross-section noise source.

(15)

4. APPLICATION TO THE LOCALISATION OF A NOISE SOURCE

In the preceding case, the flux noise was calculated assuming that the noise source was known (both its strength and its location). Even if being able to estimate the flux noise in a reactor is undoubtedly interesting, determining the location of an unknown noise source from the neutron detector readings is even more challenging. Such an inverting capability could be directly used for diagnostic purposes. In the following, a localisation algorithm allowing to locate a noise source (not its strength) is presented. Several test cases are also presented, so that the validity of the algorithm can be assessed. Finally, the localisation procedure is used in a practical case, namely the Forsmark-1 local instability event.

4.1 Localisation algorithm

The localisation algorithm is the one developed previously by Karlsson and Pázsit (1999). Therefore, only the basic principles of this procedure are recalled in the following. More details can be found in the original paper.

If one assumes that there is only one noise source located in the node (I_0, J_0), the flux noise can be calculated from Eq. (22). This can be written in a condensed form as follows:

$$\begin{bmatrix} \delta\phi_{1, I, J}(\omega) \\ \delta\phi_{2, I, J}(\omega) \end{bmatrix} = G(I_0, J_0 \to I, J;\omega) \times \begin{bmatrix} S_{1, I_0, J_0}(\omega) \\ S_{2, I_0, J_0}(\omega) \end{bmatrix} \tag{37}$$

$G(I_0, J_0 \to I, J;\omega)$ is in fact the discretised two-group Green's function of the system, which is evaluated by the noise simulator. Conceptually, $G(I_0, J_0 \to I, J;\omega)$ is a 2x2 matrix and the multiplication in Eq. (37) is a matrix multiplication. Since neutron detectors are most often sensitive to the thermal flux, one can write also:

$$\delta\phi_{2, I, J}(\omega) = G_2(I_0, J_0 \to I, J;\omega) \times \begin{bmatrix} S_{1, I_0, J_0}(\omega) \\ S_{2, I_0, J_0}(\omega) \end{bmatrix} \tag{38}$$

or more simply:

$$\delta\phi_{2, I, J}(\omega) = G_2(I_0, J_0 \to I, J;\omega) \times S_{I_0, J_0}(\omega) \tag{39}$$

where $G_2(I_0, J_0 \to I, J;\omega)$ is the second row of the $G(I_0, J_0 \to I, J;\omega)$ matrix and correspondingly the multiplication in Eqs. (38) and (39) is a scalar product.

Usually the noise source $S_{I_0, J_0}(\omega)$ can be factorised into a part depending only on the noise source location s_{I_0, J_0} and a part independent of the location (source strength $\gamma(\omega)$):

$$S_{I_0, J_0}(\omega) = \gamma(\omega) \times s_{I_0, J_0} \tag{40}$$

Estimating the ratio between the flux noise measured at two different locations A and B allows eliminating the noise source strength:

$$\frac{\delta\phi_{2, I_A, J_A}(\omega)}{\delta\phi_{2, I_B, J_B}(\omega)} = \frac{G_2(I_0, J_0 \to I_A, J_A;\omega)}{G_2(I_0, J_0 \to I_B, J_B;\omega)} \tag{41}$$

The left-hand-side of Eq. (41) can be obtained from measurements. The right-hand-side contains the unknown of the problem, namely the location of the noise source. When there is equality between the left-hand-side and the right-hand-side, the noise source has been correctly located. The localisation algorithm will calculate the right-hand-side of Eq. (41) for all possible locations of a single noise source within the core and will retain the one giving the ratio of the detector signals, i.e. the left-hand-side of Eq. (41). If one has access to several detectors, the following quantity can be evaluated for each detector combination (A, B):

$$\Delta_{A, B}(I, J) = \frac{\delta\phi_{2, I_A, J_A}(\omega)}{\delta\phi_{2, I_B, J_B}(\omega)} - \frac{G_2(I, J \to I_A, J_A;\omega)}{G_2(I, J \to I_B, J_B;\omega)} \tag{42}$$

so that the minimum of the following function should correspond to the location of the noise source (I_0, J_0):

$$\Delta(I, J) = \sum_{A, B} \Delta_{A, B}^2(I, J) \tag{43}$$

Since it is common practice to use the Auto- and Cross-Power Spectral Densities (APSDs and CPSDs respectively) of the measured signals instead of their Fourier transform, Eqs. (42)-(43) have to be written as follows:

$$\Delta_{A, B, C, D}(I, J) \tag{44}$$

$$= \frac{CPSD(A, B, \omega)}{CPSD(C, D, \omega)} - \frac{G_2(I, J \to I_A, J_A;\omega) \times G_2^*(I, J \to I_B, J_B;\omega)}{G_2(I, J \to I_C, J_C;\omega) \times G_2^*(I, J \to I_D, J_D;\omega)}$$

and

$$\Delta(I, J) = \sum_{A, B, C, D} \Delta_{A, B, C, D}^2(I, J) \tag{45}$$

Despite the apparent high number of possible detector combinations, the number of detectors quadruplets that need to be taken into account can be significantly reduced if the redundant combinations are discarded. For the sake of brevity, these simplifications are not presented here. We refer to the original paper instead (Karlsson, 1999).

4.2 Sensitivity of the algorithm

The localisation algorithm described previously can be easily tested since the noise simulator allows generating the flux noise for a given noise source. More precisely, one will assume a given location of a noise source within the core, and calculate the corresponding flux noise. The flux noise will be used as detector signals and the localisation algorithm, if successful, should return the location of the noise source. The sensitivity of the localisation algorithm with different parameters can therefore be assessed. These parameters are the number of detector signals used, the position of the noise source, the possibility of having several noise sources, the contamination of the detector signal by external noise, and finally the transfer function used for the localisation.

In the following, the results will be presented on two types of Figures, one depicting the $\Delta(I, J)$ function in a 3-D plot, and another one depicting also the $\Delta(I, J)$ function but in a 2-D plot (core map). In this latter case, the detectors are also positioned via crosses ('X'). The white ones indicate the detectors used in the localisation, whereas the black ones the detectors not used. The noise source is marked by a white asterisk ('*'), and the result of the localisation algorithm is denoted by a white circle ('O'). The core layout and the location of the detectors correspond to the Forsmark-1 BWR (Sweden).

The first Figure (Fig. 5) represents the effect of using a reduced number of detectors. The peaks in the $\Delta(I, J)$ function correspond to the detector locations. At these spots, the accuracy is better than away from the detectors. Therefore, since the noise source is not located at any of the detector position, a local maximum of the $\Delta(I, J)$ function is expected. As pointed out previously, although eliminating the redundant detectors combinations allows reducing significantly the calculation time, taking all the detectors into account still requires too much CPU effort. Furthermore, in most cases only a few number of detector signals are actually available from measurement campaigns. Therefore, the localisation algorithm was tested in two cases: first assuming that all the detectors were available, second by using only the four detectors surrounding the noise source. As can be seen on Fig. 5, the noise source is correctly located when using a reduced set of detectors. Even if using as many detectors as possible is encouraged, in the following test cases only the detectors neighbouring the noise source are used. This does not affect the success of the localisation algorithm.

The right-hand-side of Fig. 5, together with Fig. 6, allows also noticing that the precision of the localisation algorithm is perfectly insensitive to the location of the noise source. A noise source located close to the core boundary is as successfully detected as a central one.

Although the localisation algorithm has been designed for locating one single noise source, it is very unlikely that only one noise source is present in the core when actual measured signals are used. Nevertheless, as can be seen on Fig. 7, choosing a set of detectors positioned close to one of the noise sources allows detecting successfully the corresponding one, as long as the two noise sources are not close enough.

(18)
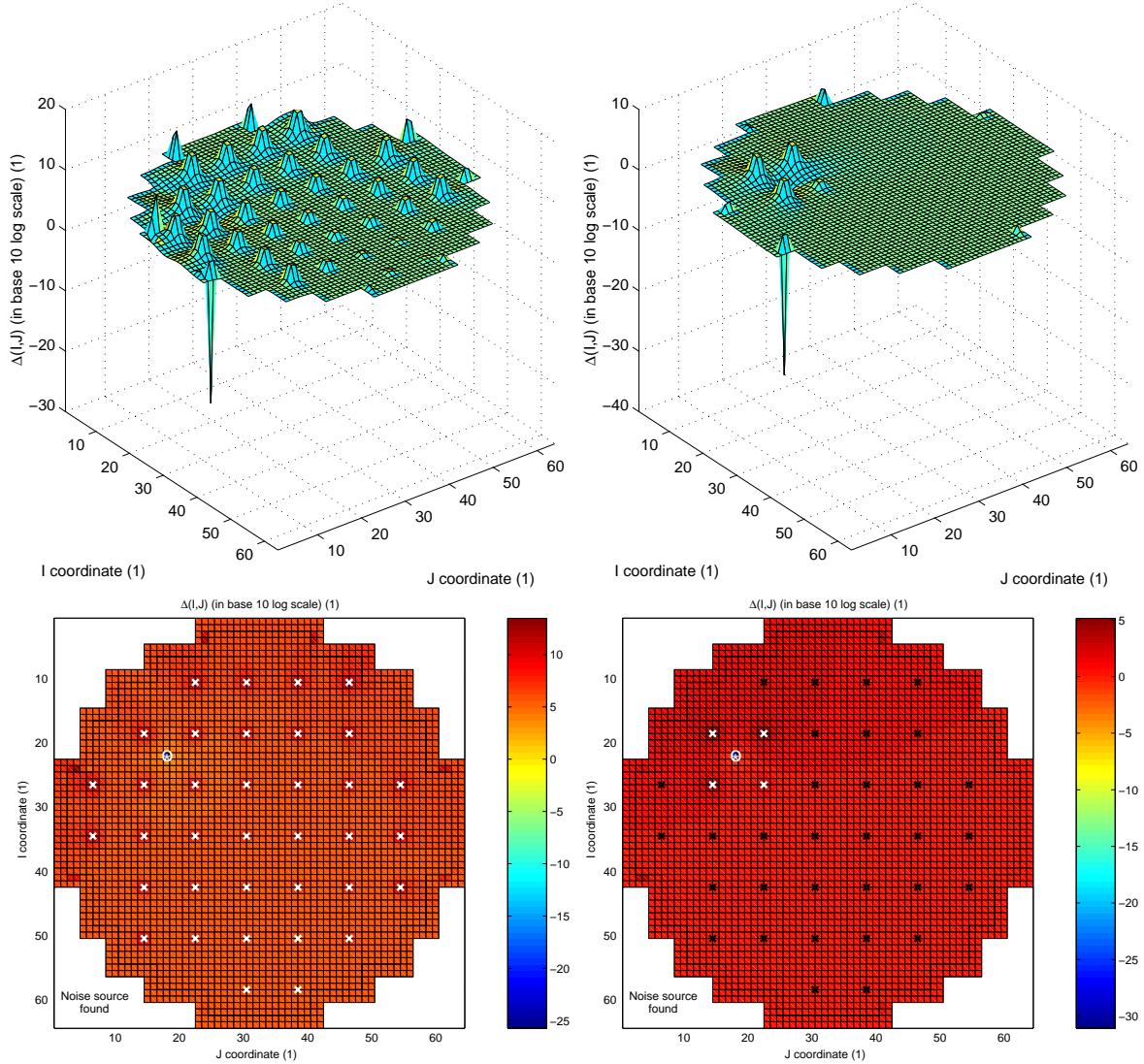
Fig. 5 Result of the localisation algorithm when all the detectors are used (left-hand-side) and when only the detectors surrounding the noise source are used (right-hand-side).

The algorithm needs also to be tested when extraneous random noise is added to the detector signals before performing the noise source localisation. As can be seen on Fig. 8, the noise source is still correctly located even with as much as 10% of extraneous noise. One could notice nevertheless that the dip in the $\Delta(I, J)$ function is less accentuated with noise than without noise (see for instance Fig. 5). The main reason that could explain why the noise source is still correctly located with a relatively high level of background noise is that in all previous estimations, the noise source was assumed to be given as a perturbation of the removal cross-section. As can be seen on Figs. 2-4, the thermal flux noise decreases much more rapidly away from the source for a removal cross-section noise source than for a fast or thermal absorption cross-section noise source. Therefore, using the transfer function between the removal cross-section noise and the thermal flux noise in the localisation algorithm is expected to provide a more pronounced minimum in
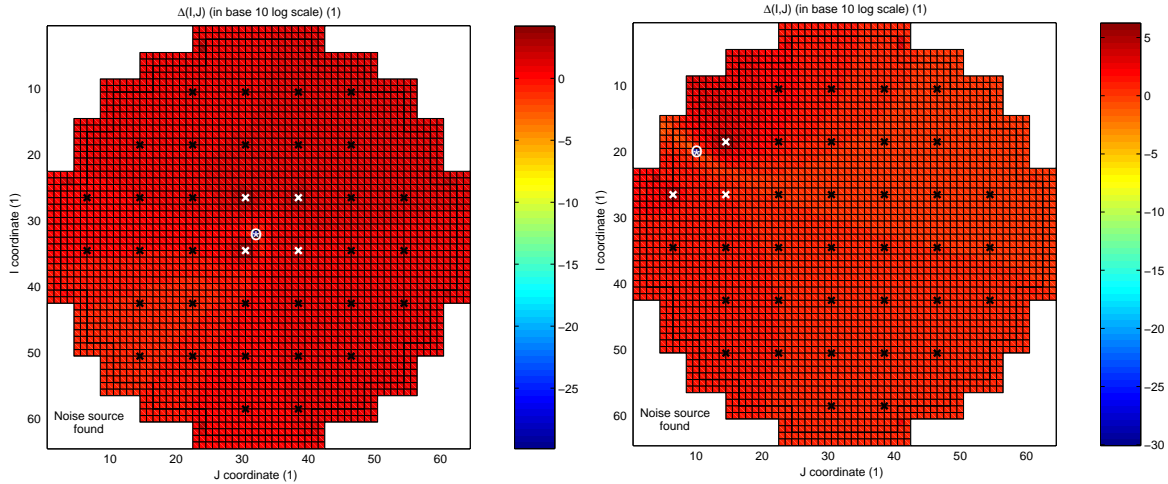
Fig. 6   Result of the localisation algorithm for a central noise source (left-hand-side) and a peripheral noise source (right-hand-side).
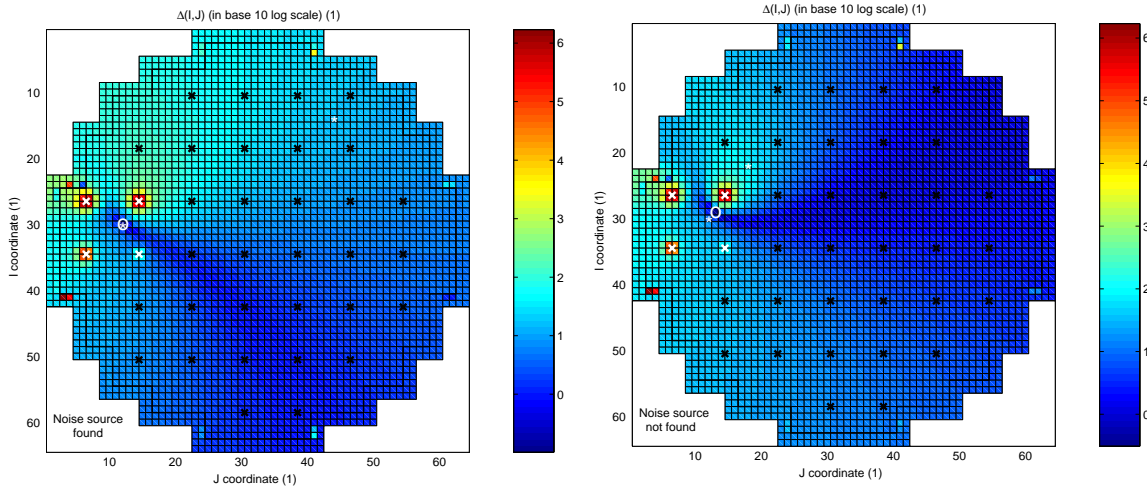
Fig. 7   Result of the localisation algorithm for two noise sources located far away from each other (left-hand-side) or close to each other (right-hand-side).

the $\Delta(I, J)$ function at the location of the noise source. The use of this transfer function is consequently less sensitive to the background noise. For the sake of brevity, the 3-D plot of the $\Delta(I, J)$ function is not depicted if one assumes that the noise source is defined in terms of the fast or thermal absorption cross-section noise, but one would have noticed that the dips corresponding to the actual noise source location are less obvious in these cases (this is particularly true for the thermal absorption cross-section case).

4.3 The Forsmark-1 local instability event

In 1996, during the start-up tests of the Forsmark-1 BWR (Sweden) for the fuel cycle 16, local instabilities were detected at reduced power and reduced core-flow. Although BWRs are known to become less stable at reduced power/core flow, the appearance of this instability event could not be understood and was not predicted by the
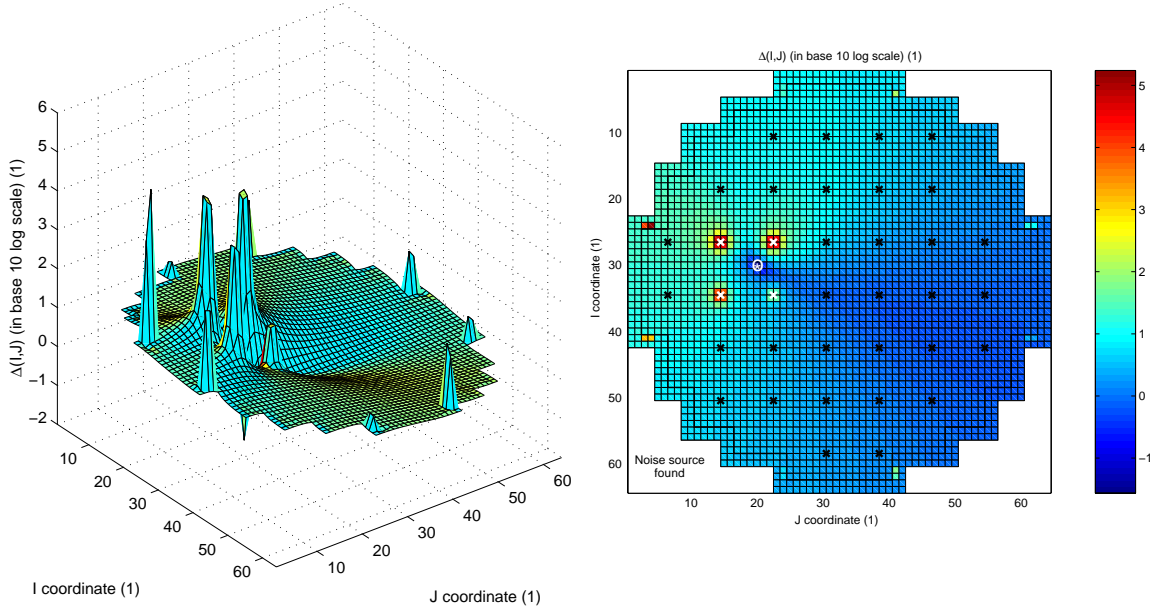
Fig. 8   Result of the localisation algorithm when the detector signals are contaminated by 10% of extraneous noise.

stability calculations. The corresponding operating point in the power/flow map was therefore avoided. In January 1997, at approximately Middle Of Cycle conditions (MOC), stability measurements were carried out in order to study the local instability discovered previously. The core was thus brought to 63.3% of power and to a core flow of 4298kg/s. Again local instability conditions were encountered, at a frequency of roughly 0.5Hz. An examination of all the LPRM signals available during this measurement campaign clearly shows a peak in the APSD of the LPRMs at this given frequency.

