Wavelet and Spectral Analysis of Some Selected Problems in Reactor Diagnostics

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Abstract

Both spectral and wavelet analysis were successfully used in various diagnostic problems involving non-stationary core processes in nuclear power reactors. Three different problems were treated: two-phase flow identification, detector tube impacting and core-barrel vibrations. The first two problems are of non-stationary nature, whereas the last one is not.

In the first problem, neutron radiographic and visible light images of four different vertical two-phase flow regimes, bubbly, slug, churn and annular flow, were analysed and classified with a neuro-wavelet algorithm. The algorithm consists of a wavelet part, using the 2-D discrete wavelet transform and of an artificial neural network. It classifies the different flow regimes with up to 99% efficiency.

Detector tubes in a Boiling Water Reactor may execute vibrations and may also impact on nearby fuel-assemblies. Signals from in-core neutron detectors in Ringhals-1 were analysed, for detection of impacting, with both a classical spectral method and wavelet-based methods. The wavelet methods include both the discrete and the continuous 1-D wavelet transform. It was found that there is agreement between the different methods as well as with visual inspections made during the outage at the plant. However, the wavelet technique has the advantage that it does not require expert judgement for the interpretation of the analysis.

In the last part two analytical calculations of the neutron noise, induced by shell-mode core-barrel vibrations, were carried out. The results are in good agreement with calculations from a numerical simulator. An out-of-phase behaviour between in-core and ex-core positions was found, which is in agreement with earlier measurements from the Pressurised Water Reactor Ringhals-3. The results from these calculations are planned to be used when diagnosing the shell-mode core-barrel vibrations in an operating plant.

Keywords: noise diagnostics, wavelet analysis, detector tube impacting, two-phase flow, core-barrel vibrations
This thesis consists of an introduction and the following papers:


Scientific publications related to the licentiate topic but not included in this thesis:


1. Introduction

Noise diagnostics has been used in Nuclear Power Plants (NPPs) since the earliest days of reactor physics [1-4], beginning fifty years ago with the start of the first reactor. Different methods have been developed during the decades up till now and still there is a lot of development going on. This thesis is a continuation of the noise diagnostic work at the Department of Reactor Physics, Chalmers University of Technology. Then, what is noise diagnostics?

In ordinary life, noise is considered to be something unwanted or disturbing, the noise in radio broadcasting or disturbing noise which can occur when speaking in a cell-phone. In order to get a clear, noise-free signal, the noise is often filtered away. However, in noise diagnostics it is the other way around, the static signal is filtered out and the noise is considered as the important part of the signal. Thus, the noise is extracted rather than filtered away. What is the use of the noise? Take, again, the cell-phone as an example. When calling to someone, there may be some disturbances (noise) in the voice transmitted over the phone. Based on the noise it could be possible to draw the conclusion that the person you are calling for example is driving a car, which is an example of a simple form of noise diagnostics. Hence, the noise is used to draw conclusions about a system, which can be very useful.

In reactor noise diagnostics, the noise in signals from detectors placed inside and outside the reactor core are used to diagnose the reactor. Compared to the simple cell-phone example the situation is somewhat different when diagnosing a reactor. First, the magnitude of the noise can be a couple of powers of ten lower compared to the signal itself. Second, it is possible to have more than one noise-source. Third, it is not always easy to get a simple response; you can not simply ask a reactor whether it is driving a car or not! Rather, continuing with the same example, it is like finding out in which of New York’s thousands of streets the car is in, what speed it has etc.

The purpose of reactor diagnostics is mainly to have control over the reactor and make sure everything is working properly. However, if something starts deteriorating, the on-line diagnostics of the reactor should detect the error and if necessary alert the operator, who in return may initiate a shut-down. The off-line diagnostics of the reactor means mainly to investigate the behaviour of the reactor and to give information about the reactor status, e.g. find components that may need maintenance or understand trends, unexpected phenomena etc. Hence, the diagnostics must be as reliable as possible. One does not want to stop the reactor by false alarm or maintain a working component, since, it is time consuming to restart the reactor and, of course, there is a loss of money if the reactor is not operating. Hence, it is important to understand and develop noise diagnostic methods, which are reliable. The methods developed in this thesis are not for on-line use in power producing nuclear plants, but more of an understanding level of the processes in a reactor and for off-line use.

At the Department of Reactor Physics, Chalmers University of Technology, reactor noise diagnostics have been used for different diagnostic tasks [5-26], such as detection of detector tube impacting, identification of core boundary (barrel) vibrations and Moderator Temperature Coefficient (MTC) determination. There are two ongoing research projects on noise diagnostics, one in cooperation with the Swedish NPP Ringhals [10-17], and one in cooperation with the Swedish Nuclear Power Inspectorate (SKI) [18-26].
In this thesis noise diagnostics is applied within the research areas of core-barrel vibrations, two-phase flow identification and detector tube impacting. One of the major goals of this thesis is to include wavelet analysis in the noise diagnostic work. Wavelets are especially suited for analysing non-stationary processes, e.g. intermittent signals. Hence, the focus of the noise diagnostics performed in this thesis is on non-stationary processes.

Since wavelets are a relatively new tool in reactor physics, a short introduction to wavelet theory is given in Section 2. The outline for the rest of the thesis basically follows the order of the appended papers. The work done on two-phase flow identification, which is described in papers I and II, is summarised in Section 3. Section 4 is a summary of the work with detector tube impacting found in paper III. Section 5 is a summary of the work with core-barrel vibrations described in papers IV and V.
2. Wavelets

One goal of this thesis and the research within the project is to use wavelet techniques in reactor noise diagnostics. Wavelets are still considered to be a new field in signal processing, even though they have been in use for almost two decades since they were first introduced in the mid-80s. The real development started in the early 90s [27], and has continued ever since by the use of wavelets in different scientific areas such as fluid dynamics, medicine, finance, physics and geophysics [28]. This Section gives a very short introduction to wavelets and their application. A more mathematical detailed explanation can be found in [27-31].

2.1 Time and frequency

The classical Fourier transform can be used to map a time signal into the frequency domain, as illustrated in Fig. 1. The Fourier transform can only be applied on stationary signals, where there are no changes in frequency over the time interval of interest. However, if the signal is non-stationary, the Fourier transform cannot be used. In that case a windowing of the signal can be done, using the so-called windowed Fourier transform or Short Time Fourier Transform (STFT). The STFT maps a time signal into a two-dimensional signal of both time and frequency. Hence, it is possible to get information about both when and at what frequency a certain event occurs. However, there is one drawback with the STFT, namely that the frequency resolution is the same for all frequencies. Often there is a need for better resolution in time at higher frequencies. The next step is to construct a tool which can map a time signal into time and frequency but with different resolution for different frequencies. The wavelet transform is able to cope with this requirement, even though it is mapping the signal into time-scale or time-level rather than time-frequency, but there is a connection between scale, level and frequency. However, the better resolution at high frequencies in time is achieved by reducing the resolution in frequency. The resolution in time becomes poorer at lower frequencies as can be seen in Fig. 1.

![Fig. 1. Schematic picture of the Fourier, short time Fourier and wavelet transforms.](image)

The Fourier transform uses an infinitely long sinusodial function as the analysing tool, hence it has no time resolution. On the other hand the wavelet transform uses a small, localized wave function (see Fig. 2) as the analysing tool, hence the name wavelet. The use of localised functions makes the wavelet transform well suited for analysing non-stationary signals such as transients and intermittent signals. The wavelet displayed in Fig. 2 is the so-called Mexican hat, which has the characteristic features of a wavelet. However, this wavelet will not be used in the rest of the Section since it does not have compact support and is not orthogonal. Without going into details, this means that it cannot be used when performing the discrete wavelet transform.
In the rest the Daubechies 4 (db4) wavelet will be used instead, since it has compact support and is orthogonal [27].

![Mexican hat wavelet](image)

Fig. 2. The Mexican hat wavelet has the form of a small localized wave, hence the name wavelet. The dotted curve is a cosine function with the same frequency as the centre frequency of the Mexican hat wavelet.

2.2 One-dimensional wavelet transform

It is possible to use the wavelet transform on one- or two-dimensional data. In this subsection the one-dimensional transform is described and in the next a short description of the two-dimensional wavelet transform is given.

There are two ways of performing the one-dimensional transform: continuous with continuous scales or levels (frequencies), or discrete with discrete scales or levels (frequencies). In this thesis the focus is on the discrete transform, hence this will be described in more details. Although part of this thesis touches upon the continuous wavelet transform, it will not be described in this Section.

The wavelet transform is based on a so-called mother-wavelet, $\psi$, which is dilated and translated (see Fig. 3), with the parameters $a$ and $b$.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$$  \hspace{1cm} (1)

At each scale $a$ there is a characteristic frequency, $F_a$, which can be calculated through the so-called centre-frequency of the mother-wavelet, $F_c$, and the sampling period, $\Delta t$, of the analysed signal:

$$F_a = \frac{F_c}{a\Delta t}$$  \hspace{1cm} (2)

The centre-frequency is the characteristic frequency of the mother-wavelet, see Fig. 2. One way of choosing the parameters in (1) are, $a = a_0^m$ and $b = nb_0a_0^m$ where $m$ is called the
level. The most common choice of $a_0$ and $b_0$ are 2 and 1 respectively. This gives the discrete one-dimensional wavelet transform of the signal $x(t)$ as:

$$T_{m,n} = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{2^m}} \psi(2^{-m} t - n) dt$$

(3)

Here $T$ is called the detailed wavelet coefficients. They contain information about the details of $x$. The coefficients $T_{m,n}$ are calculated by translating and dilating the mother wavelet along the signal, illustrated in Fig. 3, and performing the integration at each step. The transform is performed at each discrete level $m$.

With each wavelet there is an associated scaling-function, $\phi$. The wavelet and the scaling functions are orthogonal and have the following relation:

$$\psi(t) = \sum_k (-1)^k c_{1-k} \phi(2t - k)$$

(4)

Here $c$ is a scaling coefficient. The scaling functions can be used to calculate approximation coefficients of the signal, $x(t)$, in the same way as the calculation of the detail coefficients:

$$S_{m,n} = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{2^m}} \phi(2^{-m} t - n) dt$$

(5)
The approximation coefficients contain information about the mean behaviour of the signal. With the use of both the detail coefficients and the approximation coefficients the signal can be reconstructed as:

\[ x(t) = x_{m_0}(t) + \sum_{m = -\infty}^{m_0} d_m(t) \tag{6} \]

where the detail of the signal, at level \( m \), is defined as:

\[ d_m(t) = \sum_{n = -\infty}^{\infty} T_{m,n} \frac{1}{\sqrt{2^m}} \psi(2^{-m}t - n) \tag{7} \]

and the approximation of the signal at level, \( m_0 \) is:

\[ x_{m_0}(t) = \sum_{n = -\infty}^{\infty} S_{m_0,n} \frac{1}{\sqrt{2^{m_0}}} \phi(2^{-m_0}t - n) \tag{8} \]

If the input signal, \( x_0 \), is discrete and of finite length, e.g. \( N = 2^M \), as it normally is when dealing with measurements, \( m \) and \( n \) are also finite and it is possible to rewrite the above formulae as follows:

\[ x_0(t) = x_M(t) + \sum_{m = 1}^{M} d_m(t) \]

\[ x_M(t) = S_{M,n} \frac{1}{\sqrt{2^M}} \phi(2^{-M}t - n) \]

\[ d_m(t) = \sum_{n = 0}^{2^{M-n} - 1} T_{m,n} \frac{1}{\sqrt{2^m}} \psi(2^{-m}t - n) \tag{9} \]

Here \( x_M \) is the mean of the signal. From this it is possible to get a relation between approximation and detail at one level and the approximation at the next level:

\[ x_{m-1}(t) = x_m(t) + d_m(t) \tag{10} \]

With this choice of \( a \) and \( b \) the decomposition of the signal into details and approximation is called the wavelet multiresolution analysis. From the multiresolution it is clear that the wavelets, used in the discrete transform, consists of an orthogonal set of basis functions in which an arbitrary function can be expanded.

As an example of the multiresolution analysis take a chirp signal, shown at the top in Fig. 4 a), with some white noise added. It is decomposed into details and approximations at different scales using the \( db4 \) wavelet in the MatLab Wavelet Toolbox [31]. The details at each level represent the content of the signal at that level (frequency). Hence it is possible to extract
information about both when and at what frequency a certain event happens. As expected, the noise is present in the lowest level (highest frequency) details 1-3, see Fig. 4 b). The high frequency components of the regular chirp signal are visible at the middle levels, 5-6, to the right in Fig. 4 b). The low frequency components of the regular chirp signal are visible to the left at higher levels, 7-8, in the same figure. An almost noise free chirp signal is visible at approximation level 4, \( x_4(t) \) in Fig. 4 a).

\[ \begin{align*}
&\text{a) Approximation of the signal} \\
&\text{b) Details of the signal}
\end{align*} \]

**Fig. 4.** Approximation \( x_m, \text{ a)\), and details \( d_m, \text{ b), at ten different levels of the wavelet transform using the } \text{db4 wavelet. The original signal } x_0 \text{ is at the top in both plots.}**

The detail coefficients \( T_{m,n} \) contain the same information as the reconstructed details, \( d_m \), and the same is valid for the approximation coefficients \( S_{m,n} \) and the reconstructed approximation, \( x_m \). Hence it is possible to use either the coefficients or the reconstructed approximation and details when analysing the signal. The advantage with the coefficients is that the size of them are decreasing with a factor of two at each level, compared with the reconstructed signals which have the same size as the original signal.

From Fig. 4 it is possible to draw the conclusion that wavelets can be used to denoise a noisy signal. By taking the approximation at level 4 of the signal, an almost noise-free chirp signal can be obtained. However, all information from detail level 1-4 is neglected, when using this approximation. One way of performing a better denoising is to use some information from the lowest levels (highest frequencies). This can be done by thresholding the detail coefficients, in this example details at level 1-4. When reconstructing the signal, the approximation and the thresholded detail coefficients are used to calculate a denoised representation of the signal. Also it is seen that the noise can be extracted through the details. By using the three first levels of details in Fig. 4 b) it is possible to almost completely extract the noise. An application where this could be of use is the possibility to extract short intermittent parts of a signal, so-called spikes.

The one-dimensional wavelet transform (multiresolution) will be used in Section 4.