During this stability measurement, the lower plane of the core was rather well equipped with LPRMs (27 of the 36 available detector strings were actually recorded). A closer look at the phase of the measured flux noise indicated that the neutron noise was driven by a local noise source, similar to the effect of an absorber of variable strength (reactor oscillator), rather than a moving absorber, such as a vibrating control rod. The localisation algorithm presented previously allows locating a noise source of variable strength. By using the detectors in the lower plane, the 2-D representation of the core is expected to give the possibility of locating the noise source.

Such an attempt is presented in Fig. 9, where the transfer function between the removal cross-section noise and the thermal flux noise was used. Although the core layout and the detector locations are representative of the Forsmark-1 reactor, the material constants and point-kinetic parameters required by the noise simulator correspond to a generic model of a General Electric BWR/6 reactor (equilibrium core at End Of Cycle - EOC-), model which was developed previously at our Department (Demazière, 2000). It is planned to repeat the localisation procedure with a realistic set of data corresponding to the Forsmark-1 reactor in the future (and possibly with a more sophisticated discretisation scheme than the finite difference one). In the model used in this study, the fuel nodes were

(21)

spatially averaged and so were the reflector nodes. Although a two-region reactor was thus obtained, the cross-sections were adjusted in each node in order to have a critical system, as discussed previously.
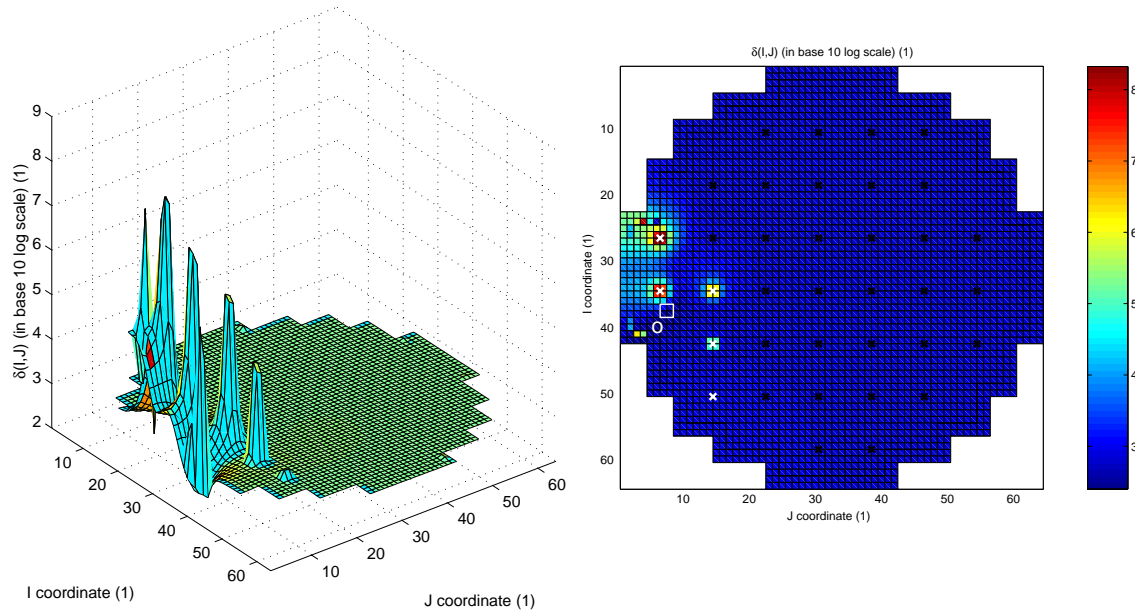
Fig. 9 Result of the localisation algorithm in the Forsmark-1 case (local instability event); the unseated fuel element is marked with a square, and the noise source identified by the localisation algorithm with a circle.

As can be seen on Fig. 9, using a suitable set of detectors, the detectors surrounding the region where a noise source is likely to be present (Karlsson, 1999), the localisation algorithm gives a global minimum located in the reflector nodes. A closer examination of the $\Delta(I, J)$ function also shows a local minimum, located in the fuel nodes. During the core outage following this instability event, a fuel assembly was found to be unseated close to the location pointed out by the localisation algorithm (Engström, 1997 and Söderlund, 1997). Consequently, a noise source of variable strength seems to be responsible for the local instability encountered in Forsmark-1. The localised character of the noise source is in favour of a channel thermal-hydraulic instability, i.e. a self-sustained Density Wave Oscillation (DWO). As pointed out by Karlsson and Pázsit (1999), when a fuel element is unseated, some of the coolant flow bypasses the fuel element and this might render the channel thermal-hydraulically unstable.

Nevertheless choosing a different set of detectors gives results which are sometimes different, i.e. the noise source is not always located at a position close to the unseated fuel element. This suggests that there are probably two (or maybe even more) noise sources located inside the core. As pointed out previously, limiting the number of detectors to a region where a noise source is suspected to be located allows successfully locating this specific noise source, as long as the other noise sources are not in the same vicinity. This is why the region around the unseated fuel element was pointed out by the localisation algorithm. Taking more detectors into account that the one used in Fig. 9 is

(22)

equivalent to take the effect of several other possible noise sources into account, whereas the algorithm has been designed for a single noise source. Finally, it is worth mentioning that only the region pointed out by the localisation algorithm was visually inspected during the core outage, i.e. other unseated fuel elements might not have been detected.

5. CONCLUSIONS

In this paper, a so-called noise simulator was presented. This noise simulator calculates the transfer function between a noise source and the resulting flux noise in the two-group diffusion approximation in 2-D. All the calculations are directly performed in the frequency domain. The simulator offers also the possibility of having several noise sources and different kinds of them simultaneously. For homogeneous cores and a given central noise source, the finite difference discretisation scheme appears to reproduce the analytical solution accurately, both regarding the amplitude and the phase of the neutron noise.

This noise simulator was then used in an inverting task, namely the localisation of an unknown noise source. Neutron noise data were generated by the simulator, and these were used subsequently in the localisation algorithm. From the detector readings at some discrete locations of the core in a 2-D plane, the algorithm pointed out a location corresponding to the suspected noise source, of which the strength is still undetermined. The algorithm was found to be perfectly insensitive to the number of detectors used, to the location of the noise source in the core, and to the presence of extraneous noise in the detector signals. These conclusions can be held as long as one single noise source exists in the core, since the localisation procedure was designed explicitly in this case. The presence of several noise sources deteriorates the accuracy of the localisation, but as long as the noise sources are well separated in space, using a set of detectors surrounding one of the noise sources gives the correct location of this specific noise source. Furthermore, the fact that the thermal flux noise diverges close to the noise source when the noise is defined from the removal cross-section makes the localisation algorithm more robust and more efficient than in the case where the noise source is defined in terms of the fast/thermal absorption cross-section noise.

This algorithm was finally applied to a realistic case, namely the Forsmark-1 local instability event. By selecting an appropriate set of detectors on the lower most LPRM level in the core, the localisation algorithm pointed out a global minimum in the reflector that had to be disregarded, and a local minimum located close to a fuel element that was discovered to be unseated during the core outage. The fact that using a different combination of detectors might give in some occurrences different results suggests that more than one noise source is responsible for the instability. Unfortunately, only 30 fuel bundles (of which one was found to be unseated) were visually inspected, i.e. some other unseated fuel bundles might not have been detected.

In the Forsmark case, a set of homogeneous cross-sections for the fuel elements and another set of homogeneous cross-sections for the reflector were used. Even if the noise simulator needs to adjust these cross-sections sets so that the system remains critical if a finite difference scheme is used, these sets are first of all rather homogeneous, and

second do not correspond exactly to the Forsmark-1 core. It is planned in the future to use a set of cross-sections representative of the Forsmark-1 core. Nevertheless, due to the presence of the control rods, the realistic set of cross-sections is expected to be highly heterogeneous. In such a case, the accuracy of the finite difference scheme will considerably deteriorate. Therefore, more efficient discretisation schemes, such as nodal methods or finite elements, are under consideration.

NOMENCLATURE

APSD Auto-Power Spectral Density
BWR   Boiling Water Reactor
CPSD Cross-Power Spectral Density
DR   Decay Ratio
DWO Density Wave Oscillation
LPRM Local Power Range Monitor
MOC   Middle Of Cycle
MTC   Moderator Temperature Coefficient
PWR   Pressurised Water Reactor

ACKNOWLEDGEMENTS

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PAPER V

A PHENOMENOLOGICAL MODEL FOR THE EXPLANATION OF A STRONGLY SPACE-DEPENDENT DECAY RATIO

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ABSTRACT

It is commonly believed that the Decay Ratio (DR), a parameter characterizing the stability of Boiling Water Reactors (BWRs), is a space-independent parameter of the reactor, i.e. it is independent of which Local Power Range Monitor (LPRM) is used in the core to perform the evaluation. This paper shows that the presence of several simultaneous types or sources of instability with different stability properties and different space dependence renders the DR also space-dependent, and even strongly space dependent. Two cases were investigated: the case of a local instability (i.e. one induced by a local noise source) coexisting with a global instability (in-phase oscillations), and the case of two local instabilities (noise sources). The results of these calculations were compared to the Forsmark-1 channel instability event, where strongly space-dependent decay ratios had been found in the measurements. Good adequacy was found between the DR model applied to the Forsmark-1 event and the corresponding measured DR. The fact that one single noise source in the core does not allow explaining a non-homogeneous DR suggests that in the case of Forsmark-1, at least two types or sources of instability had to be present in the core at the same time. According to the results obtained in this paper, these could be either a local and a global instability, or two local ones.

Key Words: Decay Ratio, neutron noise, core calculations, measurement

1. INTRODUCTION

During a measurement campaign performed during the fuel cycle 16 in the Forsmark-1 Boiling Water Reactor (BWR) in order to study BWR stability, it was noticed that the so-called Decay Ratio (DR) was strongly radially space-dependent [1]. The DR, which characterises the stability of a BWR, was always assumed until that moment to be a space-independent parameter of the core. This space-dependent character of the DR could not be understood.

A phenomenological model suggested by Pázsit in [2] was applied to the Forsmark-1 case. Originally, this model was derived to explain the discontinuous character of the DR when the operating point was changed smoothly on the power-flow map. Such a behaviour was found in the Swedish BWR Ringhals-1, where dual oscillations (local and regional) appeared simultaneously. In the study reported in [2], the space dependence of the decay ratio was not investigated, only its dependence on the operating point. However, the model takes into account the space dependence of the different oscillations, hence it was used in this study to show that the

coexistence of two types or sources of instability with different DRs and space dependence can make the DR strongly space-dependent. Two cases were investigated in this study: the case of a local noise source coexisting with a global noise source (in-phase oscillations), and the case of two local noise sources. In order to use this phenomenological model in these two cases, the calculation of the spatial structure of the neutron noise induced by the aforementioned noise sources is required.

In the following, the Forsmark-1 measurement is first described in detail. The different models used to estimate theoretically the DR are thus explained, namely the DR model is recalled and the neutron noise simulator is briefly explained. As will be seen later on, the calculation of the neutron noise in case of the global-type of oscillations is identical to the estimation of the static flux, so that a static core simulator is also required and presented in this paper. Both simulators rely on the 2-group diffusion approximation, and are able to handle 2-D heterogeneous cores. Finally, the space-dependence of the DR is estimated from the phenomenological model and compared to the measured DR.

2. DESCRIPTION OF THE FORSMARK-1 CASE

In 1996, during the start-up tests of the Forsmark-1 BWR for the fuel cycle 16, instability conditions were detected at reduced power and reduced core-flow. Forsmark-1 is a BWR of the Westinghouse Atom design (previously ABB Atom AB, or ASEA-Atom AB) built in 1980 and has a thermal core-rated power of 2700 MWth and a nominal core flow of 10450 kg/s. Although BWRs are known to become less stable at reduced power/core flow, the appearance of this instability could not be understood and was not predicted by the stability calculations. The corresponding operating point in the power/flow map was therefore avoided. In January 1997, at approximately Middle Of Cycle conditions (MOC), stability measurements were carried out in order to study the instability discovered previously. The core was thus brought to 63.3% of power and to a core flow of 4298 kg/s. Again instability conditions were encountered, at a frequency of roughly 0.5 Hz.

During this stability measurement the lower plane of the core was rather well equipped with Local Power Range Monitors (LPRMs), where signals from 27 of the 36 available detector strings were actually recorded at a sampling frequency of 12.5 Hz. One parameter that is relevant for characterizing the stability of BWRs is the DR, which is defined as the ratio between two consecutive maxima A_i and A_{i+1} of the Auto-Correlation Function (ACF) of the normalized neutron density, or alternatively two consecutive maxima of the Impulse Response Function (IRF) as calculated by using an Autoregressive Moving-Average (ARMA) or an Autoregressive model (AR) to fit the behaviour of the system. These methods are illustrated in Fig. 1. The DR gives therefore a measure of the inherent damping properties of the system. Using each detector separately allows estimating the Decay Ratio (DR) according to the following standard method [3]:

$$DR = \frac{A_{i+1}}{A_i}, \forall i \tag{1}$$

Although the DR was always assumed to be a 0-D parameter of the core, i.e. independent of the position where the DR is estimated in the core, the Forsmark-1 measurement revealed that the DR was actually strongly space-dependent, as can be seen on the following Fig. 2. This Figure shows that one half of the core exhibits a DR close to instability (higher than 0.9) and the other half has a DR close to 0.6.
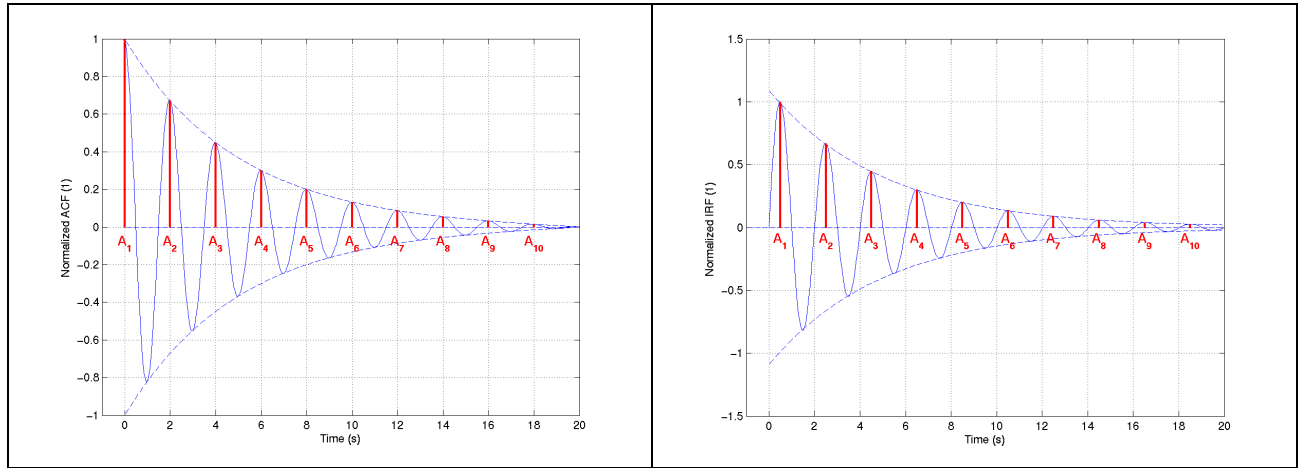
Figure 1. ACF and IRF of a second-order system (on the left-hand side and the right-hand side respectively).
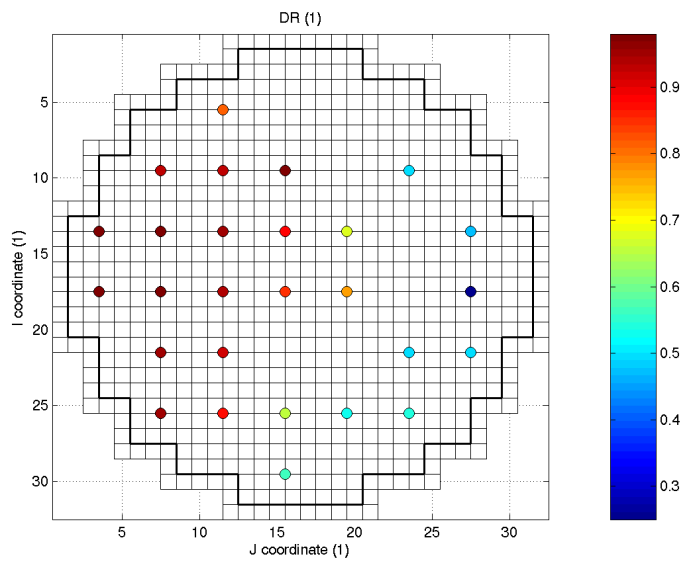
Figure 2. Measured radial space-dependence of the Decay Ratio in Forsmark-1 (derived from [4]).

A closer look at the phase of the measured flux noise indicated that the neutron noise was driven by a local noise source, similar to the effect of an absorber of variable strength (reactor oscillator). As a matter of fact, more detailed analyses of this instability event revealed that the reason of this instability was due to the presence of one or more likely two noise sources [5], [6]. In these aforementioned analyses, even a localisation of these noise sources was carried out, and one of the noise sources pointed out by the localisation algorithm was close to a fuel assembly which was found to be unseated during the fuel outage following the fuel cycle 16. As pointed out by [7], when a fuel element is unseated, some of the coolant flow bypasses the fuel element and this might render the channel thermal-hydraulically unstable (self-sustained Density Wave Oscillation or DWO [8]).

3. DESCRIPTION OF THE MODELS

The fact that the observed DR in the Forsmark-1 case exhibits a strong space-dependence suggests that two types or sources of instability are present at the same time in the core. If there was only one type or one source of instability, the DR would not be space-dependent, and would roughly be the same whatever LPRM is used to perform the DR evaluation. In the following, an analytical model that allows estimating the DR resulting from two types/sources of instability is presented and its application to the Forsmark-1 case is explained.

3.1. Analytical model of the Decay Ratio

An analytical model to calculate the DR in case of dual oscillations was proposed by Pázsit in [2]. This model was developed to explain the discontinuous character of the DR when the operating point was changed smoothly on the power-flow map, and relied on dual oscillations according to which the DR was calculated. Although the model is rather simple in order to facilitate analytical calculations with the goal of facilitating insight and understanding, its domain of validity was confirmed also in detailed core calculational models [9]. The same model will be used in this paper to study the space-dependence of the DR. The only difference with the previous investigation is the type or source of instability investigated. In Pázsit 's paper, only the case of a regional (out-of-phase) type of oscillations coexisting with a global (in-phase) type of oscillations was investigated. This model was extended here to any type or source of instability existing simultaneously in the core. More specifically, two cases are presented: the case of a local noise source coexisting with a global type of oscillations, and the case of two local noise sources. In the following, the main characteristics of the model proposed by Pázsit are recalled.

The starting point is to write that the flux fluctuations can be written as a sum between the contribution of two noise sources, each of them being factorized into a temporal part only and a spatial part only as follows:

$$\delta\phi(\boldsymbol{r},t) = \delta\phi_1(\boldsymbol{r},t) + \delta\phi_2(\boldsymbol{r},t) = \delta\psi_1(t)\varphi_1(\boldsymbol{r}) + \delta\psi_2(t)\varphi_2(\boldsymbol{r}) \tag{2}$$

where the amplitudes $\delta\psi_i(t)$ ($i=1, 2$) are second-order processes, with the same resonance frequency ω_0, but different damping properties. As will be shown later, for the case of local instability, the factorisation does not hold in a strict sense; nevertheless, it is quite well applicable in the frequency range considered. At any rate, this study relies on this basic assumption. If one further assumes that the aforementioned second-order processes are driven by the driving forces $f_i(t)$ ($i=1, 2$), the amplitudes $\delta\psi_i(t)$ ($i=1, 2$) will obey the following Eq. (3):

$$\delta\ddot{\psi}_i(t) + 2\xi_i\omega_0\delta\dot{\psi}_i(t) + \omega_0^2\delta\psi_i(t) = f_i(t) \tag{3}$$

For simplicity, we will neglect the cross-term between the two noise sources, i.e. we will assume that:

$$CPSD_{f_1,f_2}(\omega) = CPSD_{f_2,f_1}(\omega) \approx 0 \tag{4}$$

If one further assumes that the DR of any of the two types or sources of instability is larger than 0.4, the second-order terms in ξ can be neglected, and Pázsit in [2] showed that the ACF of any LPRM signal would then be given by:

$$ACF(\boldsymbol{r},\tau) = \cos(\omega_0\tau)\sum_{i=1}^{2} a_i(\boldsymbol{r}).e^{-\xi_i\omega_0\tau} \tag{5}$$

with

$$a_i(\boldsymbol{r}) = \frac{1}{1 + \dfrac{APSD_{f_j}}{APSD_{f_i}} \cdot \dfrac{\varphi_j(\boldsymbol{r})}{\varphi_i(\boldsymbol{r})} \cdot \dfrac{\ln(DR_i)}{\ln(DR_j)}}, \; i \neq j \tag{6}$$

Eq. (5) does not correspond to a pure second-order system, and therefore the definition of its DR is not unique. From a measurement viewpoint, it is practical to define the DR as the ratio between the first and the second maxima of the ACF, i.e.:

$$DR(\boldsymbol{r}) = \sum_{i=1}^{2} a_i(\boldsymbol{r}) \cdot e^{-2\pi\xi_i} = \sum_{i=1}^{2} a_i(\boldsymbol{r}).DR_i \tag{7}$$

where

$$DR_i = e^{-2\pi\xi_i} \tag{8}$$

As explained by Pázsit in [2], the cross-term can be explicitly accounted for. In such a case the expressions that we would obtain would be more complicated, but the model of the DR presented previously would be essentially the same, only the weighting coefficient $a_i(\boldsymbol{r})$ between the two types or sources of instability would be different. Furthermore, Eq. (8) allows verifying the other

hypothesis used in this model, i.e. that neglecting ξ^2 besides unity is justified for any DR larger than 0.4.

3.2. Numerical estimation of the neutron noise

In order to calculate the space-dependence of the DR when two types or sources of instability coexist in the core, one needs to estimate the $a_i(\mathbf{r})$ coefficients used in Eq. (7), i.e. one needs to define the f_i parameters and to calculate the functions $\varphi_i(\mathbf{r})$. The f_i parameters (or more exactly their ratio) can be chosen freely. The functions $\varphi_i(\mathbf{r})$, which represent the spatial dependence of the induced neutron noise, depend on the type or source of instability.

In case of a global-type of oscillations, it is well known that the induced neutron noise is spatially distributed according to the static flux $\phi_0(\mathbf{r})$, i.e. is given in the frequency domain by:

$$\delta\phi(\mathbf{r},\omega) = \phi_0(\mathbf{r}) \cdot G_0(\omega)\delta\rho(\omega) \tag{9}$$

where $G_0(\omega)$ and $\delta\rho(\omega)$ are the zero-power reactor transfer function and the reactivity noise (noise source) respectively, so that:

$$\varphi_i(\mathbf{r}) = C_i\phi_0(\mathbf{r}) \tag{10}$$

where C_i is a scaling coefficient.

In case of a local noise source, the induced neutron noise is given in the frequency domain by:

$$\delta\phi_i(\mathbf{r},\omega) = G(\mathbf{r},\mathbf{r}_0,\omega) \cdot S_{r_0}(\omega) \tag{11}$$

where $S_{r_0}(\omega)$ and $G(\mathbf{r},\mathbf{r}_0,\omega)$ are the noise source localised at the position $\mathbf{r}_0$ and the corresponding Green's function (transfer function) of the noise equations, respectively [5]. Eq. (11) shows that, in general, for the local oscillations, the space and frequency (or space and time) dependence of the neutron fluctuations does not factorise. However, one can make use of the fact that the transfer function $G(\mathbf{r},\mathbf{r}_0,\omega)$ depends on frequency only through the zero-power reactor transfer function $G_0(\omega)$. This latter, on the other hand, depends very weekly on the frequency in the so-called plateau region, roughly between 0.05 and 15 Hz. Since the oscillation frequency of the local instability was about 0.5 Hz, one can use the values of $G(\mathbf{r},\mathbf{r}_0,\omega)$ at the resonance frequency ω_0. Hence, one can write:

$$\varphi_j(\mathbf{r}) \approx C_j\left|G(\mathbf{r},\mathbf{r}_0,\omega_0)\right| \tag{12}$$

where C_j is another scaling factor. Since it is only the ratio of the scaling factors in Eqs. (10) and (12) that counts, we shall assume $C_j = 1$ in Eq. (12).

Consequently, one needs to estimate both the static flux and the neutron noise induced by the localised noise source(s) in order to use the phenomenological model given by Eqs. (6) and (7). Since this study investigates the case of the Forsmark-1 BWR, i.e. a strongly heterogeneous core, the static flux was calculated by a 2-D 2-group static core simulator and the neutron noise by a 2-D 2-group noise simulator. In the following, the basic properties of these two simulators, which were developed at the Department of Reactor Physics, Chalmers University of Technology, are briefly presented.

The static core simulator solves the following matrix equation in the two-group diffusion approximation:

$$
\left[\overline{\overline{D}}(\boldsymbol{r}) \nabla^2 + \overline{\overline{\Sigma}}(\boldsymbol{r}) \right] \times \begin{bmatrix} \phi_1(\boldsymbol{r}) \\ \phi_2(\boldsymbol{r}) \end{bmatrix} = 0 \tag{13}
$$

where

$$
\overline{\overline{D}}(\boldsymbol{r}) = \begin{bmatrix} D_1(\boldsymbol{r}) & 0 \\ 0 & D_2(\boldsymbol{r}) \end{bmatrix} \tag{14}
$$

$$
\overline{\overline{\Sigma}}(\boldsymbol{r}) = \begin{bmatrix} \dfrac{\nu\Sigma_{f,1}(\boldsymbol{r})}{k_{eff}} - \Sigma_{a,1}(\boldsymbol{r}) - \Sigma_{rem}(\boldsymbol{r}) & \dfrac{\nu\Sigma_{f,2}(\boldsymbol{r})}{k_{eff}} \\ \Sigma_{rem}(\boldsymbol{r}) & -\Sigma_{a,2}(\boldsymbol{r}) \end{bmatrix} \tag{15}
$$

All the notations have their usual meaning. Finite differences were used to carry out the 2-D spatial discretization of the system according to the so-called "box-scheme" [10]. Eq. (13), which is a homogeneous equation, was solved by using an iterative scheme, more exactly the power iteration method [10], [11]. This static core simulator was successfully benchmarked against SIMULATE-3 [12] (after axial homogenization) for both PWR [13] and BWR [14] cases.

The neutron noise simulator solves the following matrix equation in the 2-group diffusion approximation at a given frequency ω, for fluctuations of the macroscopic removal cross-section[1]:

---

[1] The neutron noise simulator is actually able to handle any type of noise sources (namely fluctuations in the fast or thermal macroscopic absorption cross-section, fluctuations in the macroscopic removal cross-section, fluctuations in the fast or thermal macroscopic fission cross-section). In the case of DWO, the fluctuations of the macroscopic removal cross-section are the most relevant ones, and therefore the neutron noise simulator in this study only calculates the corresponding induced neutron noise. We refer to [6] for the derivation of Eq. (16) in the most general case.

$$\left[\overline{\overline{D}}(\boldsymbol{r})\nabla^2 + \overline{\overline{\Sigma}}(\boldsymbol{r},\omega)\right] \times \begin{bmatrix} \delta\phi_1(\boldsymbol{r},\omega) \\ \delta\phi_2(\boldsymbol{r},\omega) \end{bmatrix} = \overline{\phi}_{rem}(\boldsymbol{r})\,\delta\Sigma_{rem}(\boldsymbol{r},\omega) \tag{16}$$

where the matrix and vector are given as:

$$\overline{\overline{\Sigma}}(\boldsymbol{r},\omega) = \begin{bmatrix} -\Sigma_1(\boldsymbol{r},\omega) & \nu\Sigma_{f,2}(\boldsymbol{r},\omega) \\ \Sigma_{rem}(\boldsymbol{r}) & -\Sigma_{a,2}(\boldsymbol{r},\omega) \end{bmatrix} \tag{17}$$

$$\overline{\phi}_{rem}(\boldsymbol{r}) = \begin{bmatrix} \phi_1(\boldsymbol{r}) \\ -\phi_1(\boldsymbol{r}) \end{bmatrix} \tag{18}$$

and the different coefficients are defined as:

$$\Sigma_1(\boldsymbol{r},\omega) = \Sigma_{a,1}(\boldsymbol{r}) + \frac{i\omega}{v_1} + \Sigma_{rem}(\boldsymbol{r}) - \nu\Sigma_{f,1}(\boldsymbol{r})\left(1 - \frac{i\omega\beta_{eff}}{i\omega + \lambda}\right) \tag{19}$$

$$\nu\Sigma_{f,2}(\boldsymbol{r},\omega) = \nu\Sigma_{f,2}(\boldsymbol{r})\left(1 - \frac{i\omega\beta_{eff}}{i\omega + \lambda}\right) \tag{20}$$

$$\nu\Sigma_{a,2}(\boldsymbol{r},\omega) = \nu\Sigma_{a,2}(\boldsymbol{r}) + \frac{i\omega}{v_2} \tag{21}$$

As for the static core simulator, finite differences were used to carry out the 2-D spatial discretization of the system according to the so-called "box-scheme" [10]. Eq. (16), which is an inhomogeneous equation, was solved by direct matrix inversion. The neutron noise simulator is thus able to calculate the spatial distribution of the neutron noise induced by any localized (or even spatially distributed) noise sources. This neutron noise simulator was successfully benchmarked against analytical solutions in case of homogeneous cores with a central noise source [6].

The only data required in order to use the static core simulator and the neutron noise simulator are the 2-D 2-group material constants, and the point-kinetic parameters of the core. In the case of the Forsmark-1 BWR, these data were obtained from calculations performed by Vattenfall Fuel AB with the SIMULATE-3 code. The data were homogenized from 3-D to 2-D by preserving the reaction rates according to the following formulae:

$$XS_{G,I,J} = \frac{\sum_K XS_{G,I,J,K}\,\phi_{G,I,J,K}\,V_{I,J,K}}{\sum_K \phi_{G,I,J,K}\,V_{I,J,K}} \tag{22}$$

and

$$\phi_{G,I,J} = \frac{\sum_K \phi_{G,I,J,K} V_{I,J,K}}{\sum_K V_{I,J,K}} \tag{23}$$

with XS_G having a broad meaning, i.e. being D_G, $\Sigma_{a,G}$, Σ_{rem}, or $\nu\Sigma_{f,G}$. All the other symbols have their usual meaning with $V_{I,J,K}$ representing the volume of the node (*I,J,K*) and *G* being the group index. In order to get a 2-D system equivalent to the 3-D system, the leakage rate of the 3-D system in the axial direction was added to the absorption cross-section in the 2-D system, both in the fast and thermal groups. The neutron noise simulator requires itself the static fluxes and the corresponding eigenvalue. Although these data are directly available after homogenization from SIMULATE-3, they have to be obtained from the 2-D 2-group static core simulator that is compatible with the neutron noise simulator. Otherwise using the SIMULATE-3 results, i.e. results that were calculated using a discretization scheme different from the one used in the neutron noise simulator – nodal methods for SIMULATE-3 and finite differences for the 2-D 2-group static core and neutron noise simulators –, would be equivalent to make the system non-critical.

4. RESULTS

In the following, the results of the previous phenomenological model applied to Forsmark-1 are presented. Two cases are investigated: the case of a local noise source coexisting with a global noise source, and the case of two local noise sources.

4.1 The case of a local noise source and a global noise source

The results corresponding to the case of a local noise source coexisting with a global noise source (in-phase oscillations) are presented in Fig. 3. The DR corresponding to the local noise source was set to 0.99, whereas the DR corresponding to the in-phase oscillations was set to 0.4. The local noise source was located at the position pointed out by a noise source localization algorithm applied previously in [6] to the case of the Forsmark-1 channel instability event. Finally, as explained earlier, the ratio f_2/f_1 can be chosen freely and was determined so that the DR calculated throughout the core matched the measured DR. As can be seen on Fig. 3, the DR calculated by using the phenomenological model given by Eqs. (6) and (7) in case of a local noise source and a global noise source is strongly spatially dependent, and reproduces quite well the behavior of the measured DR in Forsmark-1. The reason for the relatively sharp boundary between the two values of the DR is the fast spatial decay of the local oscillations. Thus there are two different regions in the core, one in which the local oscillations dominate, and one in which the global ones dominate, with a relatively narrow transition region. Such a case would not occur

with concurrent global and regional oscillations, only when at least one local component is involved.
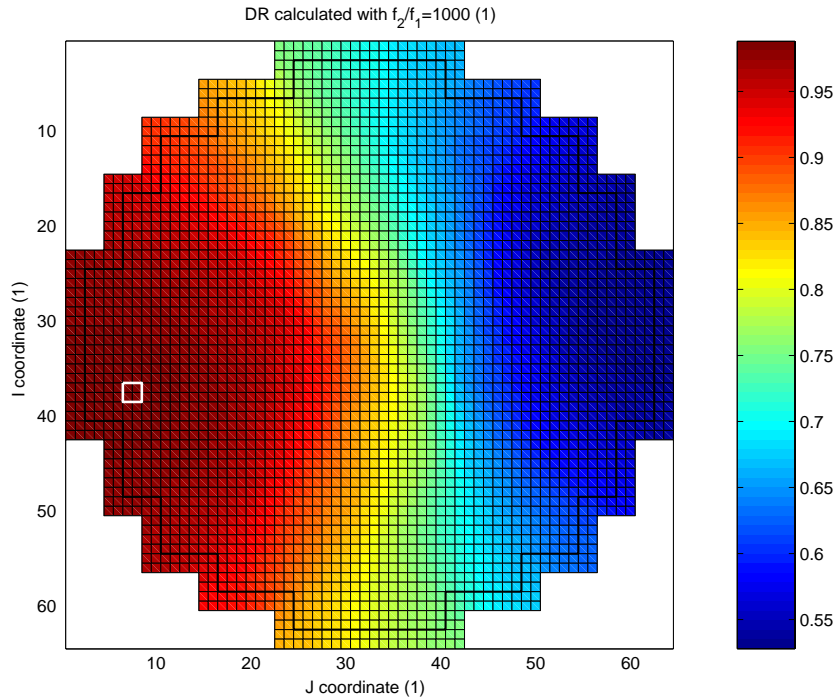
Figure 3. Simulated radial space-dependence of the Decay Ratio in Forsmark-1 in case of a local noise source and a global noise source (the white square represents the location of the local noise source).

4.2 The case of two local noise sources

The results corresponding to the case of two local noise sources are presented in Fig. 4. The local noise source with a DR of 0.99 was located at the position pointed out by the noise source localization algorithm applied previously in [6] to the case of the Forsmark-1 channel instability event. The noise source with a DR of 0.4 was positioned on the opposite side from the other noise source. The ratio f_2/f_1 was chosen so that the DR calculated throughout the core matched the measured DR. As can be seen on Fig. 4, the DR calculated by using the phenomenological model given by Eqs. (6) and (7) in case of two local noise sources is strongly spatially dependent, and reproduces again rather well the behavior of the measured DR in Forsmark-1. Again, the reason of the sharp boundary between the two stability regions is the fast spatial decay of the amplitude of the local oscillations.
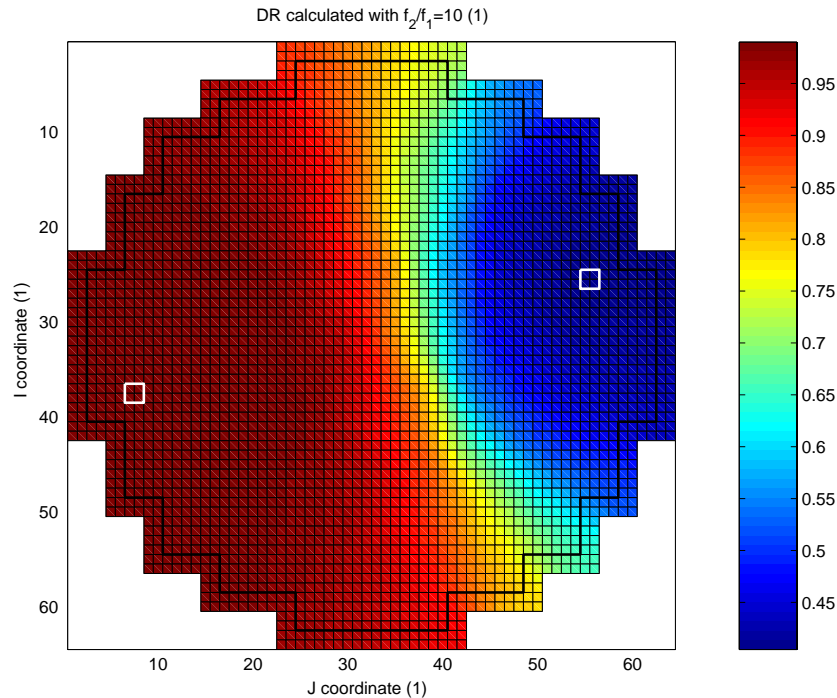
Figure 4. Simulated radial space-dependence of the Decay Ratio in Forsmark-1 in case of two local noise sources (the white squares represent the location of the local noise sources).

5. CONCLUSIONS

The purpose of this paper was to construct a simple model, with the help of which the experimentally found space-dependence of the Decay Ratio (DR) is possible. In the early cases of BWR instability, the DR appeared to be a space-independent parameter of a BWR core, characterizing its global stability. Nevertheless, it was noticed during the Forsmark-1 channel instability event (January 1997) that the DR measured throughout the core using LPRM signals was radially strongly space-dependent, ranging from 0.6 on the right-hand side of the core to values higher than 0.9 on the left-hand side of the core, i.e. values close to limit-cycle oscillations.

The phenomenological model of the DR developed by Pázsit in [2] was used in this paper in order to calculate the space-dependence of the DR in case of several types or sources of instability, each of them having different stability properties and space dependence. More specifically, two cases were investigated: a local noise source coexisting with a global noise source (in-phase oscillations), and two local noise sources. It was shown, via the use of a 2-D 2-group static core simulator and a 2-D 2-group neutron noise simulator applied to realistic data corresponding to the Forsmark-1 instability event, that the space-dependent character of the measured DR could be reproduced. This therefore confirms our original idea that in case of dual

oscillations with different space-dependence, i.e. when several types or sources of instability coexist at the same time in the core, the DR itself becomes necessarily space-dependent. In the case when at least one local oscillation is involved, the DR may appear as discontinuous in space, which is the case that was observed in Forsmark.