### 2.3 Two-dimensional wavelet transform

If the input data are two-dimensional, i.e. an image, it is possible to use a two-dimensional wavelet transform. The principles of the transformation are the same as for the one-dimensional transform, but instead of having one detail at each level there are three. The
information in each of the three details are from three directions of the two-dimensional input data, horizontal, vertical and diagonal details. The first equation in (9) can then be expressed as

$$x_0(x, y) = x_M(x, y) + \sum_{m=1}^{M} d_m^h(x, y) + d_m^v(x, y) + d_m^d(x, y)$$ (11)

This is the reconstruction for the two-dimensional discrete wavelet transform. The details, the detail coefficients, the approximations and the approximation coefficients are calculated in the same way as for the one-dimensional transform, except that different two-dimensional wavelets are used for the different directions and a two-dimensional scaling function is used. All the two-dimensional wavelet and scaling functions can be calculated from the one-dimensional ones.

$$\psi^h(x, y) = \phi(x)\psi(y)$$
$$\psi^v(x, y) = \psi(x)\phi(y)$$
$$\psi^d(x, y) = \psi(x)\psi(y)$$
$$\phi(x, y) = \phi(x)\phi(y)$$ (12)

Depicted in Fig. 5 b)-e) are the reconstructed approximation and details at the first level (highest frequency) of a two-dimensional wavelet transform, using the db4 wavelet, on the image in Fig. 5 a). In the horizontal detail c), the horizontal high frequency parts (sharp transitions) are clearly visible, e.g. the line in the middle. In the same way the vertical high frequency parts (the trees) are clearly visible in d), the vertical details, as expected. There are no clear features in the diagonal details e), since the original images have no sharp diagonal transitions. The two-dimensional transform can be used to characterise an image by identifying different features of the image at different details levels (scales) and/or different directions. This is used in Section 3.

Another feature of the wavelet transform, which is not used in this thesis, is the possibility to compress images. The differences between the original image, a), and the first level approximation, b), are hardly noticeable. However, the approximation contains only 25% the amount of data as the original image. Hence, it can be used as a compressed version.

For further information on wavelet theory and applications the interested reader is referred to [27-31]. There all the mathematical details, which are left out in this thesis, can be found.
Fig. 5. Approximation, b), and details, c)-e), from a two-dimensional wavelet transform, using the $db4$ wavelet, at the first level. The original image is depicted in a).
3. Two-phase flow identification

The first application of wavelet techniques in this thesis is in the area of two-phase flow identification. It is very important to classify the different flow regimes in a reactor, since they have quite different flow properties. Before using a flow equation, the regime must be determined in order for the right expression to be chosen for e.g. the interfacial shear coefficient or some other coefficients like the heat transfer coefficient. The task at hand is to classify two-phase flow regimes with image analysis.

The approach in this Section is to use wavelets to pre-process flow images and then extract statistical features to use as inputs to an Artificial Neural Network (ANN), see Fig. 6. The use of dynamic images as the “signal” from the flow means that the method is non-intrusive. Both non-intrusive methods, wavelets and ANN have been used previously when classifying two-phase flow regimes [32-37]. However, they have not been used in combination before. A more detailed description of the two-phase flow identification is found in paper I and II.

In nuclear technology the two-phase flow is water and vapour which flow within the core of a boiling water reactor. In vertical two-phase flow there are four main regimes, bubbly, slug, churn and annular flow. Bubbly flow is the flow of dispersed vapour in continuous liquid, with small bubbles of vapour in the water. In slug flow the bubbles of vapour have formed larger regions, with a size of approximately the size of the pipe diameter. If even more vapour is present in the pipe the bubbles break and there is an unstable regime of liquid mix with vapour, churn flow. In the last type, annular flow, the pipe is almost filled with vapour, only a thin part, close to the wall, contains liquid. The geometrical structure of the regimes is very different, but at the same time, it is difficult to express this difference in quantitative terms. This is why two-phase flow identification is difficult to perform with algorithmic methods.

3.1 Flow images

Two different sets of flow images have been analysed. The first set included dynamic neutron radiographic images of two-phase flow within a metallic water loop, see Fig. 7 a). This experiment was performed at the Kyoto University Reactor Research Institute (KURRI) [38]. By continuously increasing the heating of the water in the loop, all four flow regions were created in sequence. One drawback with these images is that they had to be converted into digital format since the recording was available on an analogue VHS in NTSC format. This conversion somewhat increases the noise in the images.

The second set of images, recorded at our department with the use of an ordinary digital camcorder, used visible light instead of neutron radiography. In the experiment the two-phase flow was simulated by injecting air into a thin rectangular pipe filled with coloured water, see Fig. 7 b). Due to the simple set-up it was only possible to simulate bubbly and slug flow. On the
other hand, a much better image quality was achieved as compared to the neutron radiographic images.

![Image of four flow regimes](image)

**Fig. 7.** a) Images of the four different flow regimes using neutron radiography, flicker noisy images.

b) Images of bubbly and slug flow regimes using visible light and coloured water, almost noise-free images.

### 3.2 Wavelet pre-processing

Before extracting inputs for the ANN, the images (2-D matrix with gray scale pixel intensity) were pre-processed with the two-dimensional wavelet transform. Wavelets are suitable to highlight features of different length scales (frequencies) in an image. It is also possible to extract information about features in different directions in an image, see Fig. 5 (Section 2.3). The different flow regimes are assumed to have different features at different length scales and in different directions. The idea is to use some features of the transformed wavelet coefficients which are characteristic for each regime. One possible feature is the energy content of the detail coefficients. In this case the first level of detail coefficients are considered and one way of expressing the energy content is given in [28] as:

\[
E_1^x = \sum_i |T_{1,i}^x|^2
\]  

(13)

Here \(x\) stands for the different directions (horizontal, vertical and diagonal) in the two-dimensional transform, see Section 2.3 for details. The energy feature was used to characterise the visible light images. Unfortunately, using the energy feature did not work in the case of the radiographic images, probably due to the flicker noise. Instead, the mean value of the first level approximation coefficients was used. Another possible feature, which could be used, is the variance of the very same coefficients. Hence, one value for the neutron radiographic images, the variance of the approximation coefficients, and three values for the visible light images, one for the details coefficients in each direction are also used. This gives a total of two features extracted for the radiographic images and six features for the visible light images. These features are, in the next step of the classification process, used as inputs for an ANN.

### 3.3 The classification algorithm

The classification task is solved by using an ANN with the wavelet pre-processed features as inputs. The principle of an ANN is to have a set of inputs with known output values. The ANN is fed with these inputs which propagate through the nodes of the network. The output is
3. Two-phase flow identification

compared with the known output values and some error parameters are calculated which are used to change the nodes. There are different ways in changing the nodes, i.e. different training procedures. The training is repeated until some pre-defined minimal criterion of the error is reached. Each repetition is called an epoch. After the training it is possible to feed the ANN with inputs whose outputs are to be determined. A properly trained ANN will generate the correct output values. For the two-phase flow classification the right flow regime type is searched.

The ANN used in this thesis is constructed from the Neural Network Toolbox in MatLab. Different types of networks and training algorithms were tested and by trial and error the best combination was selected for the task at hand. A feed-forward network with an input layer, an output layer and one hidden layer trained with the resilient backpropagation (BP) algorithm was found to be the most effective one. The number of input nodes used depends on which type of flow images is used. Two input nodes were used for the radiographic images and six for the visible light images. The number of nodes in the output layer also depends on the images, one output for each flow regime that is classified. Hence, four output nodes were used for the radiographic ones and two for the visible light images. The log-sigmoid transfer function was used for the output layer, giving values between 0 and 1. For both types of images 40 hidden nodes with tan-sigmoid transfer functions were used. The training target values were set to 0.9 for the correct regime and 0.1 for the other outputs.

The input data consist of 200 images from each of the flow regimes in the case with neutron radiography and 75 images from each of the two regimes in case of the visible light. A 5-fold cross-validation over the training data was used, i.e. 1/5 of the input data was used as a test set to verify the classification success of the network trained with the remaining 4/5 of the inputs. This was repeated five times by using different images in the test set. All images were used only once in the test set. When classifying the test set, the outputs were thresholded in order to get either 1 for the right flow or 0 for the wrong ones. If more than one output is 1 or none is 1 the image is classified as unknown flow regime.

3.4 Results from the classification

To investigate the advantage with the wavelet pre-processing, the mean value and the variance from the raw image pixel intensity were also fed in to the network, and the results were compared to those with the wavelet pre-processed input data. Six different discrete wavelets were used for the pre-processing, Haar, Daubechies 8, Coiflet 4, Symmlet 6 and Biorthogonal 3.1 all available in the Wavelet Toolbox in MatLab. In the case of the noisy radiographic images, the success ratio of the classification was around 95% for the test set of images for all the different wavelets and the same for the raw data input. In Fig. 8 the result for the input pre-processed with the Daubechies 8 wavelet in shown. Hence, there is no advantage, from the point of view of success ratio, in pre-processing the images. When using the visible light images the success ratio was even higher, around 99%, for both the pre-processed and the raw inputs. However, the number of epochs used during the training procedure is reduced with a factor of 100 when using wavelet pre-processing. For the raw data input the maximum number of epochs, set to 30 000, was always reached before the target value of the Mean Square Error (MSE), set to $10^{-3}$ was reached. In the pre-processed case the MSE target was reached within approximately 300 training epochs. Hence, the use of wavelet pre-processing has large advantages from the practical point of view.
Also worth mentioning is that 100% of the annular flow images were classified correctly in the neutron radiographic case. As expected, the slug flow and the churn flow were the ones most likely to be mixed up, i.e. classified wrong or as unknown.

The next step in the project will be to analyse neutron radiographic images, recorded in digital format. Hopefully, this will give images with the same quality as the visible light images, but with all four flow regimes available. It is expected that in the end this will lead to better performance of the wavelet pre-processing when used on all four regimes.

![Classification ratio of the neutron radiographic images using a threshold of 0.5 after the ANN. A total of 200 images from each regime were classified and the average success ratio was 95%.

**Fig. 8.** Classification ratio of the neutron radiographic images using a threshold of 0.5 after the ANN. A total of 200 images from each regime were classified and the average success ratio was 95%.
4. Detector tube impacting

Methods for detection of detector tube impacting in Boiling Water Reactors (BWRs) with noise diagnostic methods, applied to signals from neutron detectors, have been used for a long time. The first methods used were based on classical Fourier analysis [3, 39]. In these methods the Auto Power Spectral Density (APSD), coherence and phase curves are used to identify impacting tubes. Typically, a broadening of the eigenfrequency peak in the APSD and/or a distorted phase curve give information about an impacting tube. More about the spectral methods will be mentioned below. At our Department a new method, based on wavelet analysis, has been developed. First, simulations and measurements from the Swedish nuclear power plant Barsebäck-2 were analysed with both spectral and wavelet methods [5-6]. Later, measurements taken from Oskarshamn-2 were analysed and the result was compared with visual inspections made before the analysis [24-25, 40]. Between the two different analyses the wavelet algorithm was modified. In this Section a continuation of these analyses is summarised. For a detailed description see paper III. This time, measurements from Ringhals-1 are analysed both with the traditional spectral methods and the new wavelet based method. Recently the continuous wavelet transform has also been applied to the same problem, and it is also described briefly in the thesis.

4.1 Physical model

The task is to identify detector tubes which not only vibrate but also impact on the neighbouring fuel assemblies. If a detector tube hits a fuel assembly it can damage the fuel box which may cause also damage to the fuel cladding. Any such event must be avoided in an operating plant.

![Diagram of a detector tube in a BWR core with surrounding fuel assemblies](image)

*Fig. 9.* Illustration of a detector tube in a BWR core with surrounding fuel assemblies, three out of four shown. Some typical data of interest are also shown.

Fig. 9 shows a general outline of the physical setup of a detector tube together with the surrounding fuel assemblies. The vibrations arise from the strong flow of coolant water in the reactor and the fact that the detector tubes, which are roughly four meters long, are fixed only in their ends. The eigenfrequency is around 2 Hz, [3]. If the vibration is large enough, the tube
may impact on the nearby fuel assemblies, which in return will execute a short, damped oscillation after each hit, with an eigenfrequency of 10 to 20 Hz, [3]. Detectors are placed at four different axial levels in the tube. From the Ringhals-1 measurements, analysed here, signals from two of the four detectors in each tube were available. There are 36 detector tubes evenly distributed in the core giving a total of 72 signals in the Ringhals-1 case.

The signals from vibrating and impacting detector tubes are assumed to contain three parts. First a global part with some broad-band noise, \( N(t) \). The second part is a regularly oscillating part from the detector tube vibration, \( S(t) \), and thirdly an intermittent part due to the vibration of the fuel-assembly, \( T(t) \). The intermittent structure is due to the damped, randomly occurring vibration of the fuel-assembly, see Fig. 10. In general, the amplitude or the root mean square value of \( T(t) \) is much smaller than that of \( N(t)+S(t) \). Therefore, in a spectral analysis, the effect of \( T(t) \) is not visible. Hence, the classical spectral method focuses on the vibration of the detector tubes, whereas the wavelet method tries to identify the intermittent signal from the fuel assembly vibrations. Based on this fact the sampling frequency used in the wavelet method needs to be higher, since the eigenfrequency of the fuel assemblies is higher than for the detector tubes.

\[
\Phi(t) = N(t) + S(t) + T(t)
\]

\( \Phi(t) \): Total signal

\( N(t) \): Global neutron noise

\( S(t) \): Detector string noise (Stationary)

\( T(t) \): Fuel box noise (Transient)

**Fig. 10.** Schematic view of the signal, \( S(t) \), from a detector vibrating in a flux gradient with background noise, \( N(t) \), and fuel assembly impacting, \( T(t) \).

### 4.2 Analysing the measurements

As mentioned above, measurements from the BWR Ringhals-1 were analysed. In cooperation with plant personnel two different measurements were made. Both measurements were taken during full power (109%) and full core flow (~11 000 kg/s). The first measurement was carried out on 6th of September, 2002. In the rest of this thesis this measurement is referred to as measurement 1. The second measurement was made on 27th of October, 2003, referred to as measurement 2.

In measurement 1 the sampling frequency was too low, 12.5 Hz, for the wavelet method to be performed and the measurement time was approximately 11 min. For measurement 2, on the other hand, the sampling frequency was raised to 64 Hz to better use the information from
the fuel assembly vibrations (10-20 Hz). This time the duration of the measurement was 5 min. At the moment a third measurement is planned with even higher sampling frequency, 200 Hz.

4.2.1 Measurement 1

Since measurement 1 was not suited for wavelet analysis, the judgment on which tubes that may impact on the fuel assemblies is based mainly on the spectral method. The spectral method is based on the features of the Auto Power Spectral Density, APSD, the coherence, and the phase between two detectors in the same tube. The following four criteria are used to classify a detector tube as an impacting one [5]. First, broadening of the eigenfrequency peak in the APSD compared to a vibrating but non-impacting tube. Second, multiple peaks in the APSD, especially at the double eigenfrequency. Third, high coherence at the eigenfrequency, and finally, distorted (zero) phase over a large frequency range. The decision of whether a tube is impacting or not is made by qualitatively taking all four criterions into account. This is more or less a relative method. The idea with the wavelet method is to calculate a so-called Impact Rate index (IR-index). The IR-index gives the number of intermittent signals due to the fuel assembly vibration. A high value means severe impacting. By using the IR-index, there is no need for an expert judgment which is the case for the spectral method. Hence, the wavelet method is an absolute method.

As mentioned above the detector signals are assumed to consist of three parts:

\[ \phi(t) = N(t) + S(t) + T(t) \]  

(14)

where \( S(t) \) is analysed in the spectral case, described above, and \( T(t) \) is extracted in the wavelet method which will be described below.