ACKNOWLEDGMENTS

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NOMENCLATURE

ACF   Autocorrelation Function
APSD   Auto-Power Spectral Density
AR   Autoregressive (model)
ARMA   Autoregressive Moving-Average (model)
BWR   Boiling Water Reactor
CEA   Commissariat à l'Energie Atomique
DR   Decay Ratio
DWO   Density Wave Oscillation
IRF   Impulse Response Function
LPRM   Local Power Range Monitor
MOC   Middle Of Cycle
SKI   Swedish Nuclear Power Inspectorate (Statens Kärnkraftinspektion)

PAPER VI

STUDY OF THE MTC ESTIMATION BY NOISE ANALYSIS

IN 2-D HETEROGENEOUS SYSTEMS

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ABSTRACT

The effect of a heterogeneous distribution of the temperature noise on the MTC estimation by noise analysis is investigated. This investigation relies on 2-group diffusion theory, and all the calculations are performed in a 2-D realistic heterogeneous core. It is shown, similarly to the 1-D case, that the main reason of the MTC underestimation by noise analysis compared to its design-predicted value lies with the fact that the temperature noise might not be homogeneous in the core, and therefore using the local temperature noise in the MTC noise estimation gives erroneous results. A new MTC estimator, which was previously proposed for 1-D 1-group homogeneous cases and which is able to take this heterogeneity into account, was extended to 2-D 2-group heterogeneous cases. It was proven that this new estimator is always able to give a correct MTC estimation with an accuracy of 3%. This small discrepancy comes from the fact that the reactor does not behave in a point-kinetic way, contrary to the assumptions used in the noise estimators. This discrepancy is however quite small.

A nomenclature explaining all the abbreviations used in this paper can be found at the end (see Section 8).

1. INTRODUCTION

Monitoring the Moderator Temperature Coefficient (MTC) of reactivity will be of prime importance in the next future when using high burnup and Mixed Oxide (MOX) fuel assemblies in Pressurised Water Reactors (PWRs). The MTC estimation by noise analysis represents an interesting technique since, in contrast to the traditional methods, the reactor does not need to be perturbed.

Several attempts to monitor the MTC by noise analysis revealed that the MTC noise estimate was systematically smaller by a factor of two to five compared to its actual value (see Demazière (2000) for a complete list of References in this matter). Several factors could explain this underestimation. A recent theoretical investigation showed that the radial non-homogeneous structure of the temperature noise throughout the core could be responsible for most of the deviation between the MTC noise estimate and its actual value (Demazière and Pázsit, 2002a). Furthermore, this radial heterogeneous structure of the moderator temperature noise was experimentally noticed (Demazière *et al*, 2000). The fact that a large power reactor does not behave in a point-kinetic manner does not seem to play a significant role on the MTC estimation. Therefore, a new MTC noise estimator, which allows taking the spatial distribution of the temperature noise throughout the core into account, was proposed and was proven theoretically to give a very good estimation of the actual MTC (Demazière and Pázsit, 2002a).

Nevertheless, this previous study only investigated 1-D one-group homogeneous systems. We propose here to perform a substantially more advanced study of the same problem in 2-D two-group diffusion theory in heterogeneous cores. For that purpose, among other quantities, the reactor transfer function (between the noise source and the induced flux noise) needs to be known. Due to the heterogeneous character of the system, this transfer function can only be evaluated numerically. We have recently developed a 2-D 2-group neutron noise simulator (Demazière and Pázsit, 2001). The simulator was therefore used for the evaluation of the MTC noise estimation. In this work, one verified the results obtained with the 1-D homogeneous systems, i.e. that the deviation of the reactor response from point-kinetics does not play a significant role, whereas the radial incoherent structure of the temperature noise is responsible for most of the MTC underestimation. The weak effect of the deviation from point-kinetics of the reactor on the MTC was investigated carefully since it is known that 2-D systems present a larger deviation from point-kinetics than 1-D systems. It was found that the deviation from point-kinetics was still negligible regarding the MTC estimation, giving an accuracy of 3% with the new MTC noise estimator.

2. MODELLING OF THE NOISE SOURCES AND CALCULATION OF THE CORRESPONDING REACTOR TRANSFER FUNCTION

The starting point of the investigation is the modelling of a realistic commercial PWR via SIMULATE-3 (Umbarger and DiGiovine, 1992). The Ringhals-4 unit was considered since the Department of Reactor Physics investigated previously an MTC measurement using the boron dilution method performed in that reactor on May 5th, 1999, for the fuel cycle 16 and at a core average burnup of 8.767 GWd/tHM (Demazière *et al*, 2000). From this modelling, the MTC was also directly evaluated using SIMULATE-3. The material data were therefore obtained from SIMULATE-3 and properly homogenized in order to be used in the 2-D 2-group simulations.

The moderator temperature noise may have many different sources (inlet temperature noise, coolant velocity noise, heat generation noise, heat transfer noise, etc.). At any rate, the moderator temperature noise causes density fluctuations, which in turn causes removal macroscopic cross-section noise. The removal macroscopic cross-section fluctuations were thus assumed to be directly proportional to the moderator temperature fluctuations via a space- and time-independent coefficient K as follows:

$$\delta\Sigma_{rem}\left(\boldsymbol{r},t\right) = \frac{1}{K} \times \delta T_{m}\left(\boldsymbol{r},t\right) \tag{1}$$

One therefore supposes that the discrepancy of the MTC noise estimate has its origin in the possible non-homogeneous distribution of the moderator temperature noise throughout the core (the purpose of this paper is to assess how large this deviation is).

As presented in Demazière and Pázsit (2002a), the noise source was defined directly through its spatial statistical properties, i.e. its Cross-Power Spectral Density (CPSD), and was assumed to be factorised into a frequency-dependent only part and a space-dependent only part. The spatial part was further assumed to

be factorised into a shape function $\sigma^2(\hat{r})$ and an exponential decay function of the separation distance, expressing the fact that for increasing distance the correlation between two radial points decreases. The frequency content of the noise source was supposed to be a white noise in the frequency region of interest for the MTC estimation by noise analysis, i.e. between 0.1 and 1.0 Hz. In the frequency domain, the CPSD of the macroscopic removal cross-section noise can thus be written as a function independent of the frequency as follows:

$$CPSD_{\delta\Sigma_{rem}}(r,r') = \sigma^2(\hat{r})e^{-\frac{|r-r'|}{l}} \tag{2}$$

with

$$\sigma(\hat{r}) = \frac{1}{1-\left(\dfrac{\hat{r}}{R+\Delta R}\right)^2} \tag{3}$$

and

$$\hat{r} = \frac{r+r'}{2}. \tag{4}$$

R is the core radius and $\Delta R = R/5$. $\sigma^2(\hat{r})$ thus represents the noise source strength, i.e. its Auto-Power Spectral Density (APSD). The shape function given by Eq. (3) corresponds to some experimental evidence that the temperature noise is larger close to the core boundary than at the core centre (Karlsson, 2000). It has nevertheless to be pointed out that 1-D calculations (see Demazière and Pázsit (2002a)) showed that the results were completely independent of the choice of the shape function. In this model, l is called the correlation length of the temperature fluctuations and is supposed to be space independent. The correlation length indicates roughly the maximum distance between two points that can be considered as having a coherent behaviour. For greater distances, their behaviour can be assumed to be completely uncorrelated. Several correlation lengths were investigated in this study, but only the results corresponding to $l=150$ cm will be presented.

Then the 2-D 2-group noise simulator was used to calculate the flux noise $\delta\phi_i$ in r and at a frequency $f = \omega/2\pi$ induced by a unit macroscopic removal cross-section noise source $\delta\Sigma_{rem} = 1$ located at r' at the same frequency. $i = 1,2$ represents the group index, i.e. the fast and thermal groups respectively. Consequently, the neutron noise simulator actually estimates the 2-D 2-group discretised Green's function $G_{\delta\Sigma_{rem}\to i}(r,r',\omega)$. For the sake of brevity, the neutron noise simulator is not presented here in detail. We refer instead to the paper published by Demazière and Pázsit (2001). More precisely, based on such an expression, auto- and cross-spectra between the neutron and temperature noise can be calculated by multiple integrals, as will be shown in Section 3.2.

3. DERIVATION OF THE MTC NOISE ESTIMATORS

The reactivity variation induced by the change of the moderator temperature change is simply given in the frequency domain as:

$$\delta\rho(\omega) = MTC \times \delta T_m^{ave}(\omega) = MTC \times K \times \delta\Sigma_{rem}^{ave}(\omega) \tag{5}$$

One has to choose a weighting function for calculating the spatial average temperature and removal cross-section noise.

3.1 Estimation of the weighting function for the estimation of the average temperature noise

Assuming that first-order perturbation theory is applicable, one has:

$$\delta\rho(\omega) = \frac{\int -\delta\Sigma_{rem}(r,\omega)\left[\phi_1^+(r)\phi_1(r) - \phi_2^+(r)\phi_1(r)\right]dr}{\int\left[\nu\Sigma_{f,1}(r)\phi_1^+(r)\phi_1(r) + \nu\Sigma_{f,2}(r)\phi_2^+(r)\phi_2(r)\right]dr} \tag{6}$$

where the superscript "+" represents the adjoint problem. All the symbols have their usual meaning. The static flux and the adjoint flux were calculated via appropriate 2-D 2-group simulators[1] developed by us.

Since the MTC is independent of the spatial structure of the temperature noise throughout the core, one has in case of a spatially homogeneous temperature perturbation:

$$MTC = \frac{1}{K}\times\frac{\int -\left[\phi_1^+(r)\phi_1(r) - \phi_2^+(r)\phi_1(r)\right]dr}{\int\left[\nu\Sigma_{f,1}(r)\phi_1^+(r)\phi_1(r) + \nu\Sigma_{f,2}(r)\phi_2^+(r)\phi_2(r)\right]dr} \tag{7}$$

This means that in case of spatially inhomogeneous temperature fluctuations, the spatial average of the temperature or removal cross-section noise in Eq. (5) has to be estimated by using the following weighting function:

$$w(r) = -\left[\phi_1^+(r)\phi_1(r) - \phi_2^+(r)\phi_1(r)\right] \tag{8}$$

so that:

$$\delta\Sigma_{rem}^{ave}(\omega) = \frac{\int -\delta\Sigma_{rem}(r,\omega)\left[\phi_1^+(r)\phi_1(r) - \phi_2^+(r)\phi_1(r)\right]dr}{\int -\left[\phi_1^+(r)\phi_1(r) - \phi_2^+(r)\phi_1(r)\right]dr} = \frac{\int \delta\Sigma_{rem}(r,\omega)w(r)dr}{\int w(r)dr} \tag{9}$$

3.2 Derivation of the MTC noise estimators in the 2-D 2-group approximation

Two MTC noise estimators[2] have to be calculated. The first one, given by Eq. (10) below, represents the one that was used in all the experimental work so far. It is based on the assumption that the temperature noise is spatially homogeneous throughout the core, and also that therefore the reactor behaves in a point-kinetic way.

$$H_1^{biased}(r,\omega) = \frac{1}{G_0(\omega)\phi(r)}\frac{CPSD_{\delta\phi,\delta T_m}(r,\omega)}{APSD_{\delta T_m}(r,\omega)}$$

$$= \frac{1}{G_0(\omega)K\left[\phi_1(r) + \phi_2(r)\right]}\frac{\int\left[G_{\delta\Sigma_{rem}\to1}(r,r',\omega) + G_{\delta\Sigma_{rem}\to2}(r,r',\omega)\right]CPSD_{\delta\Sigma_{rem}}(r',r,\omega)dr'}{APSD_{\delta\Sigma_{rem}}(r,\omega)} \tag{10}$$

This formula, in which **r** represents the common location of temperature and flux measurements, was obtained by using Eq. (1) and the Wiener-Khinchin theorem. The space-dependent flux noise was replaced by the spatial integral of the product between the Green's function and the noise source.

The second MTC noise estimator that is worth calculating is the one given by the following Eq. (11). In this estimator, the reactor is still assumed to behave in a point-kinetic way, but the spatial heterogeneity of the temperature noise is taken into account.

_____

[1] These simulators use a spatial discretization scheme compatible with the neutron noise simulator, and calculate the direct and adjoint fluxes and the corresponding eigenvalues.
[2] In this study, the MTC noise estimators rely on the contribution of both the fast and thermal fluxes, whereas it is more likely that in-core neutron detectors are only sensitive to the epithermal flux.

$$\tilde{H}_1^{biased}\left(\boldsymbol{r},\omega\right) = \frac{1}{G_0\left(\omega\right)\phi\left(\boldsymbol{r}\right)} \frac{CPSD_{\delta\phi,\delta T_m^{ave}}\left(\boldsymbol{r},\omega\right)}{APSD_{\delta T_m^{ave}}\left(\omega\right)}$$

$$= \frac{\int w\left(\boldsymbol{r}\right)d\boldsymbol{r}}{G_0\left(\omega\right)K\left[\phi_1\left(\boldsymbol{r}\right)+\phi_2\left(\boldsymbol{r}\right)\right]} \frac{\iint\left[G_{\delta\Sigma_{rem}\rightarrow1}\left(\boldsymbol{r},\boldsymbol{r}',\omega\right)+G_{\delta\Sigma_{rem}\rightarrow2}\left(\boldsymbol{r},\boldsymbol{r}',\omega\right)\right]CPSD_{\delta\Sigma_{rem}}\left(\boldsymbol{r}',\boldsymbol{r}'',\omega\right)w\left(\boldsymbol{r}''\right)d\boldsymbol{r}'d\boldsymbol{r}''}{\iint CPSD_{\delta\Sigma_{rem}}\left(\boldsymbol{r}',\boldsymbol{r}'',\omega\right)w\left(\boldsymbol{r}'\right)w\left(\boldsymbol{r}''\right)d\boldsymbol{r}'d\boldsymbol{r}''} \tag{11}$$

This formula, in which $\boldsymbol{r}$ represents the location of the flux measurement only, was obtained by using Eqs. (9) and (1), and the Wiener-Khinchin theorem. As before, the space-dependent flux noise was replaced by the spatial integral of the product between the Green's function and the noise source.

4. RESULTS

The ratios between the traditional $H_1^{biased}\left(\boldsymbol{r},\omega\right)$ MTC noise estimator and the true MTC, and between the new $\tilde{H}_1^{biased}\left(\boldsymbol{r},\omega\right)$ MTC noise estimator and the true MTC, obtained by dividing Eq. (10) by Eq. (7) and Eq. (11) by Eq. (7) respectively, are plotted in Fig. 1. In this Figure, the horizontal coordinates represent either the common location of temperature and flux measurements or the location of the flux measurement only, for the $H_1^{biased}\left(\boldsymbol{r},\omega\right)$ and $\tilde{H}_1^{biased}\left(\boldsymbol{r},\omega\right)$ MTC noise estimators respectively. All the calculations were performed at a frequency of 1 Hz.
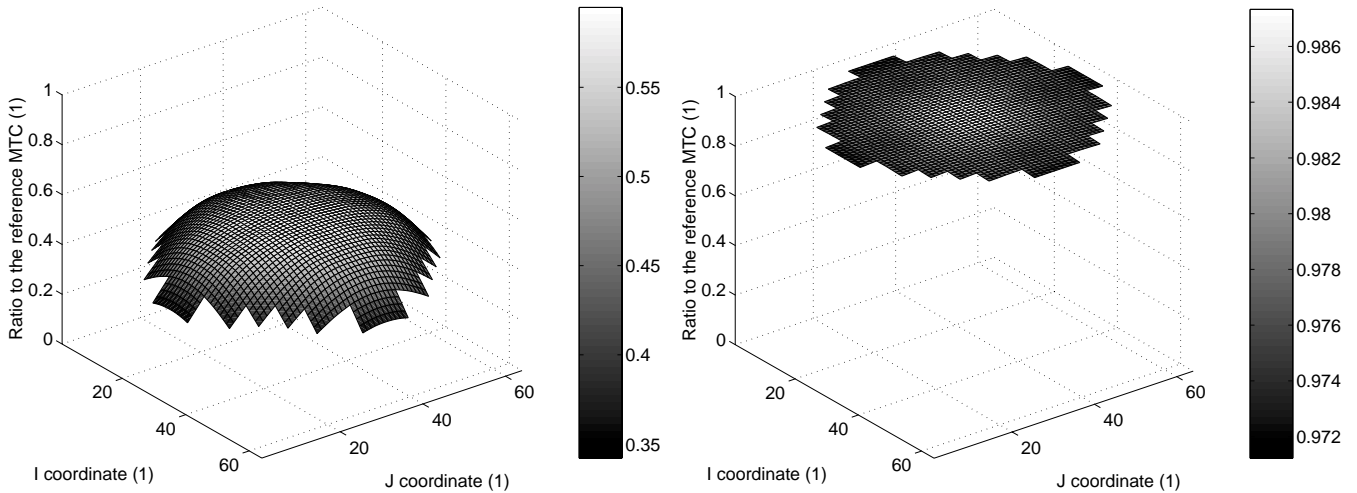

Fig. 1. Ratios between $H_1^{biased}\left(\boldsymbol{r},\omega\right)$ and the actual MTC value (left hand figure), and between $\tilde{H}_1^{biased}\left(\boldsymbol{r},\omega\right)$ and the actual MTC value (right hand figure)

As can be seen in these Figures, the new $\tilde{H}_1^{biased}\left(\boldsymbol{r},\omega\right)$ MTC noise estimator always correctly estimates the actual value of the MTC, whatever the location of the measurement of the neutron noise is. Since this noise estimator still relies on a point-kinetic behaviour of the reactor, this suggests that the deviation of the reactor response from point-kinetics is negligible for a perturbation defined by Eq. (2) with respect to the MTC determination. Consequently, measuring the total flux noise instead of only its point-kinetic component does not seem to affect significantly the accuracy of the noise analysis technique (as can be seen on the Figures, the discrepancy due to the fact that the reactor does not behave perfectly in a point-kinetic manner is less than 3%).

In contrast to the new MTC noise estimator, the traditional $H_1^{biased}\left(\boldsymbol{r},\omega\right)$ noise estimator is systematically biased low compared to the actual MTC value. Other simulations showed that the smaller the correlation length of the temperature noise is, the bigger the discrepancy is. This traditional MTC noise estimator is also strongly space-dependent. Compared to the previous 1-D study in a homogeneous reactor (see

Demazière and Pázsit (2002a)), the underestimation seems to be larger in 2-D than in 1-D for the same correlation length. Furthermore, the discrepancy in the 2-D case seems to be also more homogeneous than in the 1-D case. If one tries to relate these theoretical investigations to the experimental MTC noise studies performed so far, it seems that in the centre of the core a realistic correlation length seems to be around 100 - 150 cm in the 2-D system, whereas it is more likely that it is shorter in the 1-D system. It has to be pointed out that a radial correlation length of 100 – 150 cm seems to be longer than one would expect in a PWR core.

5. CONCLUSIONS

In this study, the effect of a non-homogeneous distribution of the moderator temperature noise on the MTC estimation by noise analysis was investigated. All the models relied on the 2-group diffusion approximation, and realistic data corresponding to a commercial reactor were axially condensed in 2-D.

It was found that the main reason why the traditional MTC noise estimator systematically underestimates the actual value of the MTC lies with the fact that the temperature noise might be radially heterogeneous, whereas this traditional MTC estimator only uses the temperature noise at the same radial location as the neutron noise. Another noise estimator (already proposed in Demazière and Pázsit (2001)) was tested and was proven to always give the correct MTC value within an accuracy of 3% of the design-predicted MTC value. This slight discrepancy results from the deviation of the reactor response from point-kinetics, an approximation on which this new MTC noise estimator still relies. The main difference between the new and traditional MTC noise estimators is that the new one takes the heterogeneous structure of the temperature noise into account.