To reduce the high frequency noise present in the signal, a wavelet denoising is performed. The denoising is done by applying a level-dependent threshold to each of the detailed coefficients, \( T_{m,n} \) as described in Section 2, in the multiresolution analysis. The wavelet decomposition is made down to a level, \( M \), corresponding to the eigenfrequency of the fuel assemblies (10-20 Hz). The denoised signal is then reconstructed by using the thresholded detailed coefficients in (9), giving \( Den(\phi(t)) \).

Then the approximation, \( X_M(t) \), which consists of the low frequency part of the signal, \( S(t) \), is removed from the denoised signal. The result is the intermittent signal from the fuel assembly vibration.

\[ V(t) = Den(\phi(t)) - X_M(t) \approx T(t) \]  

(15)

The IR-index is calculated as the number of peaks in \( V(t) \). Each peak corresponds to the start of an intermittent fuel assembly vibration.

In order to use the wavelet method, a value of the eigenfrequency of the fuel assemblies has always to be given to perform the denoising to the correct level. Since the sampling frequency of measurement 1 only allows investigation up to frequencies of 6.25 Hz, the assumed eigenfrequency of the fuel assemblies is set to 5 Hz. Even though this value is far from realistic, it is the best that can be used when performing the wavelet analysis on this measurement.

As is commonly known when using wavelets, it is not easy to know which kind of wavelet to use for a certain analysing task. Using the Meyer wavelet in the MatLab Wavelet Toolbox
gave best agreement with the result from the spectral method. However, due to the low sampling frequency the judgment of which tubes that was most likely to impact is mainly based on the spectral analysis in measurement 1.

Fig. 11 shows the result from the spectral analysis of detectors at position 22, LPRM 22.2 and LPRM 22.4. LPRM stands for Local Power Range Monitor and is the notation of the neutron detectors used. LPRM 22.4 is the upper detector and LPRM 22.2 is the lower one. Clearly there is a broad peak in the APSD around the expected eigenfrequency of the detector tubes (1-2 Hz). Multiple peaks are also visible in the APSD. The coherence is high at the very same frequency and the characteristic distortion of the phase is present (non-linear and almost zero). Hence, this detector position was classified as one of the most likely to impact.

![Fig. 11. Autospectra (APSD), coherence and phase for the detectors at LPRM position 22, measurement 1.](image)

After the analysis was performed, the plant personnel made visual inspections of some fuel assemblies, which were pointed out by the analysis, during the outage of the plant in August 2003. They inspected fuel assemblies around detector (LPRM) positions 1, 9, 10 and 22 out of which 1 had been classified by us as non-vibrating in the analysis and the three others were the ones that we judged most likely to impact. The result of the analysis and the inspection is presented in Table 1. As can be seen, LPRM 10 and 22 were pointed out by the visual inspection. They showed some wear marks on the corner of the fuel boxes. This is in good agreement with the prediction from the analysis.
4.2.2 Measurement 2

The analysis of measurement 2 was performed in the same way as that of measurement 1. However, this time the sampling frequency, 64 Hz, was better suited for the wavelet method. The eigenfrequency of the fuel assemblies was set to a more realistic value of 10 Hz in this case. In Fig. 12 a) the APSD, coherence and the phase of LPRM 16.2 and 16.4 is depicted and in b) $V(t)$ from equation (15) for LPRM 16.4 is shown. As can be seen, all the criteria for the spectral method are fulfilled and several spikes are visible in $V(t)$. Hence, this detector tube was pointed out as most likely to impact.

Fig. 12. a) Auto spectra (APSD), coherence and phase of detectors at LPRM position 16 from measurement 2. In b) the intermittent signal after the wavelet analysis of detector 16.4, measurement 2. The spikes indicating impacting are clearly visible.
Unfortunately, there was no time for visual inspections during the outage in 2004, since every second year there is a short outage. The results of the spectral and wavelet analysis are presented in Table 2. In the two groups of detectors with highest probability of impacting four out of six are pointed out by both methods, LPRM 16, 24, 34 and 35. Thus, there is a good agreement between the two methods.

**Table 2: Results of measurement 2**

<table>
<thead>
<tr>
<th>Impacting status</th>
<th>By spectral analysis</th>
<th>By wavelet analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>most likely impacting</td>
<td>15, 16, 24 and 35</td>
<td>16, 23 and 24</td>
</tr>
<tr>
<td>probably impacting</td>
<td>12 and 34</td>
<td>4, 34 and 35</td>
</tr>
<tr>
<td>small chance of impacting</td>
<td>27 and 32</td>
<td>--------</td>
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</table>

**4.3 Continuous wavelet transform**

There is also a possibility to use the continuous wavelet transform in the analysis. In the case with detector tube impacting it could be of interest to calculate the so-called wavelet coherence, [28], between detectors in the same tube. Fig. 13 shows the wavelet coherence and the spectral coherence for two different detector positions.

![Fig. 13. Spectral, a), and continuous wavelet, b), coherence for detectors in position 1 and 16.](image)

Clearly, there is a much larger difference between the wavelet coherence in the frequency band between 10 to 20 Hz than it is in the spectral case. The presence of the large values in the wavelet coherence around 10-20 Hz is not fully understood. One guess is that it could be the
fuel assembly vibrations that are seen. Detector positions pointed out by the wavelet IR-index also display large values in the frequency band between 10 and 20 Hz in the wavelet coherence. Even though the result from the continuous wavelet transform is not fully understood, this first result implies that further use may turn of to be of interest when investigating detector tube impacting.

### 4.4 Conclusions of the detector tube impacting

The spectral method is in good agreement with visual inspections. For the measurements with a sampling frequency that is suitable for the new wavelet based method, it is in agreement with the spectral one. One may wonder if there is need for a new method if a working one already exists. However, the advantage with this new wavelet method is that there is no need for comparison with a non-vibrating sample. It is an absolute method compared to the more relative spectral method, and its use requires no expertise, in contrast to the spectral based method, which cannot be performed e.g. by the control room personnel.

It is expected that a new measurement, with a higher sampling frequency that will be performed during the year of 2004/2005, and a visual inspection during the outage in late summer 2005 will confirm the agreement between the two methods and the real damage in the plant. The continuous wavelet coherence will be further investigated and hopefully better understood with the new measurement.

Up to now the wavelet method has been tested on signals from three Swedish plants, Barsebäck, Oskarshamn and Ringhals. It worked satisfactorily in all three cases, even though it had to be modified between its use in different plants.
5. Core-barrel vibrations

Core-barrel vibrations are, to a large extent, stationary processes, and have been successfully treated with spectral methods. The 2-D pendular vibrations, however, show sometimes intermittent properties (alternating isotropic - unilateral vibrations). Therefore, core-barrel vibrations will also be analysed with wavelet methods in the future. Accordingly, preparations are made by getting involved in the modelling of the in-core neutron noise induced by core-barrel vibrations. However, in this thesis, only classical, i.e. spectral analysis based, applications will be included. This analysis, still, has led to interesting new results in the understanding of certain experimental results.

Analyses of ex-core neutron noise of both beam-mode, ~8 Hz, and shell-mode, ~20 Hz, core-barrel vibrations have been used a long time, see Fig. 14, [3, 9-17]. It has also been noticed that the vibrations may lead to in-core noise which also can be used to analyse the vibrations. When analysing the shell-mode vibrations the use of in-core noise is especially important in Westinghouse reactors, since the ex-core detectors carry the same information, due to the 90° spacing. Therefore, the amplitude and the direction of the vibrations can not be determined at the same time. For this reason, in-core detectors have been included in the analysis of the shell-mode vibrations within a research project in cooperation with the Swedish NPP Ringhals [17]. However, the small number of in-core detectors hindered confirmation of the theory as well as use of the result. In order to have consistent interpretations it was necessary to assume that noise from in-core and ex-core detectors lying on the same azimuthal position have opposite phase. To confirm this out-of-phase behaviour, two different 1-dimensional analytical calculations of the noise induced shell-mode vibrations have been performed: an adiabatic approximation and a full space-frequency dependent solution. A comparison between the analytical results and the results from a numerical simulator, developed at the department [41], was also made. These calculations are described in detail in paper IV and V and a summary is given in this Section.

5.1 Description of the model

A 1-dimensional 2-group 2-region model is used with the boundary of the core set to $b=161.25 \text{ cm}$ and the boundary of the reflector set to $a=279.5 \text{ cm}$. The reactor parameters, e.g. cross-sections, are calculated from the core of the Pressurised Water Reactor (PWR)
Ringhals-4, using the in-core fuel management code SIMULATE-3 (homogenization from 3D to 1-D). The choice of which reactor to use is arbitrary. One only needs some realistic parameters to visualize numerical results in the end. The fluxes are assumed to be symmetrical around the centre of the core.

Since a 1-D 2-group 2-region diffusion model is used, there are two diffusion equations in the core region, \( c \), and two diffusion equations in the reflector region, \( r \), for the static fluxes. One in each region for the fast flux, \( I \), and one for the thermal flux, \( 2 \).

\[
\begin{align*}
D_1^c \frac{d^2}{dx^2} \phi_1^c(x) - (\Sigma_{a,1}^c + \Sigma_R^c) \phi_1^c(x) + \frac{1}{k} (\Sigma_{f,1}^c \phi_1^c(x) + \nu \Sigma_{f,2}^c \phi_2^c(x)) &= 0 \\
D_2^c \frac{d^2}{dx^2} \phi_2^c(x) - \Sigma_{a,2}^c \phi_1^c(x) + \Sigma_R^c \phi_1^c(x) &= 0 \\
D_1^r \frac{d^2}{dx^2} \phi_1^r(x) - (\Sigma_{a,1}^r + \Sigma_R^r) \phi_1^r(x) &= 0 \\
D_2^r \frac{d^2}{dx^2} \phi_2^r(x) - \Sigma_{a,2}^r \phi_1^r(x) + \Sigma_R^r \phi_1^r(x) &= 0
\end{align*}
\]

The cross-section notations are standard, i.e. \( a \) stands for absorption, \( f \) for fission and \( R \) for removal (scattering) from the fast group to the thermal. All the cross-sections and the diffusion coefficients are assumed to be constant within each region. The first step in calculating the phase behaviour of the noise, induced by shell-mode vibrations, is to solve the static equations (\( k \)-eigenvalue equations). That is easily done by using the symmetry, the interface (continuous fluxes and currents) and boundary (zero flux) conditions. Once a normalised and critical solution of the static fluxes is obtained the noise can be calculated. Criticality is achieved by adjusting the fission cross-sections.

5.2 The adiabatic approximation

In order to calculate the noise from shell-mode core-barrel vibrations, equations for a time-dependent system have to be used. One average group of delayed neutron precursors, \( C \), is assumed. The time-dependent diffusion equations now read

\[
\begin{align*}
\frac{1}{\nu_1} \frac{d \phi_1}{dt} &= \frac{\partial}{\partial x} D_1 \frac{\partial \phi_1}{\partial x} - (\Sigma_{a,1} + \Sigma_{s,1}) \phi_1 + (1 - \beta) (\nu \Sigma_{f,1} \phi_1 + \nu \Sigma_{f,2} \phi_2) + \lambda C \\
\frac{1}{\nu_2} \frac{d \phi_2}{dt} &= \frac{\partial}{\partial x} D_2 \frac{\partial \phi_2}{\partial x} - \Sigma_{a,2} \phi_1 + \Sigma_{s,1} \phi_1 \\
\frac{\partial C}{\partial t} &= -\lambda C + \beta (\nu \Sigma_{f,1} \phi_1 + \nu \Sigma_{f,2} \phi_2)
\end{align*}
\]

The arguments are left out, but everything except \( \beta \), \( \nu \) and \( \lambda \) is both time- and space-dependent. The equations are valid in both the core and the reflector regions. The shell-mode vibration is modelled by letting the boundary between the core and reflector oscillate around the static positions, \( b \) and \(-b\), in a symmetrical way, \( b(t) = b + \epsilon(t) \) and \(-b(t) = -b - \epsilon(t) \).
The equations in (18) are solved by an adiabatic approach, i.e. splitting the fluxes into a time-dependent part, amplitude factor \( P(t) \), and a space-time dependent part, shape function \( \psi(x,t) \), as follows.

\[
\begin{align*}
\phi_1(x, t) &= P(t) \cdot \psi_1(x, t) \\
\phi_2(x, t) &= P(t) \cdot \psi_2(x, t)
\end{align*}
\]

\[
P(0) = 1
\]
\[
\psi_{1,2}(x, 0) = \phi_{static}^{1,2}(x)
\]

When using the adiabatic approximation a new normalization is needed. The shape function can be normalized by using the adjoint function, \([42]\). Using (19) in (18) and multiplying with the adjoint flux, subtracting the static equations and integrating over the reactor volume one ends up with the following equations after using the normalization condition.

\[
\begin{align*}
\frac{dP(t)}{dt} &= \frac{\rho(t) - \beta}{\Lambda(t)} P(t) + \lambda_c(t) \\
\frac{dc(t)}{dt} &= \frac{\beta}{\Lambda(t)} P(t) - \lambda_c(t)
\end{align*}
\]

Linearising by splitting all time-dependent quantities into a static and a small deviation part, neglecting second order terms, \( \delta P \) can be expressed in the frequency domain as:

\[
\delta P(\omega) = G_0(\omega) \cdot \delta \rho(\omega)
\]

Here \( G_0(\omega) \) is a transfer function, which is similar to the ordinary zero power reactor transfer function. Since the frequency of interest for the shell-mode vibrations, 20 Hz, is within the so-called plateau region the transfer function is constant and equal to:

\[
|G_0(\omega)| \approx \frac{1}{\beta}
\]

Using this it is possible to express \( \delta P \) in the time-domain and by inserting this in (19) and linearising, splitting into static and small deviation parts, neglecting second order terms. The expression of the space-time dependent neutron noise is:

\[
\delta \phi_1(x, t) = \frac{\delta \rho(t)}{\beta} \cdot \phi_{static}^{1}(x) + \delta \psi_{ad}^{1}(x, t)
\]

\[
\delta \phi_2(x, t) = \frac{\delta \rho(t)}{\beta} \cdot \phi_{static}^{2}(x) + \delta \psi_{ad}^{2}(x, t)
\]

Here the perturbed adiabatic shape function is used. The adiabatic approximation is valid if the perturbation is assumed to be small. Then the variation in time of the shape function is relatively small. It is then possible to calculate the perturbed adiabatic shape function at any time, as \( \delta \psi_{ad}^{i}(x, t) = \psi_{ad}^{i}(x, t) - \phi_{static}(x) \), where \( \psi_{ad}^{i}(x, t) \) is a static flux calculated from new k-eigenvalue equations, i.e. (16) and (17) with a larger core. \( \delta \rho(t) \) is calculated, using the new \( k \), as:

\[
\delta \rho(t) = 1 - \frac{1}{k(t)}
\]
5. Core-barrel vibrations

All this gives the neutron noise induced by shell-mode core-barrel vibrations in a 1-D 2-group 2-region reactor in the adiabatic approximation.