Although the theoretical estimation of the new MTC noise estimator is based on many calculations, this MTC noise estimator can be easily used in practice since only an in-core neutron detector and as many as possible in-core thermocouples are required, for the flux noise measurement and the core average moderator temperature noise measurement respectively. The only calculation that is necessary to perform in order to be able to estimate the MTC is, as usual, the zero-power reactor transfer function $G_0(\omega)$, which in the frequency band of interest for the MTC measurement by noise analysis, i.e. 0.1 – 1.0 Hz, is very well approximated by $1/\beta_{eff}$. Regarding the core average moderator temperature noise, a proper weighting function has to be used.

Such a weighting function was tested since a noise measurement was recently performed in Ringhals-2, measurement in which 12+10 Gamma-Thermometers (GTs) were used together with a couple of in-core neutron detectors and a core-exit thermocouple. It was proven that GTs are actually working as ordinary thermocouples in the frequency range of interest for the MTC investigation by noise analysis (Demazière and Pázsit, 2002b), (Tosi and Haaland, 1993), (Haaland *et al*, 1991). Therefore, these GTs offer a unique opportunity to test this new MTC noise estimator, and to compare it to the traditional one. If the heterogeneous structure of the temperature noise is actually responsible for the underestimation of the MTC noise estimation via the traditional noise estimator, then the new one should give the correct MTC value. The Ringhals measurement and its analysis are reported in further detail in Demazière and Pázsit (2002c).

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8. NOMENCLATURE

APSD Auto-Power Spectral Density
CPSD Cross-Power Spectral Density
GT Gamma-Thermometer
MOX Mixed Oxide
PWR Pressurised Water Reactor
MTC Moderator Temperature Coefficient

PAPER VII

On-Line Determination of the MTC (Moderator Temperature Coefficient) by Neutron Noise and Gamma-Thermometer Signals

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The estimation of the Moderator Temperature Coefficient of reactivity (MTC) by noise analysis is investigated theoretically. It is shown that the main reason of the MTC underestimation by noise analysis that was noticed experimentally previously lies with the heterogeneous structure of the moderator temperature noise throughout the core. The resulting deviation of the reactor response from point-kinetics only accounts for a negligible part. Therefore a new MTC noise estimator is proposed. This estimator relies on the core average temperature noise that has to be evaluated with a proper core weighting function. The coolant temperature noise has to be measured in many points of the reactor so that the core average temperature noise could be approximated. Gamma-Thermometers (GTs) could be used for that purpose since it is demonstrated (both theoretically and via real measurements) that in the frequency range of interest for the MTC estimation they work as thermocouples.

Introduction

The Moderator Temperature Coefficient of reactivity (MTC) is an important safety parameter in Pressurized Water Reactors (PWRs). Defined as the variation of reactivity induced by a change in the coolant temperature and divided by the core-averaged temperature change [1], the MTC plays a major role in the feedback mechanism of the reactor and has to be negative so that the reactor is in all circumstances stable[1]. Due to the decrease of the boron content through the fuel

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[1] The feedback mechanism in a PWR relies mainly on the Doppler and MTC effects. Therefore, a slightly positive MTC could be permitted if the MTC combined with the Doppler coefficient provides an overall negative reactivity feedback. Due to the economical advantages of high burnup fuel, e.g., longer fuel cycle (fuel which has a positive contribution to the MTC), power utilities are trying nowadays to get licensed for a slightly positive MTC at Beginning Of Cycle (BOC).

cycle (as a consequence of the decrease of the reactivity excess due to the fuel burnup), the magnitude of the MTC increases. Consequently, the MTC is usually measured at Beginning Of Cycle (BOC) and Hot Zero Power (HZP) to verify that the MTC is negative (therefore preventing from the consequences of a power increase). The MTC has also to be estimated a few months before the expected End Of Cycle (EOC), to verify that the MTC is not too negative (thus preventing from the consequences of a cool-down event). This second measurement is performed at Hot Full Power (HFP) and indicates if the reactor can be operated until its pre-calculated EOC.

Whereas the BOC measurement can be considered as accurate (use of a reactivity meter and homogeneous temperature distribution throughout the core), the EOC measurement has been proven to be less reliable for many different reasons. The most important one is probably the fact that the procedure used to perform the EOC measurement takes some time during which some reactivity changes due to effects other than the MTC take place and which cannot be measured directly [4]. This compromises somehow the experimental technique since the main goal of the measurement is to verify that core calculations predict the EOC MTC accurately. Therefore the experimental technique should rely on as few as possible core calculated parameters. The other major concern for power utilities is the transient induced by the traditional measurement techniques such as the boron dilution method. This transient has to be monitored for 12 to 24 hours, and generally requires operating the reactor at a lower power level.

Another measurement technique was proposed some years ago to monitor the MTC without perturbing the reactor [21, 25]. This method relies on the correlation between the neutron and temperature noise signals. It is known that even if a system is at its steady state, small fluctuations occur around its mean value. These small fluctuations are representative of the dynamics of the system. Therefore monitoring the neutron and temperature noise should allow estimating the MTC while the reactor is at its steady state, thus avoiding the problems of the traditional measurement techniques. Several attempts to monitor the MTC via noise analysis were made in the past [1, 9, 11, 13-17, 19-23, 25-30]. Unfortunately, all of them revealed that the MTC estimated via the noise technique systematically underestimated the actual value of the MTC by a factor two to five. It was even noticed that this calibration factor seems to be constant throughout the fuel cycle and even between different fuel cycles as long as the same pair of neutron and temperature detectors are used.

Recent measurements of the amplitude of the temperature noise at the core exit revealed that the temperature noise seems to be radially non-homogeneous [12]. It was even noticed that the cross-correlation between two temperature detectors located inside the core on a same axial plane was strongly decreasing with the distance between the two detectors [7]. As will be explained later, the MTC noise estimate was derived in the hypothesis of spatially homogeneous temperature noise throughout the core. Therefore the effect of non-homogeneous temperature noise on the MTC was investigated in the following via an analytical one-dimensional model, which revealed that the discrepancy between the MTC noise estimate and its actual value seemed to be mostly due to the radial heterogeneous

structure of the temperature noise throughout the core. A new MTC noise estimator allowing taking this effect into account was thus proposed. This new estimator implicitly assumes that the core average temperature noise could be estimated. This requires being able to measure the temperature noise throughout the core. As will be demonstrated in the following, Gamma-Thermometers (GTs) could offer such a possibility since in the frequency range of interest for the MTC determination via noise analysis they work as temperature detectors.

A nomenclature explaining all the abbreviations used can be found at the end of this chapter.

2 MTC Estimation by Noise Analysis

In this section, the MTC definition and its noise estimator are recalled. Emphasis is put on the hypotheses used to derive the MTC noise estimator. The effect of these hypotheses on the accuracy of the MTC estimation is then quantitatively evaluated via a one-dimensional model and some conclusions are drawn.

2.1 MTC definition

According to the newest American Standard, the Moderator Temperature Coefficient (MTC) of reactivity is defined as the variation of the reactivity $\delta\rho$ induced by a change of the core inlet temperature of the coolant, divided by the coolant average temperature change δT_m^{ave} [3]:

$$\delta\rho(t) = MTC \times \delta T_m^{ave}(t) \tag{1}$$

where t represents the time.

If the temperature fluctuations are assumed to be proportional to the fluctuations of the macroscopic cross-sections via a time- and space-independent coefficient K as follows:

$$\delta T_m(\boldsymbol{r},t) = K \times \delta\Sigma_a(\boldsymbol{r},t) \tag{2}$$

with $\boldsymbol{r}$ representing the position, then the first-order perturbation theory allows calculating the average temperature change as:

$$\delta T_m^{ave}(t) = \frac{\int \delta T_m(\boldsymbol{r},t)\phi_0^2(\boldsymbol{r})d\boldsymbol{r}}{\int \phi_0^2(\boldsymbol{r})d\boldsymbol{r}} \tag{3}$$

where ϕ_0 is the static flux.

2.2 MTC Noise Estimator

As explained in [4], there are several noise estimators. When the coherence between the measured temperature noise and the measured average temperature noise equals unity, all the noise estimators coincide. The most commonly used noise estimator (the so-called H_1 estimator) is given in the frequency domain by:

$$MTC = \frac{CPSD_{\delta\rho,\delta T_m^{ave}}(\omega)}{APSD_{\delta T_m^{ave}}(\omega)} \tag{4}$$

where the *APSD* and the *CPSD* stand for the Auto-Power Spectral Density and Cross-Power Spectral Density respectively. Even if the right-hand side of Eq. (4) depends on the frequency $f = \omega/2\pi$, this dependence cancels out and the MTC is frequency-independent. The problem is that neither the reactivity noise nor the average temperature noise are measurable quantities.

The reactivity noise can be nevertheless inferred from the following relationship:

$$\delta\rho(\omega) = \frac{1}{G(\omega)} \frac{\delta\phi^{pk}(r,\omega)}{\phi_0(r)} \tag{5}$$

where G is the at-power reactor transfer function and $\delta\phi^{pk}$ is the point-kinetic component of the flux noise. Eq. (5) was derived from the factorization of the flux into an amplitude function P and a shape function Ψ as:

$$\phi(r,t) = P(t)\Psi(r,t) \tag{6}$$

which leads to the following expression of the flux noise in the frequency domain (second-order terms neglected):

$$\delta\phi(r,\omega) = \delta\phi^{pk}(r,\omega) + \delta\psi(r,\omega) \tag{7}$$

where

$$\delta\phi^{pk}(r,\omega) = \phi_0(r)\delta P(\omega) \tag{8}$$

and from the following definition of the at-power reactor transfer function:

$$G(\omega) = \frac{\delta P(\omega)}{\delta\rho(\omega)} \tag{9}$$

There are nevertheless two main problems if one wants to use Eq. (5) for evaluating the reactivity noise. The first one is due to the use of the at-power reactor transfer function, since this transfer function contains the reactivity feedback mechanism and more specifically the MTC. Consequently, this transfer function is unknown. If one considers high frequencies, typically frequencies higher than 0.1Hz, one may assume that the feedback mechanism does not take place due to the relatively large time constant of the power-to-coolant temperature

transfer function, i.e., one is interested in frequencies for which the reactor responds before the feedback begins to act. Therefore, the at-power (or closed-loop) reactor transfer function G can be replaced in that case by the open-loop or zero-power reactor transfer function G_0:

$$G(\omega) \approx G_0(\omega) \tag{10}$$

If one also neglects frequencies higher than 1Hz (this upper cut-off frequency has also the advantage of eliminating the effect of the damping of the fluctuations traveling from the neutron to the temperature detectors), the zero-power reactor transfer function further simplifies into:

$$G_0(\omega) \approx \frac{1}{\beta} \tag{11}$$

where β is the fraction of delayed neutrons which can accurately be estimated by core calculations.

So far, narrowing the frequency region of interest to the frequency band 0.1 to 1.0Hz allowed replacing the reactivity noise $\delta\rho$ by the amplitude function noise δP via Eqs. (9), (10), and (11), i.e., known quantities and parameters without any approximation. As can be seen from Eqs. (7) and (8), the amplitude function noise cannot be expressed as a function of the total flux noise $\delta\phi$ solely, rather as a function of the point-kinetic component of the flux noise $\delta\phi^{pk}$. Unfortunately, this component is not measurable in practice since neutron detectors are sensitive to the total flux noise $\delta\phi$. Therefore Eq. (5) cannot be used to estimate the reactivity noise, unless one approximates the point-kinetic component of the flux noise by the total flux noise. This is equivalent to neglecting the contribution due to the amplitude function noise $\delta\Psi$, i.e., to assuming that the reactor behaves in a point-kinetic way since one has:

$$
\begin{aligned}
\frac{1}{G(\omega)} \frac{\delta\psi(r,\omega)}{\phi_0(r)} &= \frac{1}{G(\omega)} \frac{\delta\phi(r,\omega)}{\phi_0(r)} - \frac{1}{G(\omega)} \frac{\delta\phi^{pk}(r,\omega)}{\phi_0(r)} \\
&= \frac{1}{G(\omega)} \frac{\delta\phi(r,\omega)}{\phi_0(r)} - \delta\rho(\omega)
\end{aligned} \tag{12}
$$

In summary, the reactivity noise can only be approximated by the following expression in the frequency range 0.1 to 1.0Hz:

$$
\begin{aligned}
\delta\rho^{biased}(r,\omega) &= \frac{1}{G_0(\omega)} \frac{\delta\phi(r,\omega)}{\phi_0(r)} \\
&\neq \delta\rho(\omega) = \frac{1}{G_0(\omega)} \frac{\delta\phi^{pk}(r,\omega)}{\phi_0(r)}
\end{aligned} \tag{13}
$$

This reactivity noise is only biased by the deviation of the reactor response from point-kinetics, i.e., by the fact that one uses $\delta\phi$ (measurable) instead of $\delta\phi^{pk}$ (non-measurable). It will be shown below that this bias is not significant.

Regarding the estimation of the temperature noise now, the main problem pertains to the fact that the core average temperature noise should be estimated. Unfortunately, all the commercial PWRs are badly instrumented with respect to temperature detectors. Usually, only core-inlet, core-outlet, and core-exit thermocouples are available. In all the previous experimental investigations, a core-exit thermocouple was used to estimate the MTC by noise analysis [1, 9, 11, 13-17, 19-23, 25-30]. In most cases, the thermocouple was located within the same fuel channel where the neutron noise was measured, but in some cases the two detectors were located in two neighboring fuel channels. Using one core-exit temperature instead of the core average temperature for evaluating the temperature noise is equivalent to assuming that the temperature noise is spatially homogeneous throughout the core.

Another problem related to the temperature noise is the fact that usually the local temperature noise is not measured at the same axial location as the neutron noise. In all the previous experimental work, the temperature was measured at the core-exit. If one assumes that the fluctuations travel upwards unperturbed from the in-core neutron detector to the core-exit thermocouple (no noise source between the two detectors), the only effect that takes place is a damping for frequencies higher than the frequency corresponding to the transit time between the two detectors, i.e.. typically 1.0Hz. Consequently, the higher cut-off frequency allows both eliminating the damping effect and simplifying the zero-power reactor transfer function into a very simple expression (see Eq. (11) above).

In summary, the average temperature can only be approximated by the following expression for frequencies lower than 1.0Hz:

$$\delta T_m^{ave,biased}(\boldsymbol{r},\omega) = \delta T_m(\boldsymbol{r},\omega)$$

$$\neq \delta T_m^{ave}(\omega) = \frac{\int \delta T_m^{ave}(\boldsymbol{r},\omega)\phi_0^2(\boldsymbol{r})d\boldsymbol{r}}{\int \phi_0^2(\boldsymbol{r})d\boldsymbol{r}} \qquad (14)$$

As a matter of fact, the MTC noise estimators were all derived previously on the assumption that the temperature noise was spatially homogeneous. In that case, the reactor necessarily behaves in a point-kinetic manner [5, 8]. This means that both Eqs. (13) and (14) are fulfilled, i.e., there is no bias in the estimation of the average temperature noise from the local temperature noise and no bias in the estimation of the reactivity noise from the total flux noise.

2.3 Study of the Approximations Used in the Noise Estimation on the MTC

The two previous biases, namely the deviation from point-kinetics of the reactor response and the non-homogeneity of the temperature noise throughout the core, were investigated via an analytical model of a one-dimensional bare reactor with a core radius a. Only the radial direction was taken into account since it is known that the effect of radially inhomogeneous temperature noise sources give stronger effects than the axial ones.

This analytical model relies on the one-group diffusion approximation. This model is described in details in [5, 8]. Only the main characteristics of this model are recalled here. As described by Eq. (1), the temperature noise sources are space-dependent and defined from the fluctuations of the macroscopic absorption cross-section. Furthermore, the noise sources are not given in a deterministic manner, rather via their statistical properties such as the spatial cross-correlation. Their *CPSD* is then directly factorized into an amplitude function σ^2, which corresponds to the space dependence of the noise source strength (or more simply their *APSD*) and an exponential decay function, which simply states that the correlation between two points decreases when the distance between these two points increases. How rapidly this exponential decay function decreases with increasing distances is related to a parameter l, called in the following the correlation length. Regarding the frequency content, the noise sources are assumed to be white noise. Consequently, their *CPSD* and *APSD* are frequency independent. In formula, this can be written as:

$$CPSD_{\delta\Sigma_a}\left(r,r',\omega\right) = \sigma^2\left(\hat{r}\right)e^{-\frac{|r-r'|}{l}} \qquad (15)$$

with

$$\hat{r} \equiv \frac{r+r'}{2} \qquad (16)$$

and

$$\sigma^2\left(\hat{r}\right)\Big|_{r=r'} = APSD_{\delta\Sigma_a}\left(r,\omega\right) \qquad (17)$$

Many different parameters were investigated in [5, 8]. In the following, a set of parameters corresponding to a realistic case is presented. More specifically, the cross-sections are representative of a PWR core at EOC, the correlation length l was chosen to be equal to 15cm (which seems to correspond roughly to what was experimentally noticed in [7]), and the calculations were performed at a frequency of 1Hz. Regarding the amplitude or shape function, the following one was studied:

$$\sigma\left(\hat{x}\right) = \left[1-\left(\frac{\hat{x}}{a+\delta a}\right)^2\right]^{-1} \qquad (18)$$

with $\delta a = a/5$. This shape function somehow expresses the fact that the temperature noise is larger close to the core boundary than at the core center and corresponds therefore to what was experimentally noticed [12].

From Eqs. (1) - (3), it is straightforward to calculate the actual value of the MTC via the first-order perturbation theory and one finds:

$$MTC = -\frac{1}{K\nu\Sigma_{f,0}} \tag{19}$$

where $\nu\Sigma_{f,0}$ represents the static macroscopic fission cross-section multiplied by the average number of neutrons per fission. The MTC can be correctly estimated by the following ideal MTC noise estimator in the frequency range 0.1 to 1.0Hz:

$$H_1^{ideal} = \frac{CPSD_{\delta\rho,\delta T_m^{ave}}(\omega)}{APSD_{\delta T_m^{ave}}(\omega)} = \frac{1}{G_0(\omega)}\frac{CPSD_{\delta\phi^{pk}/\phi_0,\delta T_m^{ave}}(\omega)}{APSD_{\delta T_m^{ave}}(\omega)} \tag{20}$$

As stated before, neither the reactivity noise (or the point-kinetic component of the flux noise) nor the core average temperature noise can be directly estimated.

If one assumes that the core average temperature noise can be measured, one can estimate the discrepancy introduced by the fact that the reactor does not behave in a point-kinetic way. This is accomplished by using in Eq. (4) $\delta\rho^{biased}$ instead of $\delta\rho$ (see Eq. (13)). This leads to the following biased MTC noise estimator:

$$\tilde{H}_1^{biased}(r,\omega) = \frac{CPSD_{\delta\rho^{biased},\delta T_m^{ave}}(r,\omega)}{APSD_{\delta T_m^{ave}}(\omega)} = \frac{1}{G_0(\omega)}\frac{CPSD_{\delta\phi/\phi_0,\delta T_m^{ave}}(r,\omega)}{APSD_{\delta T_m^{ave}}(\omega)} \tag{21}$$

The ratio between this estimator and the actual MTC value allows appreciating the error introduced by the deviation of the reactor response from point-kinetics. The space-dependence of this ratio is plotted in Fig. 1. As can be seen, this MTC noise estimator gives a relatively good estimation of the actual value of the MTC. Consequently, the deviation of the reactor response from point-kinetics does not play a significant role regarding the MTC estimation. One can therefore consider that approximating the reactivity noise $\delta\rho(\omega)$ by $\delta\rho^{biased}(r,\omega)$ does not induce any appreciable discrepancy in the MTC estimation.

Using this valid approximation, one can then study the effect of using the local temperature noise instead of the core average temperature noise. This is accomplished by using in Eq. (4) $\delta T_m^{ave,biased}$ instead of δT_m^{ave} (see Eq. (14)). This leads to the following MTC noise estimator:

$$H_1^{biased}(\mathbf{r},\omega) = \frac{CPSD_{\delta\rho^{biased},\delta T_m^{ave,biased}}(\mathbf{r},\omega)}{APSD_{\delta T_m^{ave,biased}}(\mathbf{r},\omega)} = \frac{1}{G_0(\omega)}\frac{CPSD_{\delta\phi/\phi_0,\delta T_m}(\mathbf{r},\omega)}{APSD_{\delta T_m}(\mathbf{r},\omega)} \quad (22)$$

This noise estimator corresponds to the one that was used so far in all the previous studies and contains the effect of both the deviation from point-kinetics of the reactor response and the approximation of the core average temperature noise by the local temperature noise. The ratio between this noise estimator and the actual MTC value is plotted in Fig. 2. As can be seen, there is a very strong underestimation of the actual value of the MTC, and this underestimation is noticeably space-dependent. Close to the core center, the MTC is roughly underestimated by a factor five, which corresponds to what was noticed in the experimental investigations.

Consequently, it seems that the huge discrepancy between the traditional MTC noise estimator H_1^{biased} and its actual value is due to the fact that the temperature noise is not spatially homogeneous rather than due to the deviation of the reactor behavior from point-kinetics. Consequently, estimating the average temperature noise and using instead the $\tilde{H}_1^{biased}$ noise estimator would allow getting a fairly good estimation of the actual MTC.
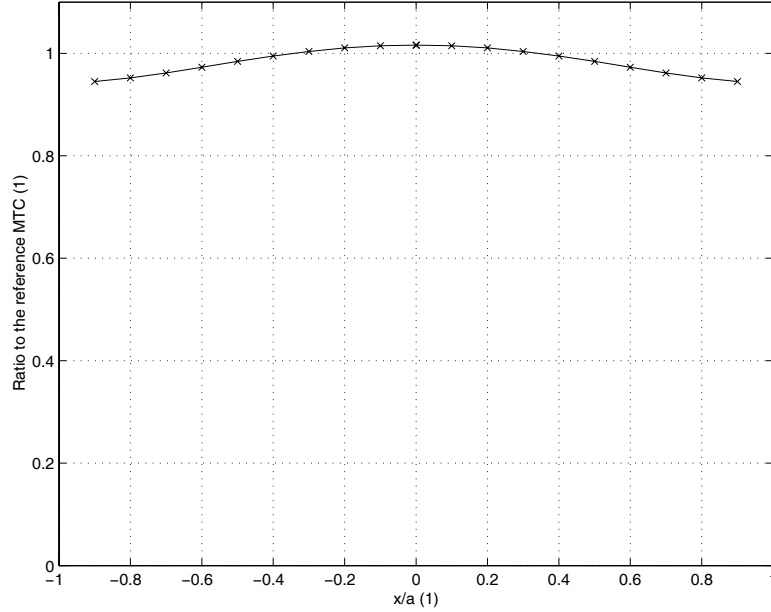
Fig. 1. Discrepancy between the MTC noise estimate $\tilde{H}_1^{biased}$ and its actual value (due to the deviation from point-kinetics of the reactor response solely)
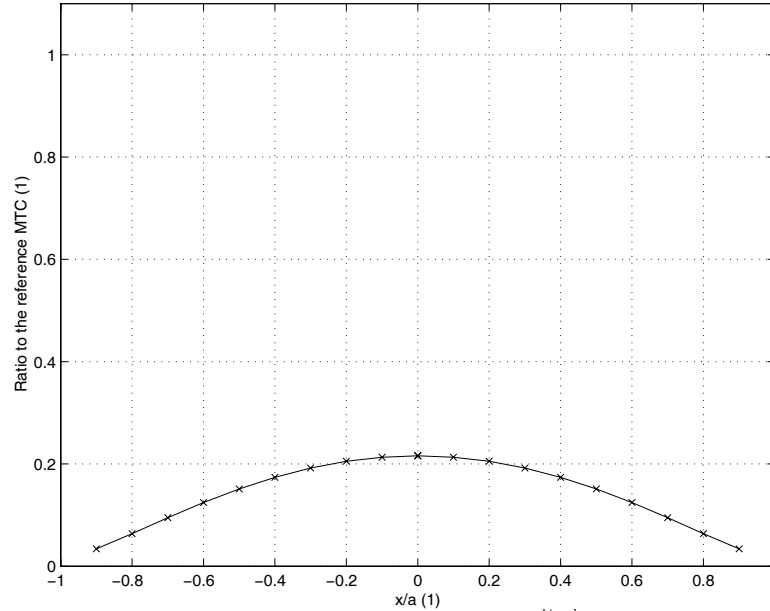
Fig. 2. Discrepancy between the MTC noise estimate H_1^{biased} and its actual value (due to the deviation from point-kinetics of the reactor response *and* the approximation of the average temperature noise by the local one)

3 Measurement of the Temperature Noise via Gamma-Thermometers

Estimating the core average temperature noise can be done by measuring the temperature noise in several points of the reactor core and by using an integration formula so that the integrals in Eq. (3) can be numerically approximated. This requires also knowing the static flux at the corresponding locations. Any static core calculator could be used for that purpose since it is well known that the static flux is nowadays accurately estimated by these codes. As pointed out previously, commercial PWRs are badly instrumented with respect to in-core temperature detectors. Nevertheless the Swedish PWR Ringhals-2 contains 108 permanent Gamma-Thermometers (GTs) installed in the core (12 strings containing each 9 GTs located at different axial levels). It was thus proposed to study the possibility of using GTs for measuring the temperature noise throughout the core. In a different setting, the use of GTs for measuring temperature noise has already been proven in the Halden Reactor Project [10, 28]. Our purpose in this work is to perform the same investigation regarding measurements taken at an operating power plant.

3.1 Description of the Halden Type Gamma-Thermometers

There are several types of GTs. The Ringhals-2 unit is instrumented with the so-called Halden type GTs [2]. In this design, a metallic pin is heated up by the gamma flux mostly (elastic and inelastic collisions) and to a lesser extent by the neutron flux ((n, γ), (n, α), (n, p) reactions, elastic and inelastic collisions). The pin is encapsulated into an outer body or housing, and the thermal insulation between the pin and the outer body is made by some xenon gas. One of the tips of the pin is therefore thermally insulated, whereas the other tip is in direct contact with the body of the GT. A differential chromel/alumel thermocouple measures the temperature drop along the pin with the hot junction located at the insulated tip, and the cold junction in direct contact with the coolant outside the GT body. Fig. 3 gives an overview of the Halden type GT. Even if GTs are designed to measure the gamma flux (or more practically the power), the cold junction of the differential thermocouple is in direct contact with the coolant and could be therefore used for monitoring the moderator temperature fluctuations.
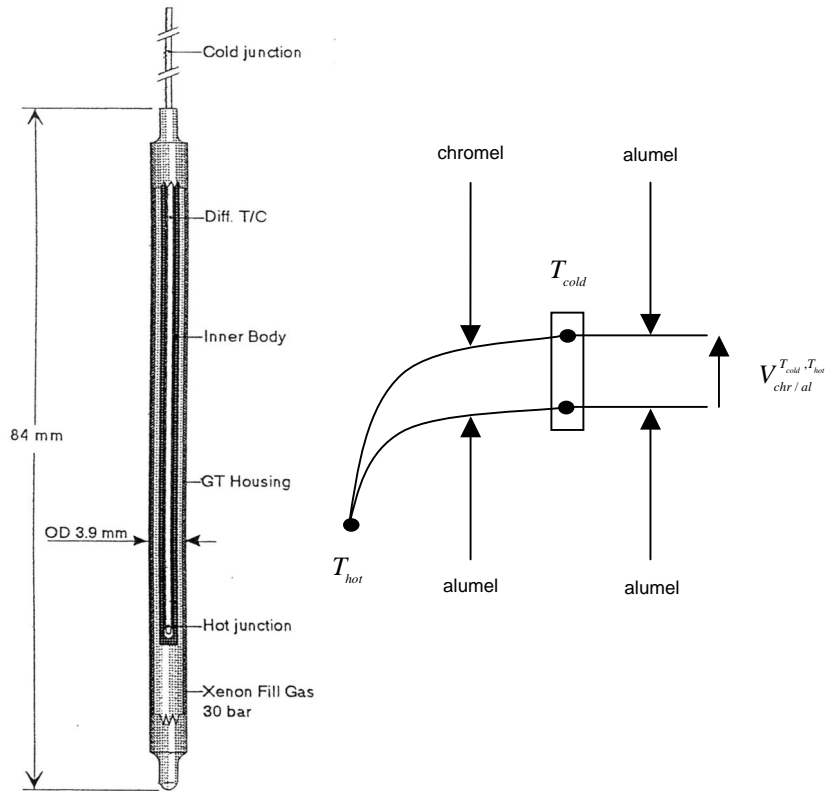

Fig. 3. Sketch of the Halden type GTs (from [10]) on the left, and the corresponding schematic description of the differential thermocouple on the right

3.2 Use of Gamma-Thermometers for Noise Analysis

As pointed out previously, the hot junction of the thermocouple measures the temperature increase due to the heating along the pin with the cold junction acting as a heat sink and which is at the same temperature as the coolant. In case of static measurements, the voltage delivered by the differential thermocouple is therefore directly proportional to the heating Q according to the following formula (in which the heat loss across the gas chamber and the radial conduction across the inner body are neglected):

$$V_{chr/al}^{T_{cold} \cdot T_{hot}} = \alpha_{chr/al} \times \left(T_{hot} - T_{cold}\right) = \alpha_{chr/al} \times \frac{QL^2}{2\lambda} \tag{23}$$

where $V_{chr/al}^{T_{cold} \cdot T_{hot}}$, $\alpha_{chr/al}$, L, λ, and Q are the voltage, the thermocouple constant (typically approximately $41\mu V/°C$ for a K type thermocouple, i.e., chromel/alumel), the separation distance between the hot and cold junctions, the thermal conductivity of the inner body, and the heat flux in the thermometer respectively.

In case of dynamical measurements, because of the different thermal characteristics of the cold and hot junctions, the thermocouple will respond differently, i.e., with different delays to phenomena associated to the hot junction and to phenomena associated to the cold junction. Since the cold junction is located above the GT body and is direct contact with the coolant, it responds very quickly to coolant temperature oscillations (with a thermal time constant typically around 0.1-1.0s). This means that the cold junction acts as a low-pass filter of the coolant temperature noise with a cut-off frequency of a few hertz. On the contrary, the hot junction is thermally insulated within the inner body and has therefore a significantly more sluggish response (with a thermal time constant typically around 10-100s). Consequently, the hot junction acts a low-pass filter of the gamma/neutron flux with a cut-off frequency of a few hundredth of hertz. If one writes the transfer function from the cold and hot junctions temperature noise ($\delta T_{cold} = \delta T_m$ and δT_{hot} respectively) to the GT signal in the frequency domain as follows:

$$\delta V_{chr/al}^{T_{cold} \cdot T_{hot}}\left(\omega\right) = \frac{\alpha_{chr/al}}{1 + j\tau_{hot}\omega} \times \delta T_{hot}\left(\omega\right) - \frac{\alpha_{chr/al}}{1 + j\tau_{cold}\omega} \times \delta T_m\left(\omega\right) \tag{24}$$

where the indices τ_{hot} and τ_{cold} refer to the time constants of the hot and cold junctions respectively, then one notices that in the frequency range of interest for the MTC estimation by noise analysis, i.e., typically from 0.1 to 1.0Hz only the signal due to the cold junction remains. The fluctuations due to the gamma/neutron heating are consequently completely filtered out by the hot junction. Only the very low frequencies contain the contribution of both the hot and cold junctions.

3.3 Checking of the GT Ability to Measure the Moderator Temperature Noise

In 2000, during fuel cycle 24, a noise measurement using GTs was performed at the Swedish Ringhals-2 PWR. The Halden type GTs were used for that purpose and three channels (J10, L11, and M03) were instrumented, as shown by the core map of Fig. 4. The detector string J10 is located close to the core center, the string M03 close to the core boundary, and the string L11 occupies an intermediate position. Each string contains nine GTs at different axial levels. For each detector string, all nine different axial levels were measured. The different levels are roughly equally spaced and cover the whole core active height. The numbering of the levels (1-9) is done from the bottom of the core active height to the top of the core active height. The duration of the measurement was almost three hours, and the signals were recorded at a sampling frequency of 8Hz. The time signals were then detrended (if a trend was found), the irregularities in the mean values eliminated by signal processing methods and data analysis was performed in the frequency domain [24]. In order to evaluate the *APSDs* and *CPSDs* of the different signals, Welch's averaged, modified periodogram method was used. The time signals were divided into overlapping sections of 256 points, then windowed by using a Hanning window. The sections were assumed to overlap by 128 points.
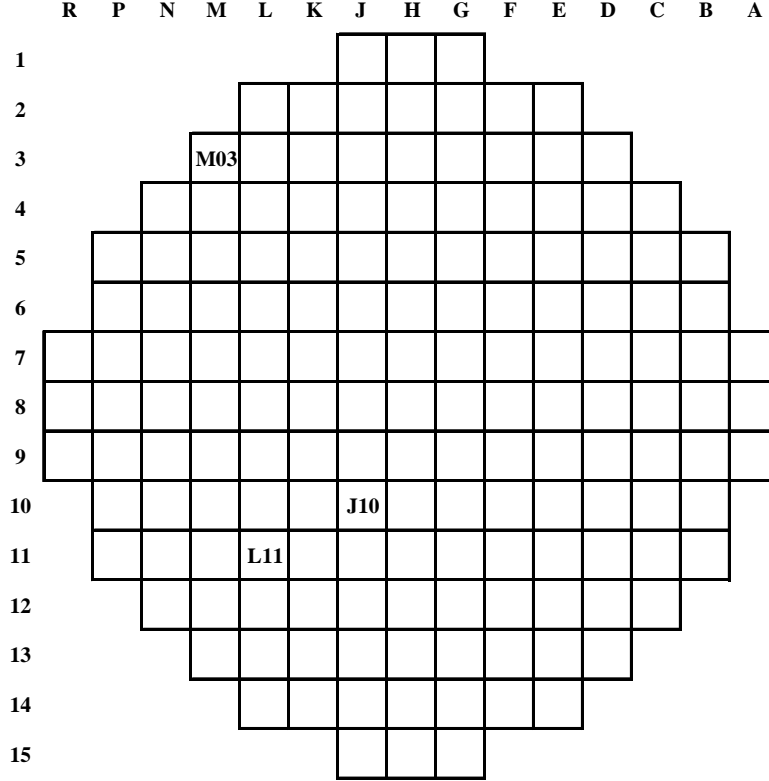
Fig. 4. Radial location of the GTs in Ringhals-2

The ability of the GTs to monitor the temperature fluctuations of the coolant can be first checked qualitatively by cross-correlating two GTs within the same fuel channel. If the GTs are sensitive to the temperature noise, then the temperature fluctuations recorded by the lowermost detector should also be present in the uppermost detector. Due to the separation distance between the two-detectors, the noise recorded by the uppermost one is simply time-shifted compared to the noise recorded by the lowermost one (if one assumes that no noise is added between the two detectors):

$$\delta V_{chr/al,uppermost}^{T_{cold},T_{hot}}(t) = \delta V_{chr/al,lowermost}^{T_{cold},T_{hot}}(t - t_0) \tag{25}$$

where t_0 is the transit time for the temperature fluctuations to travel upwards between the two detectors. In the frequency domain, this reads as:

$$\delta V_{chr/al,uppermost}^{T_{cold},T_{hot}}(\omega) = e^{-i\omega t_0} \delta V_{chr/al,lowermost}^{T_{cold},T_{hot}}(\omega) \tag{26}$$

The Wiener-Khinchin theorem allows then writing:

$$CPSD_{\delta V_{chr/al,uppermost}^{T_{cold},T_{hot}}, \delta V_{chr/al,lowermost}^{T_{cold},T_{hot}}}(\omega) \approx e^{-i\omega t_0} \left| \delta V_{chr/al,lowermost}^{T_{cold},T_{hot}}(\omega) \right|^2 \tag{27}$$

Consequently, in the frequency range where the GTs detect temperature fluctuations, the phase of the *CPSD* between the two detectors should be linear. This can be noticed in Fig. 5, where one possible pair of detectors within the fuel channel L11 was chosen as a representative example. This shows that GTs in the frequency range 0.1 to 1.0Hz are able to detect the moderator temperature fluctuations traveling upwards through the core. GTs were even used successfully to measure flow velocities within the core [7].
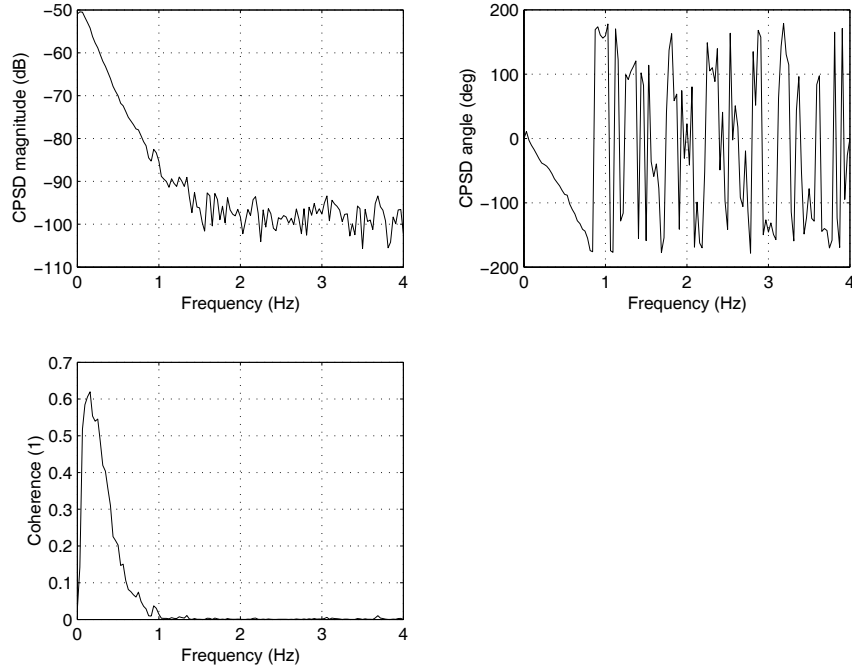
Fig. 5. *CPSD* of the detector string L11, between plane number 4 and plane number 9

Another possibility to check if GTs work as thermocouples in the frequency range of interest for the MTC estimation by noise analysis is to try to numerically estimate the transfer function of the GTs. This can be achieved by the so-called Auto-Regressive Moving Average (ARMA) modeling. If one assumes that the GTs measure a noise e, then an ARMA(n,m) model consists of trying to write the GTs transfer function as:

$$\sum_{k=0}^{n} a_k \delta V_{chr/al}^{T_{cold},T_{hot}}(t-kT) = \sum_{l=0}^{m} c_l e(t-lT) \tag{28}$$

where n and m are the order of the model, and T is the sampling interval. The a_k and c_l coefficients are estimated from the measured output signal $\delta V_{chr/al}^{T_{cold},T_{hot}}$ by

using a minimization procedure [18]. The noise e is still unknown and assumed to be completely uncorrelated, i.e., white.