5.3 The full space-frequency dependent model

The other way of calculating the noise induced by the shell-mode vibration in a 1-D model is to make a full space solution in the frequency domain. First, the static cross-sections, in both the core and reflector region, are written as:

\[ \Sigma(x) = \{ 1 - H(x - b) \} \Sigma^c + H(x - b) \Sigma^r \]  

Here \( H(x) \) is the unit step function. With a vibrating boundary \( b = b + \varepsilon(t) \), the time-dependent cross-sections are written as

\[ \Sigma(x, t) = \{ 1 - H(x - b - \varepsilon(t)) \} \Sigma^c + H(x - b - \varepsilon(t)) \Sigma^r \]  

Splitting the cross-sections into a static and a time-dependent part and making a Taylor expansion, assuming a small vibration, gives:

\[ \Sigma(x, t) = \Sigma(x) + \varepsilon(t) \delta(x - b) \delta \Sigma, \quad \delta \Sigma = \Sigma^c - \Sigma^r \]  

Using this together with a splitting into a static and a small deviation part of the fluxes and delayed neutron precursors in (18) and linearising by neglecting second order terms, equations in the frequency domain are obtained for the noise in the core and the reflector, which can be written in a condensed form as

\[ \hat{L}^c(x, \omega) \hat{\delta \phi}^c = \varepsilon(\omega) \delta(x - b) \hat{S}(x) \phi^{static} \]  

\[ \hat{L}^r(x, \omega) \hat{\delta \phi}^r = \varepsilon(\omega) \delta(x - b) \hat{S}(x) \phi^{static} \]  

Here the vector fluxes in respective regions represent both the fast and thermal fluxes. The matrices \( S \) and \( L \) are given in paper V. On the right hand side in (28) it is not indicated if the static flux is taken from the core or the reflector. But since they are equal at the interface, \( x = b \), it does not matter which one is used. The neutron noise can be complex and the solution to (28) is given as:

\[ \begin{align*} 
\hat{\delta \phi}^c(x, \omega) &= \tilde{A}_3 \begin{bmatrix} 1 \\ C_\kappa(\omega) \end{bmatrix} \frac{\sinh(\tilde{\kappa}_1(|x| - a))}{\sinh(\tilde{\kappa}_1(b - a))} + \tilde{A}_4 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{\sinh(\tilde{\kappa}_2(|x| - a))}{\sinh(\tilde{\kappa}_2(b - a))} \\
\hat{\delta \phi}^r(x, \omega) &= \tilde{A}_1 \begin{bmatrix} 1 \\ C_\mu(\omega) \end{bmatrix} \cos(\tilde{\mu}x) + \tilde{A}_2 \begin{bmatrix} 1 \\ C_\eta(\omega) \end{bmatrix} \frac{\cosh(\tilde{\eta}x)}{\cosh(\tilde{\eta}b)} 
\end{align*} \]  

The boundary conditions of zero fluxes at the extrapolated boundaries \( x = \pm b \) have been used already in (29). The coefficients, \( \tilde{A} \), are determined from the following matrix equation:
The matrix $M$ consists of the interface conditions and is given in paper V. Now, the neutron noise from the shell-mode vibrations in a 1-D reactor is calculated in two different ways. The analytical solutions for the two cases are found in equations (23) and (29).

### 5.4 Comparison of numerical and analytical solutions

The thermal noise, denoted with subscript 2, is the one of interest, since the thermal neutrons are the ones detected by neutron detectors in the power plant. The phase from the second analytical solution is easily calculated from the complex neutron noise in (29). Since the noise is complex, it is possible to have all values between $\theta$ and $-\pi$. On the other hand, in the first case, with (23), the phase is calculated as either in-phase, $\theta$, or out-of-phase, $-\pi$, i.e. only two discrete values are possible. In Fig. 15 the thermal noise from both analytical solutions is shown. The most interesting feature is the large local out-of-phase behaviour close to the boundary, where the noise also has a large absolute value. The out-of-phase behaviour means that the thermal noise close to the interface has an opposite value as in the rest of the system, i.e. a decrease of the flux at the interface and an increase in the rest of the reactor. The global increase is due to the increase in reactivity, $\delta p > 0$. The decrease at the interface arises from the fact that fission material is added into the reflector region giving more absorption of thermal neutrons, and moderator material is removed, causing less slowing down of fast neutrons to thermal ones. Hence, the local decrease of the thermal flux in the calculations can be explained.

The thermal noise from the full space-frequency dependent analytical solution and the numerical simulator is shown in Fig. 16. The phase behaviour is very much the same in both cases, but there is some deviation in the absolute values. The difference in the absolute value is probably due to the fact that the numerical simulator uses nodes instead of continuous space. Hence, the perturbation at the boundary is spread out over a whole node rather than existing in a point (Dirac-delta function).

The conclusions of these calculations are that there is a large local component of the noise close to the interface between the reflector and the core and that it is indeed possible to have an out-of-phase behaviour between in-core, close to the reflector, and ex-core detectors. However, it is not clear if this out-of-phase behaviour is present in a 2-D model or not. Hence, the next step would be to expand the calculations into 2-D. If there is a large local out-of-phase behaviour close to the interface between the core and the reflector, it could be possible to measure it with detectors placed in the outermost fuel-assemblies. If this turns out to work it could be used as a diagnose tool for the shell-mode core-barrel vibrations.
Fig. 15. The absolute value, a), and the phase, b), of the thermal noise calculated with the two different analytical approaches.

Fig. 16. Absolute value, a), and phase, b), of the thermal noise from shell-mode core-barrel vibrations. The dotted line is calculated with the numerical simulator and the full line is the full space-frequency dependent analytical solution.
6. Concluding remarks

In this thesis three different problems were investigated, two of them within the area of classical reactor noise diagnostics, core-barrel vibrations and detector tube impacting. The relatively new mathematical tool, wavelet analysis, is used in two of the problems, detector tube impacting and two-phase flow identification. Especially the use of the continuous transform and the two-dimensional transform are new concepts in the research going on at the department. Both of the new wavelet methods seem promising for further development and application within the two projects and probably in other projects as well. Whenever there is non-stationary process, wavelet analysis could be of use. Hopefully, there will be several different new as well as old problems to tackle with the wavelets in the continuing work connected to the thesis.

As mentioned above in Sections 3 to 5, there is still some work to be done in each of the projects. As regards the core-barrel vibrations, the next step would be to extend the model into two dimensions and verify the results with measurements.

Concerning the detector tube impacting, there is already a new measurement under way. This new measurement will be performed during the ongoing fuel cycle of Ringhals-1 and hopefully there will be time for visual inspections during the next outage, which can verify the result. Moreover, there is need to explore the continuous wavelet transform to get a better understanding and knowledge of its features and possible applications. A physical interpretation of the results is of major importance for further use of the continuous transform.

The next step in the two-phase flow identification project would be to apply the energy feature extraction to noise-free neutron radiographic images. However, it is not clear whether it is possible to get such images or not.

The focus of using wavelets in the noise diagnostics will continue in both projects mentioned above and in other projects as well, e.g. estimation of MTC where a so-called wavelet spectrogram could be of use.

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8. References

8. References


8. References


### 9. Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
</tr>
<tr>
<td>APSD</td>
<td>Auto Power Spectral Density</td>
</tr>
<tr>
<td>BWR</td>
<td>Boiling Water Reactor</td>
</tr>
<tr>
<td>IR-index</td>
<td>Impact Rate index</td>
</tr>
<tr>
<td>LPRM</td>
<td>Local Power Range Monitor</td>
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<tr>
<td>MSE</td>
<td>Mean Square Error</td>
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<tr>
<td>MTC</td>
<td>Moderator Temperature Coefficient</td>
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<td>NPP</td>
<td>Nuclear Power Plant</td>
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<tr>
<td>PWR</td>
<td>Pressurised Water Reactor</td>
</tr>
<tr>
<td>STFT</td>
<td>Short Time Fourier Transform</td>
</tr>
<tr>
<td>SKC</td>
<td>Swedish Centre of Nuclear Technology</td>
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<td>SKI</td>
<td>Swedish Nuclear Power Inspectorate</td>
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World Scientific
CLASSIFICATION OF TWO-PHASE FLOW REGIMES VIA IMAGE ANALYSIS BY A NEURO-WAVELET APPROACH

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A non-intrusive method of two-phase flow identification is investigated in this paper. It is based on image processing of data obtained from dynamic neutron radiography recordings. Classification of the flow regime types is performed by an artificial neural network (ANN) algorithm. The input data to the ANN are some statistical functions (mean and variance) of the wavelet transform coefficients of the pixel intensity data. The investigations show that bubbly and annular flows can be identified with a high confidence, but slug and churn-turbulent flows are more often mixed up in between themselves.

1. Introduction

Two-phase flow patterns are usually classified into four classical so-called flow regime types. These are 1) bubbly, 2) slug, 3) churn-turbulent, and 4) annular flow regimes (see Fig. 1). Recognition and, possibly control, of the flow regime types is essential in numerous practical applications. Although the flow classification can be done reliably in fully instrumented channels in which thermocouples, pressure transducers, flow-meters etc. are available, a more challenging alternative would be to use non-intrusive methods. In this field, the availability of the methods is not as wide.

Non-intrusive methods so far have been based on radiation attenuation measurements, such as X-ray [1] or gamma-rays. These methods are usually based on the detection of collimated rays penetrating the flow, and the processing of the variation of the intensity, modulated by the flow, by various statistical methods (probability distributions, auto- and cross-correlations and spectra). A qualitatively different approach, which will be pursued in this paper,

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is to use image analysis. After all, the concept of flow regime arises from an intuitive judgement of the topology of the flow, based on visual observation.

Such images of two-phase flow can be produced in transparent pipes easily with visible light, but in metal pipes neither X-rays or gamma-rays are applicable. X-rays do not penetrate the wall, and gamma-rays can not, in general, be produced with desired intensity such that an image with good contrast and dynamics can be achieved. However, dynamic neutron radiography has been developed to produce images of two-phase flow in metal pipes at the Kyoto University Research Reactor Institute [2]. Some of these measurements were made available for us, and were used in the present analysis. Some sample images are shown in Fig. 1.

2. The classification algorithm

Our objective in this work was to find an algorithmic identification method. Artificial neural networks (ANNs) were selected for this purpose. Static images for training and test data were provided as follows. IN the recording at our disposal, the heating was gradually increased starting with cold water. During the measurement, all four flow regimes occurred in sequence, with a smooth transition between the different regimes. The sections of the four regimes were identified. From each sequence, 200 tiff images were extracted and used in the classification. Each image consisted of about 60 000 pixels (411*153 pixels).

In this work a simple feed-forward network with back-propagation was used consisting of an input layer, an output layer and one hidden layer (Fig. 2). As the figure shows, 40 nodes were used in the hidden layer, two different types of transfer functions between the layers, and four output nodes, representing the four different regimes. Before the output nodes, thresholding was used; values about the threshold were converted
to unity and values below to zero. An identification was considered definite (but obviously not necessarily correct) if and only if one of the output nodes showed unity and the others zero. The regime type was defined by which output node was “fired”.

3. **Generation of the input data with wavelet transform**

The information content in the images, represented by the about 60,000 pixels, needs obviously reduced in dimension before using it for input. Wavelet transform seems to be a very effective tool to achieve this goal ([3], [4]). Wavelet transform coefficients are often better (more sensitive) features in pattern recognition tasks than the original data. What regards two-phase flow, the four flow regimes have structures that show up spatial variations at different scales. Hence, wavelet coefficients from a multi-resolution analysis (or the original input data, after a wavelet multiresolution analysis) seem to be suitable input data. To further reduce the number of input data, at each resolution scale, one can condense the coefficients further, i.e. use their first two statistical moments (mean and variance).

As it turned out, the quality of the radiography recordings had a relatively poor quality in terms of quantitative usage for the present purpose such that a full 2-D multiscale resolution analysis was not practical. After the step of splitting the data into approximations and details as

\[
S_{ij}(x, y) = S_{i}(x, y) + D_{ij}^h(x, y) + D_{ij}^v(x, y) + D_{ij}^d(x, y)
\]  

no useful information content remained in the details \(D_{ij}\). Thus, only the data of the first level approximation were used. These were further condensed into two parameters, the mean and the variance. Hence the network used in this work had two input nodes. This will be improved significantly in the future, as work is going on with better quality input data from new measurements.

4. **Results**

Fig. 3 shows the mean of the wavelet coefficients corresponding to the first approximation of the multiresolution analysis for various wavelets, as functions of the sample number. The figure shows that this parameter has a relatively good discrimination power. The variance of the same values, not shown here, shows more overlapping, and hence less discrimination.

The results are shown in Fig. 4 where tests were made with 200 samples for each regime. The results show that the identification was quite successful. In particular the success ratio was 100% for annular flow, and very close to 100%
Fig. 3. The mean of the wavelet coefficients for the different flow regimes as functions of the sample number.

Fig. 4. The results of the identification procedure with different wavelets.

also for bubbly flow. Slug and churn-turbulent flow were mistaken for each other in a few percents of the cases.

This pilot study shows the possibilities of the method, even if the quality of the input data did not make the full power of the wavelet pre-processing process visible. Further work is going on to test and develop the method further with better quality input data.

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References

PAPER II
Classification of Two-Phase Flow regimes via Image Analysis and a Neuro-Wavelet Approach

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Classification of Two-Phase Flow regimes via Image Analysis and a Neuro-Wavelet Approach

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ABSTRACT

A non-intrusive method of two-phase flow identification is investigated in this paper. It is based on image processing of data obtained partly from dynamic neutron radiography recordings of real two-phase flow in a heated metal channel, and partly by visible light from a two-component mixture of water and air. Classification of the flow regime types is performed by an artificial neural network (ANN) algorithm. The input data to the ANN are some statistical moments of the wavelet pre-processed pixel intensity data of the images. The pre-processing used in this paper consists of a one-step multiresolution analysis of the 2-D image data. The investigations of the neutron radiography images, where all four flow regimes are represented, show that bubbly and annular flows can be identified with a high confidence, but slug and churn-turbulent flows are more often mixed up in between themselves. The reason for the faulty identifications, at least partially, lies in the insufficient quality of these images. In the measurements with air-water two-component mixture, only bubbly and slug flow regimes were available, and these were identified with nearly 100% success ratio. The use of the wavelet preprocessing, as compared to using the raw data without pre-processing, did not have an influence on the success ratio of the identification; however, it decreased the training time (number of epochs) with about a factor 100.

KEYWORDS: Two-phase flow classification; Image analysis; Neural networks; Wavelet analysis.

1. INTRODUCTION

Two-phase flow patterns are usually classified into four classical so-called flow regime types. These are 1) bubbly, 2) slug, 3) churn-turbulent, and 4) annular flow regimes (see Fig. 1.). Recognition and, possibly control, of the flow regime types is essential in numerous
practical applications. For instance, in two-phase flow calculations, in order to use the correct algorithm in a fluid dynamics code, the regime type in question must be known in advance, so that the right expression can be chosen for e.g. the interfacial shear coefficient or some other coefficients like the heat transfer coefficient.

Although the flow classification can be done reliably in fully instrumented channels in which thermocouples, pressure transducers, flow-meters etc. are available, a more challenging alternative would be to use non-intrusive methods. In this field, the availability of the methods is much more limited. Non-intrusive methods so far have been based on radiation attenuation measurements, such as X-rays (Vince and Lahey, 1982) or gamma-rays (Chan and Banerjee, 1981; Chan and Bzovey, 1990; Kok et al., 2001). These methods are usually based on the detection of collimated rays penetrating the flow, and the processing of the variation of the intensity, modulated by the flow, by various statistical methods (probability distributions, auto- and cross-correlations and spectra).

A qualitatively different approach, which will be pursued in this paper, is to use image analysis, in combination with a neural network based identification method with a wavelet pre-processor. After all, the concept of flow regime arises from an intuitive judgement of the topology of the flow, based on visual observation. Correspondingly, image analysis, although not combined with so-called intelligent computing methods, has quite recently been recognised as one possible avenue of flow identification (Ulbrich et al., 2002; Kashdan et al. 2004a, b). Images of two-phase flow are though not easy to produce for real high-pressure high-temperature flows. Ulbrich et al. (2002) used a water-air two-component flow with visible light, and this is one of the alternatives that will also be investigated in this paper. Kashdan et al. (2004b) investigated particle beam in air. Images of a flow can be produced in transparent pipes easily with visible light, but in metal pipes, needed for real two-phase flow, neither X-rays or gamma-rays are applicable. X-rays do not penetrate the wall, and

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Fig. 1. Schematic drawings of the different phases of vertical two-phase flow.
gamma-rays can not, in general, be produced with desired intensity such that an image with good contrast and dynamics can be achieved.

However, in the past one and a half decade, dynamic neutron radiography has been developed to the extent that it can produce dynamic images of two-phase flow in metal pipes. Such measurements were made at the Kyoto University Research Reactor Institute (Mishima et al., 1982). Some of these measurements were made available for us, and were used in the present analysis. However, the quality of these recordings turned out to be insufficient for the purpose of effective image analysis. Hence we have set up model experiments with coloured water - air loop in a plexiglas slab pipe. In these experiments visible light and an ordinary digital video camcorder was used. This way images with much better contrast and sharpness could be obtained. The purpose of these latter experiments was to check the algorithms, elaborated first in connections with the radiography images, and see if the improvement in the input data quality led to improvement in the flow regime classification. More details on the data acquisition and some image examples will be shown in the next section.

The image analysis, leading to the flow regime identification, is based on wavelet transform, i.e. multiresolution analysis, and neural identification. Such methods have been used for two-phase flow identification (Wu et al., 2001), but the input signals were simple process signals and not images. Multiresolution analysis is capable of extracting information from a one- or two-dimensional structure, corresponding to different scales. Such a procedure leads also to the reduction of the dimensionality of the information represented by the image, and can be used as input data to an Artificial Neural Network (ANN) for the classification. This requires training samples (images) being available to train the network, where the flow regime type is known. The trained network can be fed by similar, but unknown type, images as input, and used to classify the type or regime. It is this program that has been performed in the present paper and which will be described more in detail below.

2. THE FLOW IMAGES

Two different types of images were used. The first one was obtained with neutron radiography, and it comprises all four flow regimes (Fig. 2). The second one was obtained with visible light of only bubbly and slug flow (Fig. 3).

The way of producing dynamic neutron radiographic images, and in particular for two-phase flow, can be found in the literature (Mishima et al. 1988, 1999a; Balasko et al., 1987, 1999). The essence is to use a collimated beam of thermal neutrons from a reactor to illuminate the object, in this case the two-phase flow in the pipe. After penetrating the pipe and the flow, where some neutrons are removed by scattering and absorption in the sample, the beam is directed to a neutron converter, which converts the neutron beam into visible light. This light, after a reflection by a mirror (in order to filter out gamma-rays that accompany the neutrons for most neutron sources) are recorded with an ordinary CCD video camera.

The images used in this work were recorded at the Kyoto University Reactor Research Institute (KURRI), by the Division of Nuclear Engineering (Mishima et al., 1998). During the experiment, the water was circulated in a loop with an overflow tank, and was heated up. The experiment started with pure water in the loop, and the heating was increased. With
increasing heating, all four flow regimes occurred in sequence. The measurement results were available for us in form on an analogue VHS recording in NTSC format. Form the whole sequence, four sections, corresponding to the four different regimes, were selected, and for each regime a number of individual frames were selected. Each frame was then saved as an individual jpg file.

Although, by a visual inspection of the video, the various flow regimes can be clearly seen, the contrast and sharpness of the static images obtained by the above procedure was relatively poor. Some examples are shown in Fig. 2. One reason for the poor quality is the digitization of the original analogue recording; the fact that the pictures were not recorded with digital technology leads to a high level of background noise.

To obtain better quality images, a simpler experiment was set up at our department. In this case a thin transparent plastic pipe, filled with coloured water, was used to generate the images. The images were recorded with a digital video camera. The two-phase flow was simulated by injecting air in the bottom of the pipe. By this way only two-component flow could be created, with only bubbly and slug flow regimes. The same way as with the radiography recordings, the a number of frames corresponding to the (now only two) available regime types were selected, and converted as individual jpg files. Some examples are shown in Fig. 3. It is seen that due to the digital signal handling throughout, and the better contrast and sharpness of the original image to be recorded, the image quality of these files is much superior to the radiography ones.

It has to be noted that the difficulty we encountered with the neutron radiography images is not due to the technique itself. Very high quality images of two-phase flow, and even steam explosions, have been produced lately with pulsed reactors and/or high speed digital cameras (Mishima et al., 1999b). It is the intention of the present authors to repeat the analysis presented here also on better quality radiography images when and if they will become available for us.

Fig. 2. Images of two-phase flow using neutron radiography.
In the present study, from the neutron radiographic images a total of 200 frames from each of the four regimes were used for the identification and classification process, whereas 75 frames from each of the two regimes were used from the experiment with visible light.

3. WAVELET DATA PROCESSING

The information in each image in the digital (jpg) files is contained in approximately 60,000 pixels. Naturally, this information content needs to be reduced in dimension before using it as input to the neural network. Wavelet transform is known to often be an effective tool to achieve this goal (Addison, 2002; Bergh et al., 1999). Apart from data compression, wavelet transform coefficients are also often better (more sensitive) features in pattern recognition tasks than the original data. What regards two-phase flow, the four flow regimes have structures that show up spatial variations at different scales. Hence, wavelet coefficients from a multi-resolution analysis (or the original input data, but reduced by a one-step wavelet multiresolution analysis) seem to be suitable input data.

In order to improve the classification process of the flows, it is advisable to pre-process the images using wavelet techniques before extracting the input data for the Artificial Neural Networks (ANNs), (Hazarakia et al., 1997; Fadhel and Bhattacharyya, 1999; Verma et al., 2002; Kandaswamy et al., 2003). The advantages using wavelet transform include some noise reduction and feature extracting at different scales and directions of the images.

The one-dimensional wavelet transform maps a time signal into a time and frequency signal at different frequency levels, $N$, (Addison, 2002; Bergh et al., 1999; Mallat, 1999). At each level the signal is decomposed into an approximation and a detail. It is possible to do both a discrete (with discrete frequency levels), and a continuous (with continuous frequency levels), transform. In the case with an image, one uses the two-dimensional wavelet

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Fig. 3. Images of two-phase flow using visible light and coloured water.
transform, which maps a two-dimensional signal (image), in spatial rather than time coordinates, into the two coordinates at different frequency (wave number) levels. The two-dimensional transform can also be done both discrete and continuous in the frequency coordinate or level.

At the first level of a 2-D discrete wavelet transform, the coarsest level of approximation coefficients, $A_1$, contains 25% of the information of the original image, $S_0$. The approximation coefficients, $A_1$, at level 1, can be used to make a reconstruction of an approximation, $S_1$, of the original image. In the same way the detail coefficients, $T^h_1$, $T^v_1$, and $T^d_1$ (Fig. 4) containing the high frequency information of the image, at level 1 can be used to reconstruct horizontal, $D^h_1$, vertical, $D^v_1$, and diagonal, $D^d_1$, details of the original image. Adding the details to the approximation one can reconstruct the original image completely without any loss of information.

$$S_0(x, y) = S_1(x, y) + D^h_1(x, y) + D^v_1(x, y) + D^d_1(x, y)$$  \hspace{1cm} (1)

It is possible to do the transformation into lower levels where the details at each level contain the information of the signal corresponding to the frequency of that level. But in this work, only the first level of transformation was found to be useful to apply. The coefficients and the corresponding reconstructed images contain the same information, hence it is possible to use the coefficients when extracting input data for the ANNs. The reconstructed images are useful when displaying the transform.

![2D Wavelet Transform Diagram](image)

**Fig. 4.** First level of the 2D wavelet transform. $S_0$ is original image or data, $A_1$ is the set of the first level approximation coefficients and $T_1$ are the first level detail coefficients in each of the three directions, vertical, horizontal and diagonal.

### 4. IDENTIFICATION WITH ANNS

As mentioned above, it is possible to improve the classification if the input data are pre-processed with wavelets before using them in the ANN. With the wavelets it is possible to extract features which are not visible in the original data. One set of features, mentioned in Hazarika et al. (1997), is the mean and variance of the first level approximation coefficients. Another possibility would be to use the energy of the different wavelet details. The energy of each detail is defined as the sum of the square of the absolute value of the detail coefficients, (Addison, 2002).

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In the first run, the neutron radiography images were analysed. Input data from the raw images were extracted and compared with the wavelet pre-processed input data. In this case the mean and variance from both the raw images and the first level approximation coefficients were used first, as reference input data. It turned out that, due to the poor image quality, no other features could be successfully used in the classification procedure.

In the second case the images produced with visible light, that had much better quality, were used. Again the mean and variance of the raw data were used as reference. Here, however, wavelet transform pre-processing did lead to improvement. With the wavelet pre-processing, the energy of the first level of detail coefficients and their variances were used, giving a total of 6 inputs for the ANN (see Fig. 5).

\[
E_1^d = \sum |T_{1,i}^d|^2
\]  

(2)

As usual with wavelet analysis, the choice of the right wavelet for the task at hand is not self-obvious. One guideline of choosing a suitable wavelet is to select one, which has the same features as the data analysed. For this classification task the following wavelets were tested: Daubechies of order 8 (db8), Symmlet of order 6 (sym6), Coiflet of order 4 (coif4),
Daubechies of order 1 (Haar) and biorthonormal (bior3.1). These are all available in the Wavelet Toolbox in MatLab (Anon, 2000).

4.1 Classification using artificial neural networks

ANNs are capable to tackle very complicated tasks, including non-linear classification problems. The backpropagation (BP) algorithm is the most frequently used algorithm for the training of such networks (Pazsit and Kitamura, 1997). We have used the multi-layered perceptron (or simply the feed-forward network) consisting of an input layer, an output layer and one hidden layer. The network receives input through the nodes in the input layer, from which the signals propagate forward to the nodes of the consecutive layer and output signals are produced in the output layer. In the backward phase, error signals are propagated backward through the network and some parameters are adjusted in reference to the error signals.

The performance of ANNs depends on the proper choice of the input parameters. We have investigated the performance of the ANN for various input parameter sets. The number of the input and output nodes is defined by the problem itself. For the radiographic images, the number of input nodes was 2 and for the visible light images it was 6, as mentioned before. All the input feature vectors were normalized so that they fall in the range [-1,1]. For each type of flow, a corresponding output class is associated. The ANN has 4 output nodes for the radiographic case and 2 output nodes for the visible light case, corresponding to the 4 or 2 different flow types (Fig. 6). The target value, during training, for each class contains value of 0.9 for the correct category and three or one dummy variables with value of 0.1.

Since the output layer chosen for this classification task has a \( \log\text{-}sigmoid \) transfer function, the output values range from 0 to 1, and thresholding has to be performed on the output data, to get 0 or 1, when classifying. Two different threshold levels were used, 0.5 and 0.7. All output values larger than the threshold are set to unity and all other to zero. If the output data, after the thresholding, are all four zeros, the corresponding image is classified as unknown or unclassified flow regime. The same applies for the case of more than one non-zero value. Lower threshold makes the classification less certain but a too high threshold will classify many images as unknown.

Fig. 6. The Artificial Neural Network used in the classification process of the neutron radiograpic images. The thresholding is only used during testing.
The optimal number of nodes in the hidden layer, the training algorithm and the activation functions were determined by trial and error. Tan-sigmoid function was used for the hidden layer and log-sigmoid for the output layer. A few of the modified backpropagation (BP) algorithms such as adaptive learning rate, resilient BP, scaled conjugate gradient and gradient descent algorithm with momentum were examined for training the ANN. Cross-validation was used to estimate which learning ANN model will perform the best on the problem at hand. For each of the models a 5-fold cross-validation over the training set was used, which means that 1/5th of the training data was used as a validation set and the process was repeated with non-overlapping rotations. In the case of the neutron radiographic images, each learning model was trained with 640 samples for the four various types of flow, and then they were tested on one subset of 160 samples which was not used during training. For the visible light images, the training set consisted of 120 samples and the test subset of 30 samples (Fig. 5). The resilient backpropagation (RP) algorithm was found to have the highest average test set score. Namely, the classification efficiency with RP algorithm was 100% (the percentage of the flows that were correctly classified) when the recall test was performed on the training set, and 95% when the recall test was made on the subset not used during training. The best performance was obtained for the training and validation test set with an ANN structure consisting of one hidden layer having 40 nodes. The default performance function for feedforward networks was the mean square error, i.e. the average squared error between the network outputs and the target outputs. The ANN training was performed until the maximum number of epochs, set to 30000, was reached or the mean square error, MSE, target value of $10^{-3}$ was achieved. All calculations were carried out by using the toolboxes available with the technical computing software MATLAB (Anon, 2000).

5. RESULTS AND DISCUSSION

5.1 Neutron Radiographic images

The performance of different wavelets, db8, sym6, coif4, haar and bior3.1, on the classification efficiency has been investigated. The classification efficiency is defined as the percentage ratio of the number of flow pictures correctly classified to the total number of pictures corresponding to one type of flow. Average efficiency of the flow classification for each type of the chosen wavelets by using ANN with mean values and variance as input and for threshold levels of 0.5 and of 0.7, is depicted in Fig. 7. The average efficiency is shown with error bars (standard deviation). Almost the same result is achieved independent of which wavelet type that is used. Though, coif4 has a slightly lower percent of correct classified regimes and Haar has the best classification efficiency.

Clearly a threshold of 0.5, with accuracy of about 95%, is better than a threshold of 0.7, which only has about 87% correctly classified images. But even by using the mean and variance of the original image values, the efficiency is the same. So there is not much improvement, in classification, using the wavelet transform in this case.