Once the coefficients a_k and c_l are estimated, the time constants of the corresponding transfer function can be estimated since any real pole p of the transfer function defines a specific system dynamics first-order mode and can therefore be associated to a time constant τ as [31]:

$$\tau = -\frac{T}{\ln p} \tag{29}$$

The corresponding cut-off frequency is thus:

$$f = \frac{1}{2\pi\tau} \tag{30}$$

When comparing Eq. (28) with Eq. (24), it can be noticed that the noise e in Eq. (28) is a combination of both the cold and hot junction signals given in Eq. (24). This represents a simplification and an approximation to the actual GT transfer function. Nevertheless, in such a case an ARMA(2,1) model could suffice to estimate the two time constants. Simulations were therefore carried out to verify that the time constants given by the ARMA(2,1) model estimated from simulated data using Eq. (24) and two independent white noise sources for the cold junction and hot junction temperature noise were equal to the two actual time constants, i.e., the one corresponding to the cold junction and the one corresponding to the hot junction.

Due to the hardware/software processing of the signals such as the DC removal and the anti-aliasing filter, an ARMA(2,1) model was found to be in some cases unsatisfactory when applied to measured data. Simulations were then carried out in order to determine what order the ARMA model should be. It was assumed that both the DC removal and the anti-aliasing filter were equivalent to two second-order digital high-pass and low-pass filters respectively with two different cut-off frequencies. Depending on the ratio between the cut-off frequencies of the anti-aliasing filter and of the cold junction, and on the ratio between the cut-off frequencies of the DC remover and the hot junction, several cases can be encountered. The ARMA model can detect the anti-aliasing filter, and/or the cold junction, and/or the DC remover, and/or the hot junction (with *or* having a non-exclusive meaning). This corresponds to the following ARMA models: (1,0), (2,0), (2,1), (3,1), (3,2), (4,2), (4,3), (5,3), (6,4), and (6,5). As before, simulations were carried out in order to see if the ARMA models were able to correctly estimate the time constants, using Eq. (24) and two independent white noise sources for the cold junction and hot junction temperature noise. It was noticed that due to the presence of the DC remover and the anti-aliasing filter, the time constants were sometimes biased, but they still remain in the same range as their actual values.

The criteria for choosing the best ARMA model were thus the following [18]:

- the ARMA model must correspond to the hypotheses for which the model was developed, i.e. if the anti-aliasing filter, and/or the cold junction, and/or the DC remover, and/or the hot junction are assumed to be detected by the model, then their respective zeros/poles locations and values should match realistic cases;
- a zero/pole cancellation means that the model order can be reduced;
- any identical poles/zeros pattern between different ARMA models generally expresses the fact that this specific pattern probably contains the actual system dynamics;
- the fit between the measured data and the data simulated by the ARMA model should be as high as possible;
- the residuals, i.e. the part of the output that the model could not reproduce, should be as low as possible;
- the 99% confidence interval of the zeros and poles should not be too large.

The results of the ARMA modeling are summarized in Table 1, Table 2, and Table 3 for the detector strings J10, L11, and M03 respectively. In some occurrences, it was not possible to detect either the cold junction time constant, or the hot junction time constant, or both. As the simulation revealed, the removal of the DC component and the anti-aliasing filter could explain such a behavior. Other cases show that two time constants can clearly be determined. Even if it was noticed with the test cases that the numerical values of the estimated time constants could have somewhat large uncertainty, it seems that one of the time constant lies in the range 0.1 to 1.0s, and the other time constant lies in the range 2.0 to 20.0s. The first one corresponds therefore to the cold junction, whereas the second one corresponds to the hot junction.

Table 1. Results of the ARMA modeling for the detector string J10

Level	ARMA	Fit [%]	τ_{cold} [s]	f_{cold} [Hz]	τ_{hot} [s]	f_{hot} [Hz]
1	(2,1)	57.6	0.089	1.780	17.113	0.009
2	(2,1)	72.6	-	-	-	-
3	(1,0)	55.4	-	-	14.323	0.011
4	(2,1)	75.7	-	-	-	-
5	(2,1)	58.6	-	-	-	-
6	(3,1)	71.3	0.471	0.338	-	-
7	(3,1)	90.2	0.029	5.413	-	-
8	(5,3)	82.9	0.290	0.549	-	-
9	(5,3)	83.9	0.492	0.324	-	-

Table 2. Results of the ARMA modeling for the detector string L11

Level	ARMA	Fit [%]	τ_{cold} [s]	f_{cold} [Hz]	τ_{hot} [s]	f_{hot} [Hz]
1	(1,0)	44.1	-	-	12.123	0.013
2	(4,2)	58.2	0.224	0.712	11.235	0.014
3	(1,0)	56.7	-	-	22.990	0.007
4	(4,2)	52.1	0.150	1.063	6.070	0.026
5	(4,2)	61.3	0.409	0.389	4.405	0.036
6	(4,2)	56.4	0.309	0.514	2.919	0.055
7	(4,2)	48.6	0.279	0.571	2.626	0.061
8	(4,2)	53.0	0.424	0.375	1.755	0.091
9	(2,1)	54.2	-	-	-	-

Table 3. Results of the ARMA modeling for the detector string M03

Level	ARMA	Fit [%]	τ_{cold} [s]	f_{cold} [Hz]	τ_{hot} [s]	f_{hot} [Hz]
1	(2,1)	57.6	0.091	1.740	23.912	0.007
2	(2,1)	58.7	0.083	1.927	15.245	0.010
3	(2,1)	55.0	0.065	2.433	17.348	0.065
4	(2,1)	58.0	0.048	3.301	20.308	0.008
5	(1,0)	42.2	-	-	7.838	0.020
6	(1,0)	52.0	-	-	17.673	0.009
7	(2,1)	49.8	0.900	0.177	2.103	0.076
8	(2,1)	40.0	0.413	0.386	6.604	0.024
9	(4,2)	52.7	0.281	0.567	3.760	0.042

4 Conclusions

In the past, the bias between the MTC noise estimation and its actual value was considered to be due to the deviation of the reactor response from point-kinetics. A simple one-dimensional analytical model relying on the one-group diffusion approximation revealed that this non point-kinetic response of the reactor accounts for only a negligible part in the MTC underestimation. Most of the discrepancy comes from the fact that the temperature noise is not spatially homogeneous throughout the core. A new MTC noise estimator was therefore proposed. This estimator still relies on the point-kinetic response of the reactor, but uses a core average temperature noise. This core average temperature noise is estimated by using the square of the static flux as a weighting function. By doing so, the discrepancy between this MTC noise estimator and its actual value is very little, and in practice only due to the still remaining deviation of the reactor response from point-kinetics.

In order to use this new MTC noise estimator, one should be able to measure the local temperature noise in many points within the core, so that the average temperature noise could be numerically evaluated. Gamma-Thermometers (GTs) could be used for that purpose since in the frequency range of interest for the MTC determination by noise analysis, typically 0.1 to 1.0Hz, they work as thermocouples. The main reason for this is the large thermal time constant of the GT hot junction. This large time constant completely filters out the signal induced by the gamma/neutron noise, and only the signal associated due to the cold junction remains. This cold junction is in direct contact with the coolant and therefore monitors the moderator temperature noise. This was proven both qualitatively and quantitatively. The qualitative analysis relied on the linear phase behavior of the *CPSD* between two GTs located within the same fuel channel but at different axial elevations, this linear slope being characteristic of coolant temperature fluctuations traveling upwards and unperturbed. The quantitative analysis was carried out by ARMA modeling of the GTs transfer functions. Two time constants were in most cases found, one lying in the range 2.0 to 20.0s and representative of the hot junction, and another one lying in the range 0.1 to 1.0s and representative of the cold junction.

It is intended to test this new MTC noise estimator in the Swedish Ringhals-2 PWR, which is instrumented with 108 GTs throughout the core. The neutron noise would be monitored by an in-core neutron detector synchronized with the GTs signals. The ratio between the local temperature noise (close to the core center) and the average temperature noise is expected to give the calibration factor that was noticed experimentally previously, i.e., in the range of one half to one fifth. This means that in principle only the GTs signals should allow checking that the heterogeneous structure of the temperature noise throughout the core is responsible for the MTC underestimation.

Acknowledgements

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Nomenclature

APSD	Auto-Power Spectral Density
ARMA	Auto-Regressive Moving Average
BOC	Beginning Of Cycle
CPSD	Cross-Power Spectral Density
EOC	End Of Cycle
GT	Gamma-Thermometer
HFP	Hot Full Power
HZP	Hot Zero Power
MTC	Moderator Temperature Coefficient
PWR	Pressurized Water Reactor

PAPER VIII

ANALYSIS OF AN MTC NOISE MEASUREMENT PERFORMED IN RINGHALS-2 USING GAMMA-THERMOMETERS AND IN-CORE NEUTRON DETECTORS

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ABSTRACT

A noise measurement in the Swedish Ringhals-2 PWR was performed in January 2002 by using twelve gamma-thermometers and two in-core neutron detectors, all located on the same axial level in the reactor. The gamma-thermometers are very versatile tools since they allow estimating the core-averaged moderator temperature noise throughout the core. This core-averaged temperature noise was then used to estimate the MTC by noise analysis, via a new MTC noise estimator. It was shown that whatever the location of the neutron detector might be, the MTC is always correctly estimated by this new MTC noise estimator, without any calibration to a known value of the MTC prior to the noise measurement. For the purpose of comparisons, the MTC was also estimated by using a single gamma-thermomemeter and a single core-exit thermocouple, together with an in-core neutron detector. In such cases, the MTC was systematically underestimated, with a stronger bias for the core-exit thermocouple than for the gamma-thermometer. This shows that the main reason of the MTC underestimation by noise analysis in all the experimental work until now was due to the radially non-homogeneous temperature noise throughout the core. The resulting deviation from point-kinetics of the reactor response has a negligible effect.

KEYWORDS
Moderator Temperature Coefficient (MTC), noise analysis, Gamma-Thermometer (GT), temperature noise (structure of the), point-kinetics.

A nomenclature explaining all the abbreviations used in this paper can be found at the end (see Section 7).

1. INTRODUCTION

There have been many attempts in the past few years to estimate the Moderator Temperature Coefficient of reactivity (MTC) in Pressurised Water Reactors (PWRs) by noise analysis (see Demazière (2000) for a complete list of references in this matter). Practically, the MTC was evaluated by using the signals delivered by one in-core Neutron Detector (ND), and a core-exit Thermocouple (TC) located at the top of the same fuel assembly or of a neighbouring fuel assembly. The so-called H_1^{biased} MTC noise estimator was always used for these estimations. This noise estimator can be defined in the frequency range 0.1 – 1.0 Hz as follows:

$$H_1^{biased}\left(\boldsymbol{r},\omega\right) \approx \beta_{eff} \times \frac{CPSD_{\delta\phi/\phi_0,\delta T_m}\left(\boldsymbol{r},\omega\right)}{APSD_{\delta T_m}\left(\boldsymbol{r},\omega\right)} \qquad (1)$$

where β_{eff} is the effective fraction of delayed neutrons[1], the *APSD* and the *CPSD* stand for the Auto-Power Spectral Density and the Cross-Power Spectral Density respectively. The relative neutron noise $\delta\phi/\phi_0$ and the moderator temperature noise δT_m are both measured at the same radial position $\boldsymbol{r}$. As indicated on the l.h.s. of Eq. (1), this MTC noise estimator always gives a biased estimation of the MTC, i.e. the MTC was always observed to be underestimated by a factor of two to five.

A recent theoretical investigation performed by the authors (see Demazière and Pázsit (2002a, 2002b)) showed that there are two main reasons that could explain why the MTC is underestimated. The first one lies with the fact that the temperature noise is measured in one point of the reactor (usually at the core-exit), whereas there is no proof that the temperature noise is homogeneous throughout the core. There is even some experimental evidence that the temperature noise is strongly radially heterogeneous (Andersson *et al.*, 2002). According to the MTC definition, the core-averaged temperature noise should be used while evaluating the MTC. The other reason for the MTC underestimation is that the reactor will not behave in a point-kinetic manner due to the spatial non-homogeneous structure of the temperature noise. The use of Eq. (1) implicitly assumes that point-kinetics is applicable. As a matter of fact, a correct, i.e. non-biased, MTC noise estimator was proposed by the authors in the frequency range 0.1 – 1.0 Hz:

$$H_1^{ideal} \approx \beta_{eff} \times \frac{CPSD_{\delta\phi^{pk}/\phi_0,\delta T_m^{ave}}\left(\omega\right)}{APSD_{\delta T_m^{ave}}\left(\omega\right)} \qquad (2)$$

In this ideal MTC noise estimator, the point-kinetic component $\delta\phi^{pk}$ of the neutron noise $\delta\phi$ and the core-averaged temperature noise δT_m^{ave} should be used. In the theoretical work mentioned previously, another new MTC noise estimator was also tested. This new $\tilde{H}_1^{biased}$ MTC noise estimator supposes that the core-averaged temperature noise could be measured, but still uses the total neutron noise $\delta\phi$ instead of its point-kinetic component $\delta\phi^{pk}$. Due to the latter, this noise estimator is biased in the frequency band 0.1 – 1.0 Hz:

$$\tilde{H}_1^{biased}\left(\boldsymbol{r},\omega\right) \approx \beta_{eff} \times \frac{CPSD_{\delta\phi/\phi_0,\delta T_m^{ave}}\left(\boldsymbol{r},\omega\right)}{APSD_{\delta T_m^{ave}}\left(\omega\right)} \qquad (3)$$

Simulations showed that this new MTC noise estimator gives a fairly good estimation of the MTC (see Demazière and Pázsit (2002a, 2002b)), whatever the radial location of the ND might be. This suggests therefore that the deviation of the reactor response from point-kinetics does not play a significant role on the MTC estimation. The main reason of the MTC underestimation in all the experimental work carried out so far thus seems to be due to the radial non-homogeneous structure of the temperature noise throughout

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[1] β_{eff} is actually an approximation of the reciprocal of the zero-power reactor transfer function $G_0\left(\omega\right)$ (plateau approximation).

the core. Consequently, the MTC could be correctly estimated by noise analysis if the noise estimator given by Eq. (3) could be used, i.e. if the core-averaged temperature noise could be measured.

It is well known that Westinghouse type PWRs do not have any in-core temperature detectors, only a few core-exit TCs. At the Swedish Ringhals-2 PWR, 108 Gamma-Thermometers (GTs) are nevertheless installed permanently in the core. They are distributed in 12 detector strings, each of them containing 9 GTs located at different axial levels and covering the whole core active height. In the frequency range of interest for the MTC investigation by noise analysis, i.e. 0.1 – 1.0 Hz, these GTs were proven to work as ordinary thermocouples (see Demazière and Pázsit (2002c), Haaland *et al.* (1991), and Tosi and Haaland (1993)). Therefore, the GTs installed at Ringhals-2 could be used to measure the core-averaged temperature noise. Together with an in-core ND, the new noise estimator given by Eq. (3) could be used to evaluate the MTC.

Such a noise measurement was recently performed at Ringhals-2 on January 16th, 2002. The purpose of this paper is to give an account of this measurement and the corresponding MTC noise estimations.

2. MEASUREMENT SET-UP

A noise measurement was carried out at the Swedish Ringhals-2 PWR during the fuel cycle 26, at a core-averaged burnup of 7.30 GWd/tHM (January 16th, 2002). This measurement was performed while the reactor was at steady state, and at full power. The measurement length was about 25 minutes, and the sampling frequency was 8 Hz. In this measurement, one plane of the reactor, located at 30% of the core active height from its bottom, was fully instrumented with all the available GTs, and with two NDs. The GTs were of the so-called RADCAL type, whereas the NDs were ordinary fission chambers. The NDs were chosen so that they were located as close as possible to a GT. For the purpose of comparison, the signal of a core-exit thermocouple was also recorded. This core-exit thermocouple was an ordinary K type thermocouple, i.e., chromel/alumel, and was located at the top of a fuel assembly containing a GT, and next to a fuel assembly containing a ND. Such a measurement set-up is summarised in the following Fig. 1.
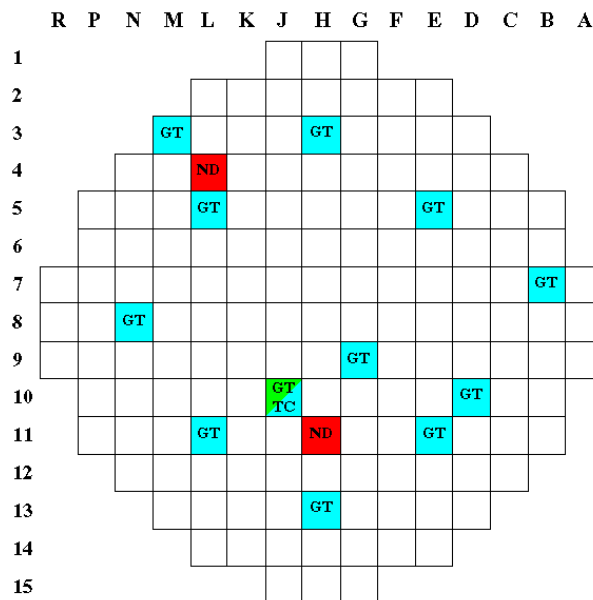

Fig. 1. Radial position of the detectors for the noise measurement in Ringhals-2 on January 16th, 2002 (GT = gamma-thermometer, ND = in-core neutron-detector, and TC = core-exit thermocouple)

Regarding the hardware processing of the signals, only the noise content of the NDs and the core-exit thermocouple were monitored by manually offsetting the mean values. These signals were then amplified. No offset and no amplification were applied to the signals of the GTs. These recorded signals were thus digitally converted. The software processing of the signals was carried out via MATLAB (The Math

Works, 2000). The time-signals were detrended (if a trend was found), and data analysis was performed in the frequency domain. In order to evaluate the APSDs and CPSDs of the different signals, Welch's averaged, modified periodogram method was used. The time-signals were divided into overlapping sections of n points, then windowed by using a Hanning window. The sections were assumed to overlap by $n/2$ points. As explained in the following, several values for n were tested: 512, 256, and 128 points.

3. ANALYSIS OF THE MEASUREMENT

As mentioned previously, the new MTC noise estimator relies on the core-averaged moderator temperature noise:

$$\delta T_m^{ave}(\omega) = \frac{\int \delta T_m(\mathbf{r}, \omega) w(\mathbf{r}) d\mathbf{r}}{\int w(\mathbf{r}) d\mathbf{r}} \tag{4}$$

where $w(\mathbf{r})$ is a weighting function. In this investigation, only the radial structure of the temperature noise is taken into account, since the axial structure is believed to have a second-order effect compared to the radial one (the axial effect is mostly the transport of the temperature noise upwards with the flow, i.e. a damping of the noise for frequencies higher than typically 1 Hz).

Assuming that first-order one-group perturbation theory prevails, Demazière and Pázsit (2002a) showed that the weighting function that has to be used to calculate the core-averaged temperature noise is the square of the static flux (referenced in the following as the W1 weighting function):

$$w_1(\mathbf{r}) = \phi_0^2(\mathbf{r}) \tag{5}$$

In this experimental work, the spatial distribution of the static flux throughout the core was obtained from core calculations performed by SIMULATE-3 (Umbarger and DiGiovine, 1992), at the core operating conditions corresponding to the measurement.