All of the different wavelets classified the annular flow with 100% efficiency, which is also the case for the original data. In Fig. 8 each bar represents the 200 images from each flow and the different colours show which regime the images were identified with. White is for unknown or unclassified images. As it is seen, a very few images turn out to be unclassified for a threshold of 0.5. The most difficult regimes to classify for both the wavelet
and the original data are the slug and churn flow. This is of course due to the fact that these flows have similar features.

5.2 Visible light images

The same five wavelets as above were used also in the case with visible light images. The results are partly similar to the radiography images. Namely, there is not much increase of the classification efficiency when using wavelets compared to the features of the raw images. However, in this case there is a large improvement in the number of epochs (iterations) used when training the ANN. When using the wavelet pre-processed data, the number of epochs is reduced with a factor of 100, compared to the raw data input (Fig. 9). With the raw data, the number of epochs always exceed the maximum number of 30 000, before reaching the \( \text{MSE} \) target values of \( 10^{-3} \). The classification efficiency is also slightly

![Fig. 7. Classification efficiency for the different wavelets and the original data, threshold 0.5 to the left and 0.7 to the right.](image1)

![Fig. 8. Result of the classification of the neutron radiographic two-phase flow images using input data pre-processed with the db8 wavelet, with a threshold of 0.5 to the left and 0.7 to the right.](image2)
better in this case, compared to the neutron radiographic case. This is presumably also attributed to the better quality of the images. For the wavelet pre-processed data one slug flow image was classified as bubbly and the rest were correctly classified. That is, $149/150 = 99.33\%$ for wavelet pre-processing, whereas success ratio of classification with the raw image inputs was $146/150 = 97.33\%$. The result was the same for all wavelets with a threshold of 0.7.

Fig. 9. Depicted is the number of epochs used when training the network. The input data are wavelet pre-processed visible light images.

6. CONCLUSIONS

The methodology used in this paper appears to be suitable to classify two-phase flow regimes by image analysis, in a non-intrusive way. The original idea was to use data from dynamic neutron radiography of two-phase flow in a heated metal pipe. Such measurements have been performed and by now good quality images can be obtained. Unfortunately no such measurements were available to us, only old analogue recordings. However, a proof-of-principle investigation was made of the applicability of the image analysis method, with positive outcome. The potential of the method were then investigated by using experiments with visible light on a water-air mixture. In this experiment only a few basic flow structures could be achieved, but the images had better quality. Hence it was possible to prove the extended efficiency of the wavelet pre-processing of the data before the neural identification. The two different measurements together show that on-line, non-intrusive identification of two-phase flow in a metal pipe can be achieved with processing dynamic neutron radiography images with a neuro-wavelet approach.

7. ACKNOWLEDGEMENT

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Investigation of detector tube impacting in the Ringhals-1 BWR

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Abstract: Neutron noise measurements were made in two consecutive fuel cycles in the Swedish BWR Ringhals-1 with the purpose of diagnostics of vibrations and impacting of detector strings. The analysis was based on two different methods. The first was the traditional spectral analysis, which uses autospectra and the coherence and phase between detectors in the same string, for a qualitative judgement. The second method was based on calculating the so-called impacting rate by a wavelet-based procedure, developed at the Department of Reactor Physics at Chalmers recently, and which was previously tested on data from the Oskarshamn BWR. Also a new method, wavelet-based coherence, was tested with success. The first measurement series was not suitable for a wavelet analysis, because of the low sampling frequency. The traditional method is more robust; however, it only gives a qualitative result and requires the subjective decision of a noise expert.

Based on the results of the analysis, with emphasis on the traditional method, the detector tubes were divided into three groups with respect to the severity and likelihood of impacting. For the first series of measurements, these conclusions could be checked against visual inspection of the fuel assemblies during refuelling after the cycle, in order to find impacting damage. A good correlation between the prediction of the analysis and the inspection results was found.

Keywords: noise analysis, neutron noise, detector impacting, spectral analysis, APSD, coherence, phase, wavelet analysis.

Biographical notes: Carl Sunde received his MSc in engineering physics from Chalmers University of Technology, Sweden, in 2002 and is working as PhD student at the Department of Reactor Physics of Chalmers. His research interests are noise diagnostics, diagnostics of two-phase flow and intelligent computing methods including artificial neural networks and wavelet analysis.

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1 Introduction
Excessive vibrations and impacting of detector strings in BWRs have been a known problem for a long time [1-6]. The situation is illustrated in Figure 1. High flow turbulence around the detector from the bypass flow at the lower core support plate can cause strong vibrations of the detector tube (also called instrument tube, or detector/LPRM string). Various methods of detecting impacting from the signals of the vibrating neutron detectors have also been elaborated and tested during the past few decades [3-9].

These methods can be divided into two categories: traditional or spectral analysis based methods, and wavelet analysis methods. The spectral analysis methods are based on assumptions concerning how the spectral properties of the APSD, and in particular the properties of the vibration peak(s) and the phase of the cross-spectra change compared to vibrating but not impacting, or not vibrating, cases. These methods assume stationarity of the signal, and they are mostly qualitative. They have nevertheless been found to work in several cases. As is clear from the description, these methods mostly rely on the knowledge of a "reference base", i.e. vibration characteristics of vibrating but not impacting strings. In other words these methods are in need of some form of calibration. The wavelet-based methods are more absolute, i.e. have less need of calibration, and supply a quantitative measure of the severity of impacting. They do not assume a stationary behaviour, rather they can handle intermittent signals as well. They are, on the other hand, based on a hypothesis, which, although being very plausible, has not been possible so far to prove or disprove in practice. The hypothesis is that each impact of the detector string against the fuel box induces a short, damped vibration of the assembly, with a frequency which is higher than the eigenfrequency of the detector string (for an illustration, see Figure 2). The neutron noise induced by this short-lived vibration is sensed by the neutron detector as a “spike” in a broad-band noise, which can be extracted efficiently by wavelet filtering methods. This latter method has also been verified on measurements taken in several Swedish BWRs [9-11]. In addition, continuous wavelet transform methods were also tested in this work as a novel method for detecting detector impacting. A time-frequency dependent wavelet coherence was calculated, which shows high values in the frequency range of the impacting detector strings around the eigenfrequency of the damped vibrations of the fuel assemblies.

When assessing the performance of the various methods, it should be noted that there is no general assessment available about the performance of the various methods at different reactors. In most cases, one kind of method was used by one diagnostic group at one type of core and corresponding instrumentation. There are also some indications that a method, which was used successfully at one reactor unit, did not perform successfully when transferred to another core. In Sweden, both spectral and wavelet based methods, that worked well at the Barsebäck reactor, did not perform so well when transferred to measurements made on the Oskarshamn reactor [5-6, 9-11]. The wavelet based method, for instance, required further development before it worked just as effectively for the Oskarshamn measurements as for the Barsebäck ones.

In other words, the experience with the transfer of methods from one core to another is relatively sparse. Therefore, it appears interesting to report on the applicability of some, already tested, methods in a new core. In Sweden, vibration analysis has been performed in the Barsebäck and Oskarshamn reactors, but this is the first case when the two methods reported on in this communication were tested on the Ringhals-1 BWR. Unfortunately, only one (the second) out of the two measurement campaigns was suitable for wavelet analysis, whereas post-cycle checking of the caused damage of impacting, which can be used for rating the performance of the diagnostics, is currently available only for the first measurement series. (As of this writing, the fuel cycle in which the second measurement series was made, is still on-going).

The wavelet-based analysis indeed showed that a direct transfer of the method that worked quite effectively for the Oskarshamn measurements, is not possible, and further modifications of the method became necessary. On the other hand, the traditional spectral analysis could be performed for both cycles, and the performance of the diagnostics in the first cycle could be checked during fuel inspection during the revision. A good correlation
was found between the prediction of the diagnostics and the actual damages found. A postcycle confirmation of the second analysis will be made during refuelling in late summer 2004.

2 Principles of analysis

2.1 The spectral analysis method

This method was elaborated and first used during the analysis of the early Barsebäck measurements in 1986 by the Studsvik noise group [5]. Its principles are described thoroughly in [6]. The essence is that in the absence of vibrations, the APSD (auto-spectrum) of the LPRMs, i.e. the individual detectors, is a smooth function of the frequency, and the coherence and phase between any two detectors follows a pattern characteristic for propagating perturbations in the presence of a local noise component (in-core BWR noise). The phase is linear up to 10 Hz, and the coherence shows a periodic peak-sink structure, whose maxima and minima coincide with the zero- and $\pi$ -crossings of the phase.

The presence of weak vibrations can be noticed in the appearance of a relatively narrow peak in the APSD at the fundamental frequency (eigenmode) of the detector string, usually in the range of 2.5 Hz. With an increasing amplitude of the vibrations, the fundamental peak increases; also, new peaks occur at frequencies of the higher harmonics (double and triple of the fundamental frequency). When impacting starts, the vibration peak gets broadened. Concurrently, the linear phase, and to a lesser extent also the sink structure of the coherence, gets distorted with increasing vibrations. The most visible is the distortion of the phase around the vibration frequency; with increasing strength of the vibrations, the phase tends to be zero over an increasing frequency region around the vibration frequency, owing to the simultaneous movement of the two detectors.

It is important that out of the above quantities, only the peak broadening is a clear indicator of impacting. The higher harmonics and phase distortion are, strictly speaking, only an indicator of increasing vibration amplitudes. Nevertheless, a large vibration amplitude is associated with a higher probability of impacting. Experience so far shows that the set of above indicators can be used to a quantitative classification of the probability and severity of impacting into three basic groups as follows:

No vibrations:
- smooth structure of the APSD and the coherence, no peaks
- linear phase between the two detectors in the same string, showing the time delay of the signal (bubble transit time).

Vibrations but no impacting:
- a single narrow peak in the APSD and the coherence
- linear phase distorted in the narrow frequency range of vibrations, the phase is zero there.

Indications of impacting:
- a broad peak in the APSD
- several peaks, mainly a second peak at the double frequency of the fundamental mode
- phase curve distorted and zero over a large frequency range
- The severity of the impacting can be judged by whether or not all three points are fulfilled simultaneously, and also on the quantitative value of the indicators (brodening, distortion etc).

2.2 The wavelet-based method

The first method that has been used in several applications before, is based on discrete wavelet transform and wavelet filtering. The principles of this method are described in [8-11]. An illustration of the assumptions is shown in Figure 2. In case of impact-free vibrations, the detector signal consists of a broadband background noise, and a narrow frequency band
component, both components being stationary. The impacting induces a short, damped vibration of the fuel assembly, which is converted into a short, decaying transient in the detector signal. It is assumed that the eigenfrequency of the transient vibrations is noticeably higher than that of the detector strings, and is in the range 10-15 Hz. These transients or spikes occur in an intermittent manner, and their contribution to the signal APSD is negligible compared to the other two components, due to their small amplitude and sparse occurrence. The task of the wavelet analysis is to detect the presence of such spikes and give a measure of the frequency of their occurrence. This quantity is named the “impacting rate”, and is calculated in an algorithmic way by wavelet filtering.

The essence of the method is a wavelet filtering, based on the discrete wavelet transform (DWT) and a thresholding of the wavelet transform coefficients before inverse transform. A usual problem in such an application is the determination of the filter threshold. Usually, the variance of the background noise is used to set the threshold in an algorithmic way. The background noise level, on its turn, is determined from the high frequency tail of the APSD, for frequencies above the fuel assembly eigenfrequency. In the first application [8] Haar wavelets were used, and the break frequency of the high-pass filter of the background extraction method was set in an ad hoc way at a frequency that was assumed above the fuel eigenfrequency.

Later this method was further developed by using scale-dependent thresholding, i.e. the thresholds are different for each of the different scales in the wavelet multiscale resolution [9-10], when analysing measurements taken in Oskarshamn-2. First a global threshold is calculated [13-15]:

$$\text{thr}_g = 0.3936 + 0.1829 \times \log_2(N)$$

(1)

where N is the length of the signal. Then a scale dependent threshold is calculated as the median absolute deviation:

$$\text{thr}_a(a) = \text{median}(|c(a)|)/0.6745$$

(2)

where c(a) are the wavelet coefficients at scale a. The final threshold at scale a is then:

$$\text{thr}(a) = \text{thr}_g \times \text{thr}_a(a)$$

(3)

The multiscale resolution is performed down to scales corresponding to the frequency of the fuel-box, 10 Hz. The connection between scale and frequency is given by the so-called centre frequency of the wavelet [13]. When analysing the Oskarshamn measurements it was also noticed that, unlike in the Barsebäck case, the choice of the mother wavelet did play a role. Moreover, the algorithm had to be improved to be competitive with other, non-wavelet based techniques. The improvement consists of a final thresholding of the difference V between the wavelet denoised signal $\text{Den}(S)$ and the approximation at the largest scale level A in the multiscale resolution

$$V = \text{Den}(S) - A$$

(4)

using the variance of the background noise.

In analysing the measurements from Ringhals-1 the discrete Meyer wavelet was used without the final thresholding of V. The result is partly a visual display of the remaining spikes that are supposed to represent the impacting effects and thus give an intuitive measure of the impacting frequency or severity, and partly a number called Impact Rate, which is the total number of spikes (impacts) in the signal.

In addition, a new method was tested in the present measurements the first time, based on wavelet coherence. The inspiration for the test of this method was taken from a recent work by Pokol and Por [16]. Similarly to the case of the traditional spectral analysis method, the wavelet coherence was calculated between the signals of two detectors in the same instrument tube. The wavelet coherence between two signals f and g is determined as follows [16]. First a short-term averaging integral of the wavelet transforms is calculated as:

$$C^w_{f,g}(f,t) = \int_{t-T/2}^{t+T/2} W^*_f(f,\tau)W_g(f,\tau)d\tau$$

(5)
Here \( W(f, \tau) \) is the continuous wavelet transform of signal \( f \) and \( g \), respectively and \( T \) is chosen to be 100 times the sampling frequency in order to get a good averaging. Note here that, according to the character of the application, the wavelet transform is given in terms of the frequency rather than as a function of the scale. The two variables are inversely related, but the correspondence is unambiguous.

The coherence is then calculated as

\[
\gamma_{f,g}^w(f, t) = \frac{|C_{f,g}^w(f, t)|}{\sqrt{C_{f,f}^w(f, t)C_{g,g}^w(f, t)}}
\]

(6)

This way a coherence function is obtained, which, similarly to the ordinary CWT transform, is dependent on both time and frequency. One can choose the averaging window in (5) to be of different length; if the integral is extended to the whole time interval of the measurement, one obtains a coherence function which, similarly to the FFT-based coherence, is only dependent on frequency. Even in such a case, as a rule, the information content of the wavelet coherence differs from that of the spectral coherence, as is demonstrated in [14]. Here the expectation is, as was shown in [16], that the wavelet coherence can identify characteristics at certain frequencies that cannot be seen in the spectral functions (APSD) or the wavelet-based spectrograms of single signals.

### 3 The measurements

Two different measurements are analysed, the first measurement, #1, was taken during start-up of Ringhals-1 on September 3-6, 2002 and the second measurement, #2, one year later on October 27, 2003, also during start-up. The first measurements consist of four different measurements, three at reduced core flow and reduced power, to investigate BWR stability, and one measurement at full flow and full power. The later one is the most interesting one regarding detector tube vibrations, hence solely this measurement was used in this analysis. Signals from a total of 36 core positions were recorded, each position containing two detectors, one at a lower axial elevation (4) and another one at a higher axial elevation (2). This gives a total of 72 different signals, but the signals from one position were defect. The measurement was around 11 min long with a sampling frequency of 12.5 Hz.

After the analysis of the first measurement it was realised that the sampling frequency was too low for the wavelet-based method. A new measurement was then taken during the start-up of the following cycle in 2003. This measurement also consists of 72 signals, but with a higher sampling frequency of 60 Hz and a length of 5 min.

The layout of the core, with the positions of the detector strings is shown in Figure 3. The positions of the LPRM strings 9, 10 and 22 that were judged in the analysis to have impacting in the first series of measurements (see Section 4), as well as string 1 which did not even indicate vibrations, are marked in the figure.

### 4 Results of the analysis

#### 4.1 The measurements made in 2002 (series #1)

Experience from earlier studies shows that classification and ranking of the probability and severity of impacting of the various strings is most effective if all cases are investigated and an internal comparison between the different strings is made. This is because the methods, at least for the traditional spectral analysis method, are not absolute, and the criteria for judgement are differing from core to core. For the first series of measurements, analysed in this section, the wavelet based analysis was not effective, due to the low sampling frequency, hence in this section we shall concentrate on the spectral analysis.

For all strings included in the measurements, the APSD of both detectors within a string, as well as the coherence and the phase between the two detectors were made. A complete list of these indicators, as well as the raw time signals and the APD functions (amplitude
probability distributions) were listed in the internal report [12]. Based on these data, the strings were classified as follows. Some strings did not show signs of vibrations and hence had no impacting at all. Some strings showed signs of mild vibrations where the probability of impacting was judged to be negligible. The next category is that of strings with definite vibrations and a non-negligible likelihood of impacting, such that the impacting was not judged to be severe. Finally some strings were judged to have severe impacting with a large confidence in the judgement, based on the experimental evidence. Examples of some of the cases, and in particular for the heavily impacting strings, will be shown below.

LPRM 1 (Figure 4) shows no signs of vibrations, hence this instrument tube has no impacting. As Figure 4 shows, the APSD has a smooth dependence on frequency, without any apparent peaks. The most convincing indicator of negligible vibrations is supplied by the phase curve. It has a clear linear shape up to 4.5 Hz, which is the maximum frequency range for Ringhals-1 where linear phase can be observed. For comparison it is interesting to notice that in Barsebäck-1 the linear phase extended well over 10 Hz, showing that the signatures are different even for non-vibrating strings for different cores. The coherence also has a structure that is associated with pure two-phase flow induced noise, with minima and maxima corresponding to the $\pi$- and zero-crossings of the phase, respectively.

The strings with the largest likelihood of heavy impacting were found to be LPRMs 9, 10 and 22. The corresponding auto-spectra, coherence and phase curves are shown in Figures 5-7. These all show one or more peaks in the auto-spectrum, and a significantly distorted phase and coherence structure. LPRMs 9.2 and 9.4 (Figure 5) both show a broad peak at around 1.5 Hz. Here, LPRM 9.2 denotes detector 2 in string 9 etc. In addition, peaks are present even at higher frequencies; in LPRM 9.2 the first higher harmonics at 3 Hz is seen, whereas 9.4 shows a peak at around 4.5 Hz, which corresponds to the second harmonics. The phase is nearly zero everywhere between 0 and 4 Hz, and the sink structure of the coherence is different from that of LPRM 1, i.e. the non-vibrating case. There is a peak in the coherence at 1.5 Hz, which is the fundamental frequency of the instrument tube vibrations.

The analysis of the other two strings that were judged to have impacting goes in a similar manner. In LPRM 10 there is a broad peak at the fundamental frequency, and traces of peaks at higher frequencies. The phase starts out in a linear manner, but above 1 Hz it drops to zero. In LPRM 22 there is a large peak at 1.5 Hz, which is not as broad as in the other two impacting strings, but several higher harmonics are seen with broad peaks. The phase and coherence are distorted, and the peak structure of the coherence is completely dictated by the positions of the peaks of the APSD, i.e. the vibration eigenfrequencies.

Finally, one case is shown, LPRM 15, with some possibility of impacting, whose likelihood and/or severity is nevertheless judged to be lower than for the above three cases (Figure 8). The same characteristic features are seen as for LPRMs 9, 10 and 22, but to a smaller extent. The peak at 1.5 Hz is smaller and is narrow, and the phase shows a linear dependence on frequency over a large part of the region 0-4 Hz, except at the vibration frequency. This case also illustrates the fact that there is no clear-cut separation between the cases of heavy and light impacting, and that the procedure is based on an expert evaluation of the calculated quantities.

As was mentioned earlier, for this measurement, the wavelet based impact detection algorithm was not applicable. The reason is the low sampling frequency of the measurement (which was primarily made for investigating BWR stability, whose characteristic frequency is about 0.5 Hz). The sampling frequency of 12.5 Hz means that the spectral quantities can only be calculated up to 6.25 Hz. This is sufficient to see the detector tube vibration peaks and the lowest higher harmonics. However, the damped vibrations of the fuel assemblies, induced by the impacting, are expected to lie in the range 10-15 Hz. The low sampling frequency causes two problems. First, the short transients (“spikes”) in the signal, corresponding to the effect of the fuel assembly vibrations, can go undetected fully or partially. Second, the variance of the background noise, which is necessary to know in order to set the wavelet filter threshold, cannot be determined since this requires access to the high frequency tail of the APSD, above 10 Hz. For this reasons, no wavelet-based wavelet analysis of measurement 1 was performed.
4.2 The measurements made in 2003 (series #2)

This measurement was performed with a sampling frequency of 64 Hz, which makes it suitable also for wavelet analysis. However, the analysis will be first performed with the traditional spectral analysis, similarly as in the previous case. The principles are the same, therefore here only a summary of the findings will be given here.

In these measurements, four strings were found to show the strongest vibrations, and these are LPRMs 15, 16, 24 and 35. As an illustration, the case of LPRM 16 is shown in Figure 9. It shows the same features as those in measurement 1. There are several broad peaks at around 2, 4 and 6 Hz, i.e. at the fundamental frequency and corresponding harmonics. One can note that these frequencies are somewhat higher than those in the previous measurement. The phase is close to zero at low frequencies up to 4 Hz and the peak structure of the coherence follows the peaks in the APSD closely. The other LPRMs judged to execute impacting show similar features. They are not shown here for brevity. A summary of all the findings with classifications is given in the next Section.

For measurement #2, the wavelet based impact diagnostics was also possible to perform. Here, however it was noticed that the method, based on wavelet filtering and thresholding, which was also used in the evaluation of the Oskarshamn-2 measurements, had to be slightly modified. This is in line with the fact that when applying the method, originally used for Barsebäck, to the Oskarshamn measurements, also a modification was necessary before a good discrimination power of the method was achieved as follows.

The threshold given in Eqn (3) was used on the wavelets coefficients of the two smallest scales, corresponding to frequencies higher than 10 Hz, of the multiscale resolution using the discrete Meyer wavelet [13]. After the thresholding the signal was reconstructed and the difference \( V \) (Eqn 4), was examined for the possible occurrence of spikes, which are supposed to represent the intermittent fuel-box vibrations. In this measurement the best discrimination was obtained at core level 4, i.e. the lowest axial elevation. The quantity \( V \) (Eqn 4) is shown in Figure 10a for LPRM 10.4 which is assumed to be non-vibrating, and on Fig. 10b for LPRM 16.4 which is assumed to be impacting according to the spectral analysis. The difference between the two signals, what regards the number of impacting spikes found, is clearly visible. The IR index for LPRM 16.4 is 4.6 impacts/min whereas for LPRM 10.4 it is only 0.2 impacts/min, which quantifies the impacting status of the two strings.

Out of the 36 LPRM signals at level 4, the following ones have IR index higher than 2 impacts/min: LPRMs 4, 16, 23, 24, 34 and 35. Out of these, LPRMs 16, 24 and 35 are identical with three of the four LPRMs, pointed out by the spectral method to be the most probable impacting ones. LPRM 24 is in the second highest category of impacting probability based on the spectral method. Thus the result from the wavelet based analysis is in good agreement with the classical spectral analysis.

As was mentioned earlier, the new method of wavelet coherence was also tested on these measurements. The wavelet coherence was calculated by using a Meyer wavelet via Eqns (5) and (6) for each string, using the two detector signals available per string. It was found that for certain strings a high value of the coherence was found between 10 and 20 Hz, which is the frequency range where the fuel box vibrations are expected to take place. For the other strings no such component was observed, so this analysis divided the LPRM tubes into two categories. A comparison with the results from the spectral and wavelet filtering analysis showed that the LPRM strings showing a large wavelet coherence around 15 Hz coincide with those that were pointed out as most likely impacting. An illustration is given in Figure 11, which shows four LPRM strings: two with high wavelet coherence at 15 Hz (LPRM 16 and 35) and two others that have an average (low) coherence at these frequencies (LPRM 1 and 17). A comparison with Table II confirms that these strings are suspected for strong impacting by the other two methods.
It is interesting to note that the ordinary FFT-based coherence has a large, sharp peak at around 15 Hz for all detector signals, with very little variation in the amplitude (Figure 12). The origin of this very narrow peak in the spectral based coherence is not understood. Although it is in the middle of the frequency range found by wavelet coherence method, which is attributed to the frequency of the impacting induced damped vibrations of the fuel assemblies, yet this appears to be given rise by a different physical phenomenon. Namely, strongly damped vibrations lead to a broad peak (peak broadening is in fact one of the traditional methods to detect impacting); the peak in the spectral based coherence is too narrow (monochromatic) to be due to impacting. The narrow peak in the spectral based coherence, on the other hand, does not show up in the wavelet coherence, due to the fact that the frequency resolution of wavelet methods are much coarser (especially for higher frequencies) than the FFT-based methods.

Hence it seems that, despite the quantitative coincidence of the frequencies where both the spectral and the wavelet coherence has peaks or high values, respectively, these correspond to physically different phenomena. It is also seen that the wavelet coherence has a much higher discrimination power for detecting impacting than the ordinary coherence. The performance of the wavelet coherence method has though to be confirmed by further tests.

5 Discussion

In measurement 1, only the traditional diagnostic procedure was applicable, so a comparison between the two methods was not possible. On the other hand, the predictions of the traditional spectral method could be compared to the results of visual inspection of several, but not all, fuel assemblies. The reason for the partial observation is that during an average refuelling, time constraints do not permit to inspect all four fuel assemblies around all 36 LPRM positions, only a few selected ones. One use of the vibration diagnostics is to restrict the number of inspected fuel assemblies into a limited set of suspected positions.

The result of the traditional diagnostics of measurement 1 can be summarized in the Table 1. In the left side the classification of the different LPRMs is given with respect to the probability or severity of impacts. This information was made available to the power plant as a recommendation on which LPRM positions should be inspected. In particular, the impacting LPRMs 9, 10, 22 and the non-vibrating LPRM 1 (as reference) were recommended to be checked. During the refuelling in October 2003, a total of ten fuel assemblies around the four positions were inspected. Around LPRMs 1 and 9 no wear damages was observed, but two assemblies around LPRM 10 and one around LPRM 22 showed marks of wear, see Table 1 to the right. It is seen that a good correlation exists between the LPRMs predicted to have impacting and the actual damage.

What regards measurement 2, no post-cycle inspections are available yet (the cycle is still on-going as of this writing). On the other hand, both the spectral and the wavelet-based analyses were possible to perform. The analysis results are summarised in Table 2 for the two methods. The Table shows that the LPRMs pointed out as the most severely vibrating are not completely identical for the two methods, but they have a large overlap between the two groups. The LPRMs 16, 24 are the ones pointed out by both methods to have the highest probability of impacting, hence they are the primary suspects for impacting. LPRM 35 is in the highest group of the spectral method and in the second highest in the wavelet method. Finally LPRM 34 is in the group with the second highest probability in both of the methods. Also the new method of using wavelet coherence was also tested for this measurements, and supplied results completely consistent with the findings of the spectral and wavelet filtering methods. The applicability of this latter method needs, however, confirmed in further measurements. It is also important to get feedback from the inspections during refuelling to confirm or deny the applicability of the methods. The inspection during a later revision will bring a very useful and important clarification regarding the performance of the two methods.

6 Conclusions
Diagnostics of impacting of detector strings was shown to be possible by both spectral and wavelet based methods. The spectral method was possible to apply to the Ringhals-1 case without any modifications compared to earlier applications. However, this method is not suitable for on-line monitoring by the operators; rather it requires expert judgement and works best off-line. The wavelet based method had to be modified compared to earlier applications before it turned out to be effective. This shows that the wavelet method, at least when first applied to a new core, does not fulfil the expectations of being suitable for an algorithmic, absolute (without calibration) and quantitative method that can be used for on-line monitoring by the operators. Nevertheless, it can be tuned to a specific core such that after tuning it can be used for on-line monitoring by non-experts. The tuning means finding the suitable mother wavelet form and the corresponding threshold values. It requires an analysis of all signals, and the optimum parameters are found by assuming that there are both non-impacting and impacting strings. In other words, the tuning of the method is just as subjective and based on expert knowledge as the traditional method. However, after having optimized for a given core, it fulfils the expectations of being an absolute, algorithmic method. This latter statement will be followed up in applying the wavelet method, optimized in this study, for later cycles in the Ringhals-1 core. In addition, a new method of wavelet-based coherence was tested which, although at this point appears to be more like empirical without deep theoretical justification, seems to be directly applicable for a new core without tuning. Its applicability has though to be tested in more applications.

Acknowledgements

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References


### Table 1  Results of measurement #1

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<th>Impacting status</th>
<th>LPRM by analysis</th>
<th>LPRM by inspection</th>
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<tr>
<td>most likely impacting</td>
<td>9, 10 and 22</td>
<td>10 and 22</td>
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<tr>
<td>probably impacting</td>
<td>4, 8, 16 and 24</td>
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</tr>
<tr>
<td>Very small chance of impacts</td>
<td>11, 12, 30 and 33</td>
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### Table 2  Results of measurement #2

<table>
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<th>Impacting status</th>
<th>By spectral analysis</th>
<th>By wavelet analysis</th>
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</thead>
<tbody>
<tr>
<td>most likely impacting</td>
<td>15, 16, 24, 35</td>
<td>16, 23 and 24</td>
</tr>
<tr>
<td>probably impacting</td>
<td>12 and 34</td>
<td>4, 34 and 35</td>
</tr>
<tr>
<td>Very small chance of impacts</td>
<td>27 and 32</td>
<td>---------------</td>
</tr>
</tbody>
</table>
**Figure 1** Illustration of the position of the detector string in a BWR core between four fuel elements, with indication of some typical data of interest.