Since a good measurement technique should rely on as few as possible calculated parameters, being able to measure the static flux could be particularly interesting. Such a possibility arises with the GTs (Glöckler, 2002). The GTs were designed primarily to monitor the static gamma flux in the reactor. It is known that the static gamma flux is directly proportional to the static neutron flux. Since only the static neutron flux relative to its core-averaged value is required in Eq. (4), the knowledge of the corresponding proportionality factor between the static gamma and neutron fluxes is not required[2]. Therefore, a weighting function, which could be used to calculate the core-averaged temperature noise, could be simply the square of the mean value of the GTs (referenced in the following as the W2 weighting function):

$$w_2(\mathbf{r}) = \left[GT\_mean(\mathbf{r}) \right]^2 \tag{6}$$

If one assumes that first-order two-group perturbation theory is applicable, Demazière and Pázsit (2002b) showed that the weighting function that has to be used to calculate the core-averaged temperature noise is a combination of the direct and the adjoint fluxes, in the fast and thermal groups. If the effect of a change in the moderator temperature is supposed to have the greatest effect on the macroscopic removal cross-

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[2] Since the GTs are located within fuel assemblies that have different burnup, the ratio between the static gamma flux and the static neutron flux might be space-dependent. In such a case, the space-dependence has to be taken into account in the evaluation of the core-averaged temperature noise. Preliminary CASMO-4 (Edenius *et al.*, 1993) modelling of a single typical PWR assembly at different burnup showed that the standard deviation of the ratio between the static gamma and neutron fluxes is less than 10% of the average value for burnup up to 60 GWd/tHM. In the case of a full core, this figure is probably lower since a GT is sensitive to the gamma flux of several neighbouring fuel assemblies with different burnup. From one GT location to another, the average burnup of the fuel assemblies that the GT is sensing is roughly the same, due to the reloading pattern. Further investigation is nevertheless needed and is currently under way.

section, one obtains the following weighting function (referenced in the following as the W3 weighting function):

$$w_3(\boldsymbol{r}) = -\left[\phi_1^+(\boldsymbol{r})\phi_1(\boldsymbol{r}) - \phi_2^+(\boldsymbol{r})\phi_1(\boldsymbol{r})\right] \qquad (7)$$

If the greatest effect induced by a change of the moderator temperature is the effect on the macroscopic thermal absorption cross-section, then the weighting function is (referenced in the following as the W4 weighting function):

$$w_4(\boldsymbol{r}) = -\phi_2^+(\boldsymbol{r})\phi_2(\boldsymbol{r}) \qquad (8)$$

These different weighting functions were tested by using the new MTC noise estimator given by Eq. (3). For the purpose of comparisons, the traditional MTC noise estimator given by Eq. (1) was also evaluated. In the latter case, the local temperature noise was used, recorded either inside the core via the closest GT to the ND, or outside the core via the core-exit TC. All the MTC estimations were therefore carried out for both of the in-core neutron detectors H11 and L04. The effective fraction of delayed neutrons, which is required in the MTC noise estimators, was estimated to be equal to 537 pcm by SIMULATE-3. The MTC was also directly evaluated by SIMULATE-3 and was found to be equal to –51 pcm/°C. This value was considered as the reference value in the rest of this study.

The MTC noise evaluations showed that the MTC was frequency dependent with rather huge variation of the MTC magnitude in the frequency range 0.1 – 1.0 Hz. Therefore the following methodology was applied. In this frequency range, the maximum of the coherence between the ND and the temperature noise (estimated either from the W1, W2, W3, or W4 weighting functions, or directly from the GT J10, the GT L05, or the core-exit thermocouple) was first determined. Then all the frequencies for which the coherence was larger than half this maximum were used for the MTC evaluation. The final MTC value was simply obtained by averaging these values at the corresponding frequencies.

It was found that the MTC estimated via the previous procedure was strongly dependent on the number of points used for the Fast-Fourier Transform (FFT). Such dependence can be seen in Figs. 2 and 3, where the W2 weighting function was used for the calculations (since this weighting function is the most practical one to use from a measurement point-of-view). In these Figures, the points used for the final MTC evaluation are circled in bold.
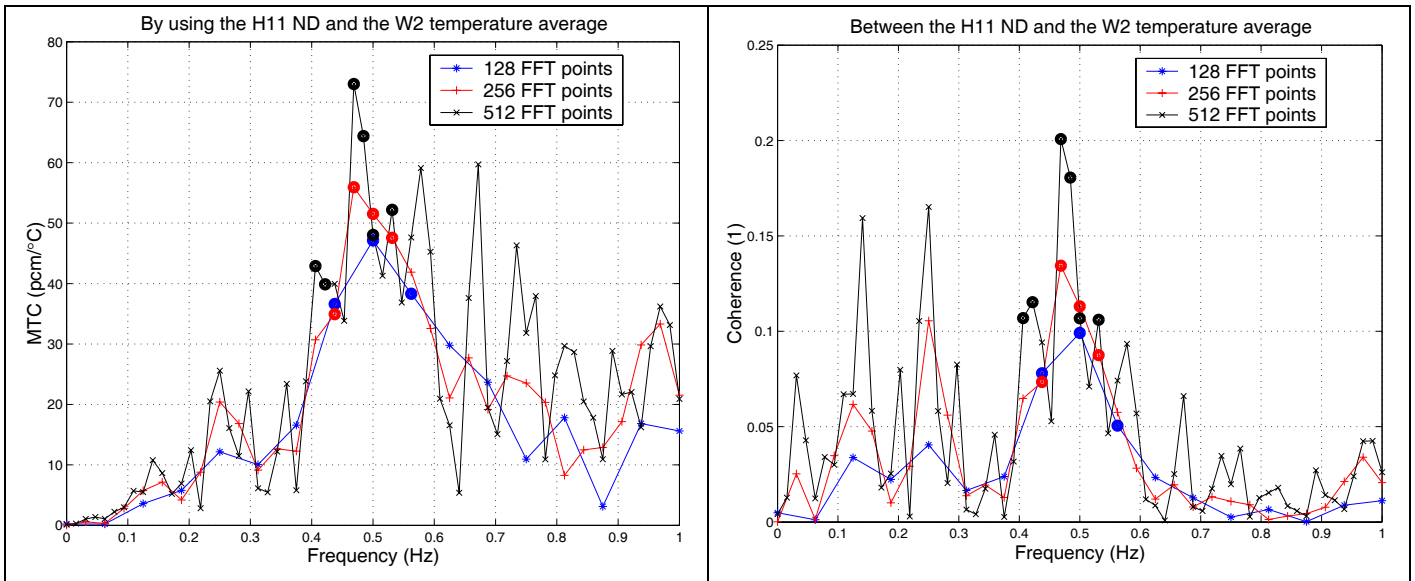

Fig. 2. Frequency dependence of the MTC noise estimation with respect to the number of FFT points used (neutron noise measured in the H11 assembly and temperature noise evaluated by using W2)
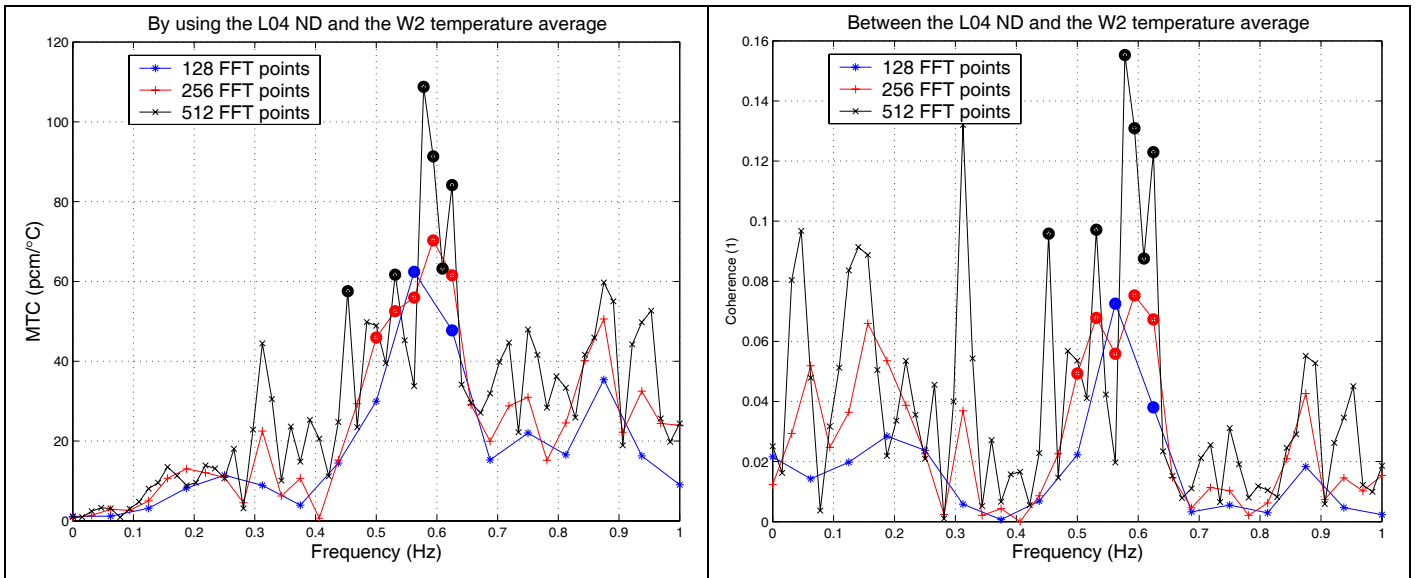
Fig. 3. Frequency dependence of the MTC noise estimation with respect to the number of FFT points used (neutron noise measured in the L04 assembly and temperature noise evaluated by using W2)

The resulting MTC values are depicted in Fig. 4, where the standard deviation associated with each MTC evaluation is also represented. As can be seen on this Figure, the 256 FFT points evaluation seem to be the most realistic one, both with respect to the reference MTC value given by SIMULATE-3 and to the relatively flat behaviour of the MTC for the selected frequencies (the peaks in the 512 FFT points MTCs are clearly non-realistic). Assuming therefore that the spectral analysis of the signals has to be carried out with 256 FFT points, one can compare the MTCs given by the different noise estimators and the different weighting functions. Such a comparison can be seen in Fig. 5 and Table 1, where the standard deviation associated with each MTC estimation is also given.
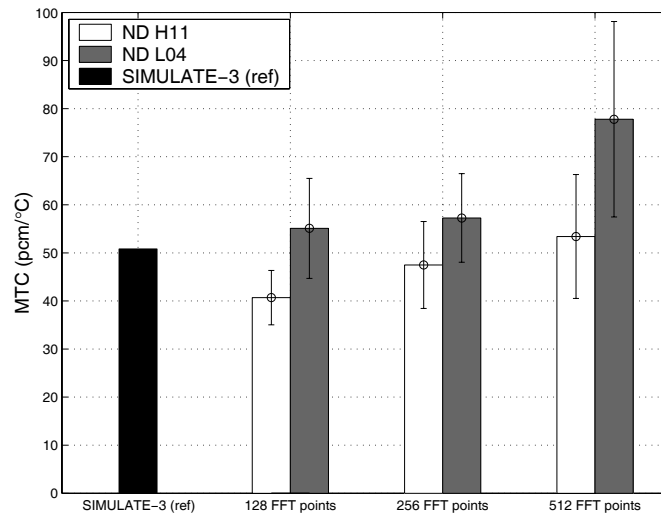

Fig. 4. Comparison of the MTC noise evaluations with respect to the number of FFT points used (temperature noise evaluated by using W2)
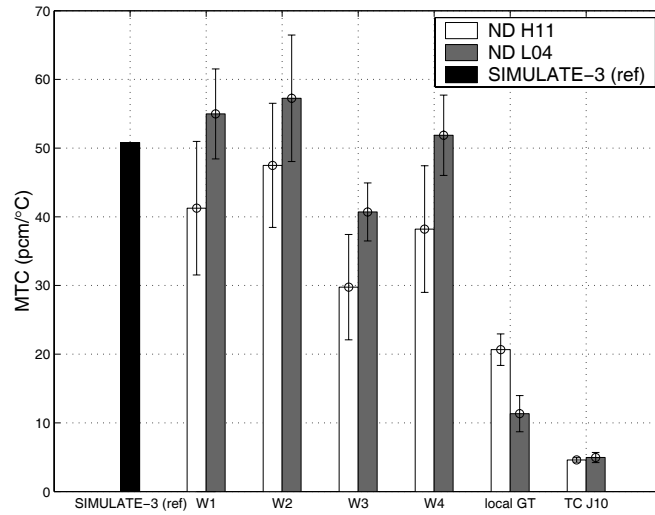
Fig. 5. Comparison between the different MTC noise estimators (estimations carried out with 256 FFT points)

Table 1 − Summary of the MTC noise evaluations (estimations carried out with 256 FFT points)

	MTC (pcm/°C)	Standard deviation (pcm/°C)	MTC (pcm/°C)	Standard deviation (pcm/°C)	Reference MTC given by SIMULATE-3 (pcm/°C)
	(flux measured in position H11)		(flux measured in position L04)		
New noise estimator with W1	-41.3	9.7	-55.0	6.6	-50.8
New noise estimator with W2	-47.5	9.0	-57.3	9.2	-50.8
New noise estimator with W3	-29.8	7.7	-40.7	4.2	-50.8
New noise estimator with W4	-38.2	9.2	-51.9	5.8	-50.8
Old noise estimator with local GT	-20.7	2.3	-11.3	2.6	-50.8
Old noise estimator with TC J10	-4.6	0.3	-5.0	0.7	-50.8

4. DISCUSSION AND CONCLUSIONS

The main conclusion from this MTC noise measurement is that using the new MTC noise estimator given by Eq. (3) gives a MTC value that is very close to the reference value given by SIMULATE-3, if one takes the confidence intervals into account. This new MTC noise estimator relies on the core-averaged moderator temperature noise, which can be evaluated in different ways (by using either the W1, the W2,

the W3, or the W4 weighting functions). As Fig. 5 and Table 1 show, if one uses the core-exit TC located above the fuel assembly J10, and consequently uses the traditional MTC noise estimator given by Eq. (1), then the MTC is strongly underestimated by a factor of approximately 10. Likewise, replacing the core-exit TC by a single GT (the nearest one to the ND used in the evaluation) systematically underestimates the MTC value by a factor of 3 to 5. As explained in Andersson *et al.* (2002), the reason why the MTC is underestimated at the core-exit is that the temperature fluctuations are larger at the core-exit than inside the core. The mixing of the coolant above the fuel assemblies is probably responsible for this effect. Similarly, the fact that the MTC is still underestimated when using one single GT instead of using all the signals of the GTs and the corresponding core-averaged temperature noise means that the temperature noise recorded in this specific point of the reactor is larger than the core-averaged one. This can be easily seen on Fig. 5 in Andersson *et al.* (2002).

The main reason of the MTC underestimation in all the experimental work so far is therefore the overestimation of the temperature noise outside the core and the overestimation of the (radially) core-averaged temperature by a local temperature value. The latter overestimation is valid for both the core-exit and the in-core measurements.

The fact that the results using the core-averaged temperature noise do not depend strongly on the radial position of the ND used in the MTC evaluation and give the actual MTC value suggests that the deviation of the reactor response from point-kinetics does not play a significant role on the MTC estimation by noise analysis (the new MTC noise estimator given by Eq. (3) still assumes a point-kinetic behaviour of the reactor). This effect was expected from the theoretical work performed previously (see Demazière and Pázsit, (2002a, 2002b)). Consequently, the conclusions drawn by this theoretical work are proven by the experimental one: the main reason of the MTC underestimation by noise analysis in all the experimental investigations performed until now lies with the fact that the moderator temperature noise is radially strongly heterogeneous in the core; the resulting deviation of the reactor response from point-kinetics is nevertheless not significant.

As can be seen on Fig. 4 and on Figs. 2 and 3, the MTC evaluation by using 256 FFT points seems to give the most realistic results. Taking the standard deviation into account gives an MTC estimated by SIMULATE-3 lying in the confidence interval of the measurement. The way the final MTC is calculated, i.e. detecting the frequency having the highest coherence and taking all the frequencies between 0.1 and 1.0 Hz having a coherence higher than half this maximum into account, is very subjective. Having a more restrictive way of choosing the frequencies for the final MTC evaluation would narrow the confidence interval and give a better MTC estimation.

As can be seen on Fig. 5 and Table 1, the MTC depends to some extent on the weighting function used to calculate the core-averaged temperature noise throughout the core. The weighting functions using the square of the static flux, either calculated by SIMULATE-3 (W1) or measured via the GTs (W2), give the best results. The W3 weighting function gives somewhat underestimated MTC values (but still much higher than using a single GT or a single TC), whereas the W4 weighting function gives also acceptable results. This means that the hypothesis on which the W4 weighting function was derived is better than the one on which the W3 weighting function was derived, i.e. the moderator temperature noise has a bigger effect on the macroscopic thermal absorption cross-section than the removal cross-section with respect to the MTC.

Using the W2 weighting function has many practical aspects, the most important one being that the static flux does not need to be calculated but can be directly measured via the GTs. The GTs are therefore very versatile tools since they can provide both the moderator temperature noise and the static neutron flux throughout the core. These are required for an accurate estimation of the core-averaged moderator temperature noise. This core average can then be used in the new MTC noise estimator that was proven, both theoretically and experimentally, to give an accurate MTC estimation, wherever the neutron noise is measured in the core. The only parameter that is needed for the MTC estimation is the effective fraction of delayed neutrons, which can be easily predicted by any static core simulator.

Although this noise measurement is very encouraging, more work needs to be done. More specifically, a few points have to be investigated in further detail, such as for instance the frequency dependence of the MTC in the frequency range 0.1 – 1.0 Hz. The axial dimension has also to be taken into account. Although it is expected that only a damping effect due to the coolant flow is taking place, it has to be verified that no noise source is actually present in the core, i.e. the moderator temperature noise is created outside the core. This can be investigated by using all the GTs in one or several detector strings. Likewise, the effect of estimating the core-averaged moderator temperature noise by using several axial planes containing the GTs (only one axial plane was taken into consideration in this study) has also to be investigated. Regarding the measurement itself, it was found that the noise signals of the GTs were not accurate enough. Manually offsetting the mean value of the GTs seems to be necessary before the analogue-to-digital conversion.

More measurements are planned in the future in order to verify the reproducibility of these results.

Finally, and most importantly, it has to be emphasized that the Ringhals-2 PWR represents a unique case in which the moderator temperature noise can be measured inside the core via the GTs. In most western-type PWRs, there are no "in-core" thermocouples. Therefore, the only core-averaged moderator temperature noise that can be estimated is the one measured at the core-exit with the use of all the available core-exit thermocouples. As pointed out previously, the moderator temperature noise measured above the core (at the level of the core-exit thermocouples) is overestimated compared to the one measured inside the core. The MTC is therefore expected to be underestimated even if one uses the average of all the core-exit thermocouple signals. Nevertheless, the bias is likely to be smaller compared to the case when a single core-exit thermocouple is used since the radial heterogeneous structure of the moderator temperature noise could be taken into account. The possibility of using the new MTC noise estimator with the core-exit thermocouples is planned to be investigated in the near future. If, as expected, the bias remains, one still has to estimate a calibration factor. The theoretical derivation of this calibration factor is a difficult task since the reason of the overestimation of the moderator temperature noise just above the core is not known. If the coolant mixing is responsible for this extraneous noise source, only a thermal-hydraulic code would allow modelling such a phenomenon, and hopefully, deriving a rigorous calibration factor.

5. ACKNOWLEDGEMENTS

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7. NOMENCLATURE

APSD   Auto-Power Spectral Density
CPSD   Cross-Power Spectral Density
FFT   Fast-Fourier Transform
GT   Gamma-Thermometer
ND   Neutron Detector (in-core)
PWR   Pressurised Water Reactor
MTC   Moderator Temperature Coefficient
TC   Thermocouple (core-exit)