**Figure 2** Schematic view of the signal of a detector vibrating in a flux gradient with background noise and impacting. For explanation see text in the paper.
Figure 3  The layout of the core of Ringhals-1 with the locations of the detector strings. String #1 did not show vibrations, whereas strings 9, 10, and 22 were found to have impacting in the measurement series #1.
Figure 4  Autospectra, coherence and phase of the detectors in LPRM 1, measurement #1.

Figure 5  Autospectra, coherence and phase of the detectors in LPRM 9, measurement #1.
Figure 6
Autospectra, coherence and phase of the detectors in LPRM 10, measurement #1

Figure 7
Autospectra, coherence and phase of the detectors in LPRM 22, measurement #1
Figure 8  Autospectra, coherence and phase of the detectors in LPRM 15, measurement #1

Figure 9  Autospectra, coherence and phase of the detectors in LPRM 16, measurement #2
Figure 10 Difference $V$ of the detectors at level 4 in LPRM 10 and LRPM 16 showing the spikes corresponding to the fuel-box vibrations, measurement #2.
Figure 11  Wavelet-based coherence, as a function of time, for four different LPRM strings. Figures on the left: LPRMs 1 and 17. Low coherence everywhere. Figures on the right: LPRMs 16 and 35. High coherence in the frequency band 10-20 Hz, indicating impacting.
Figure 12  Spectral based coherences for the four strings shown in Figure 11. In all four strings a narrow peak around 15 Hz is shown in the coherence, with somewhat lower magnitude for the strings that were not suspected as impacting by the other methods.
PAPER IV
Calculation of the neutron noise induced by shell-mode core-barrel vibrations in a 1-D 2-group 2-region slab reactor

C. Sunde and V. Arzhanov
Calculation of the neutron noise induced by shell-mode core-barrel vibrations in a 1-D 2-group 2-region slab reactor

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Abstract

The subject of this work is the calculation of the in-core neutron noise, induced by the shell-mode vibration of the core-barrel. The original motivation was to investigate whether an out-of-phase behaviour can exist between the in-core and ex-core detectors lying on the same azimuthal position. To this order a two-region two-group diffusion model was used in one dimension. The noise was calculated in the adiabatic approximation. It was found that an out-of-phase behaviour can exist between in-core detectors, close to the core boundary, and the ex-vessel detectors, for large systems. In addition, due to the use of two-group theory, the strong local noise component at the boundary of the vibrating core was also found. The paper gives details of the calculations and the results.
1. Introduction

The ex-core neutron noise, induced by core-barrel vibrations, have long been used to diagnose both beam-mode and shell-mode vibrations. In connection with the study of in-core neutron noise induced by fluctuating system boundaries, [1], it has been realized that core-barrel vibrations might lead also to in-core noise. This is especially useful for the diagnostics of shell-mode vibrations in Westinghouse reactors because all ex-core detectors carry the same information, due to the 90° spacing. Hence it is not possible to determine both the vibration amplitude and the direction from the detector signals. In particular it is not possible to find out if the reason for a change in the signal amplitude is due to the change in the vibration amplitude or the direction of the vibrations.

For this reason we have started including the in-core detector signals in the analysis of core-barrel vibrations in the collaborative research project between Ringhals and the Department of Reactor Physics at Chalmers University of Technology ([2] - [8]). However, confirmation of the theory as well as use of the results for diagnostics was hindered by the low number of in-core detectors. In particular, during evaluation of the only available measurement in Ringhals-3 when both in-core and ex-core detectors were available, in order to have a consistent interpretation, it was necessary to assume that ex-core and in-core detectors lying on the same azimuthal position have opposite phase.

This out-of-phase behaviour was contradicting the simple theory that was used so far. However, in that theory the in-core noise was only calculated in a non-reflected system. One cannot exclude the possibility that the core-barrel vibrations, if treated in a reflected system, will lead to another structure of the neutron noise than in a bare system. For instance, the outbound movement of the boundary in the unreflected system means some multiplicative material added outside the static boundary, which then would lead to both an increase of reactivity and, in addition, to a local increase of the neutron flux. Both components would be in phase with the signal of the ex-core detectors. However, in a reflected system, an outbound movement of the core boundary means a decreasing reflector thickness, and, under certain conditions, also possibly a decrease of reactivity. Hence, it is not obvious whether such a movement will lead to the increase of the in-core flux or not.

The purpose of this report is to investigate the in-core noise induced by core-barrel vibrations in reflected systems by the extension of the model used earlier. In order to be able to handle a reflected system, it was necessary to use two-group theory. To compensate for the complications that this extension incurs, the description was reduced to one spatial dimension. Density variations, induced by the 1-D shell mode vibrations, were also accounted for. The noise was calculated in the adiabatic approximation. This made it possible to handle the problem analytically throughout, with a numerical evaluation of the final formulas.

2. Description of the reactor model

A one-dimensional model of a reflected reactor is selected for this study with a central core and outer reflectors placed symmetrically around the core (Fig. 1). Two-group diffusion theory will be used to calculate the noise, with corresponding cross sections and diffusion constants in the fast, 1, and thermal, 2, group, respectively. One averaged group of delayed neutrons was used in the dynamic calculations. The shell-mode vibrations will be modelled by simultaneous,
symmetric vibrations of the core boundary around the static position \( x = \pm b \). Hence both the static and the dynamic case will be symmetric around the origin, and this simplifies the solution of the problem.

\[
\begin{array}{c|c|c}
\phi^r_{1,2}(x) & \phi^c_{1,2}(x) & \phi^r_{1,2}(x) \\
-a & -b & 0 & b & a
\end{array}
\]

Fig. 1. 1-D Core with reflector

Since the original idea was to see if there can, at least under certain circumstances, exist an out-of-phase behaviour between in-core and ex-core (ex-vessel) detectors, the boundary condition at the outer boundary of the system was chosen to be the no incoming current condition, which can be expressed by the logarithmic flux derivative and the extrapolation length, see below. This way the scalar flux does not vanish at the system boundary, and the ex-core detector signals were chosen to be equal to the flux at the boundary.

### 3. The static case

#### 3.1. Boundary and interface conditions

To start we define two functions in the core: \( \phi^c_{1,2}(x), -b \leq x \leq b \), and two functions in the reflector: \( \phi^r_{1,2}(x), -a \leq x \leq -b, b \leq x \leq a \). The subscript indicates if it is the fast, \( f \), or the thermal, \( t \), energy group. The superscript \( r \) stands for the reflector and \( c \) for the core.

Because of the symmetry of the problem we have first of all:

\[
\begin{align*}
\phi^c_{1,2}(-x) &= \phi^c_{1,2}(x), -b \leq x \leq b \\
\phi^r_{1,2}(-x) &= \phi^r_{1,2}(x), -a \leq x \leq -b, b \leq x \leq a
\end{align*}
\]  \hspace{1cm} (1)

This gives us the possibility to set up the problem only for \( x \geq 0 \).

The boundary and interface conditions used are described below in equation (2) and (3)

\[
\begin{align*}
\phi^r_1(x)|_{x = b} &= \phi^r_1(x)|_{x = b} \\
\phi^r_2(x)|_{x = b} &= \phi^r_2(x)|_{x = b} \\
D^r_1 \frac{\partial \phi^r_1(x)}{\partial x}|_{x = b} &= D^r_1 \frac{\partial \phi^c_1(x)}{\partial x}|_{x = b} \\
D^r_2 \frac{\partial \phi^r_2(x)}{\partial x}|_{x = b} &= D^r_2 \frac{\partial \phi^c_2(x)}{\partial x}|_{x = b} \hspace{1cm} \text{Interface conditions} \hspace{1cm} (2)
\end{align*}
\]
Here \( d_1 \) and \( d_2 \) are the extrapolation lengths. They are defined through the cross-sections and diffusion constants in the reflector.

\[
\begin{align*}
\frac{d}{dx} \phi_1^r(x) \bigg|_{x = a} &= -\frac{1}{d_1^r} \phi_1^r(x) \bigg|_{x = a} \\
\frac{d}{dx} \phi_2^r(x) \bigg|_{x = a} &= -\frac{1}{d_2^r} \phi_2^r(x) \bigg|_{x = a}
\end{align*}
\] Boundary conditions (3)

These are all the conditions we need to solve the problem and we are free to normalize the flux.

3.2. Equations in the reflector

The diffusion equations for the fast and thermal fluxes in the reflector region, were no fission occurs are as follows:

\[
\begin{align*}
D_1^r \frac{d^2}{dx^2} \phi_1^r(x) - (\Sigma_{\alpha,1} + \Sigma_R^r) \phi_1^r(x) &= 0 \\
D_2^r \frac{d^2}{dx^2} \phi_2^r(x) - \Sigma_{\alpha,2} \phi_2^r(x) + \Sigma_R \phi_1^r(x) &= 0
\end{align*}
\] Equations (5)

Now, the subscript for the cross sections displays which type of reaction it is, \( a \) for absorption, \( R \) (removal) for scattering from the fast to the thermal group and \( f \) for fission in the core region. The superscript \( r \) stands for the reflector and later \( c \) for the core. The general solutions of these coupled differential equations are easily derived and simplified by using (1). The constants will be determined later when solving the core equation an using the boundary and interface conditions. Although, there are six constants and six conditions one of the constants will be undetermined since an equation for the criticality appear from the six conditions. Though, we have to deal with the large exponents due to the large size of a real reflector (\( a \approx 300 \) cm and \( e^{10} \)). By multiplying the solution with \( e^{-\tilde{b}} \) and use new integration constants, the numerical values become smaller:

\[
\begin{align*}
\phi_1^r(x) &= \alpha_3 e^{\kappa_1(x)\tilde{b}} + \alpha_4 e^{-\kappa_1(x)\tilde{b}} \\
\phi_2^r(x) &= \alpha_5 e^{\kappa_2(x)\tilde{b}} + \alpha_6 e^{-\kappa_2(x)\tilde{b}} + \frac{\Sigma_R^r}{D_2^r (\kappa_2^2 - \kappa_1^2)} \phi_1^r(x)
\end{align*}
\] (6)

where \( \tilde{b} = \frac{a - b}{2} \)
Here the following notations are used:

\[
\kappa_1 = \frac{1}{L_1} = \frac{\Sigma_{r}}{D_1}, \quad \kappa_2 = \frac{1}{L_2} = \frac{\Sigma_{a,2}}{D_2}
\]

\[
\Sigma_1 = \Sigma_{a,1} + \Sigma_R
\]

### 3.3. Equations in the core

The equations in the core are a little bit more complicated due to the fact that they involve fission terms that are not present in the reflector region. This means that the fast and thermal equations are more coupled than in the reflector region. We assume that all fission neutrons are fast but one could also assume that some part of them are thermal. Then one would have to add a fission term in the equation for the thermal ones, using the spectrum of the fission neutrons.

\[
\begin{align*}
D_1^c \frac{d^2}{dx^2} \phi_1^c(x) - (\Sigma_{a,1} + \Sigma_{x,1}) \phi_1^c(x) + \frac{1}{k} (\nu \Sigma_{f,1}^c \phi_1^c(x) + \nu \Sigma_{f,2}^c \phi_2^c(x)) &= 0 \\
D_2^c \frac{d^2}{dx^2} \phi_2^c(x) - \Sigma_{a,2} \phi_2^c(x) + \Sigma_{x,1} \phi_1^c(x) &= 0
\end{align*}
\]

(8)

When solving this equation system we try with the following form of the fluxes.

\[
\begin{align*}
\phi_1^c(x) &= \phi_{1,0}^c e^{\lambda x} \\
\phi_2^c(x) &= \phi_{2,0}^c e^{\lambda x}
\end{align*}
\]

(9)

With this inserted into (8), we arrive with a matrix equation:

\[
\begin{pmatrix}
D_1^c \lambda^2 & 0 \\
0 & D_2^c \lambda^2
\end{pmatrix}
\begin{bmatrix}
\phi_{1,0}^c \\
\phi_{2,0}^c
\end{bmatrix}
+ \begin{pmatrix}
-\Sigma_1^c \\
\Sigma_R^c
\end{pmatrix}
\begin{bmatrix}
\phi_{1,0}^c \\
\phi_{2,0}^c
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\]

(10)

With

\[
\Sigma_1^c = \Sigma_{a,1} + \Sigma_R - \frac{1}{k} \nu \Sigma_{f,1}^c
\]

(11)

and if

\[
det \begin{pmatrix}
-\Sigma_1^c - D_1^c \lambda^2 & \frac{1}{k} \nu \Sigma_{f,2}^c \\
\Sigma_R^c & -\Sigma_{a,2} - D_2^c \lambda^2
\end{pmatrix} = 0
\]

(12)

there exist a solution to the equations. And the eigenvalues, \( \lambda \), can be determined and expressed
in terms of the cross sections and diffusion constants of the two energy groups as:

$$\lambda^2_{1,2} = \frac{(D_1^c \Sigma_{a,2}^c + D_2^c \Sigma_1^c) \pm \sqrt{(D_1^c \Sigma_{a,2}^c - D_2^c \Sigma_1^c)^2 + \frac{4}{k} \Sigma_{f,2}^c \Sigma_{R}^c D_1^c D_2^c}}{2D_1^c D_2^c}$$  \tag{13}$$

This gives four eigenvalues which appear in two pairs:

$$\lambda_{1,2} = \pm \eta = \pm \sqrt{\frac{\left(D_1^c \Sigma_{a,2}^c + D_2^c \Sigma_1^c\right)^2 + \frac{4}{k} \Sigma_{f,2}^c \Sigma_{R}^c D_1^c D_2^c}{2D_1^c D_2^c}}$$  \tag{14}$$

$$\lambda_{3,4} = \pm i \mu = \pm \sqrt{\frac{\left(D_1^c \Sigma_{a,2}^c - D_2^c \Sigma_1^c\right)^2 + \frac{4}{k} \Sigma_{f,2}^c \Sigma_{R}^c D_1^c D_2^c - \left(D_1^c \Sigma_{a,2}^c + D_2^c \Sigma_1^c\right)^2}{2D_1^c D_2^c}}$$

We also have a relation between $\phi_{1,0}^c$ and $\phi_{2,0}^c$ from equation 10 reading as

$$\phi_{2,0}^c = \frac{\Sigma_R^c}{\Sigma_{a,2}^c - D_2^c \lambda^2} \phi_{1,0}^c$$  \tag{15}$$

This gives us two constants:

$$C_1 = \frac{\Sigma_R^c}{\Sigma_{a,2}^c - D_2^c \eta^2}$$  \tag{16}$$

$$C_2 = \frac{\Sigma_R^c}{\Sigma_{a,2}^c + D_2^c \mu^2}$$

Together with the eigenvalues this gives us the solution of the fast and thermal flux in the core although we have to determine the criticality of the system.

$$\begin{align*}
\phi_1^c(x) &= \alpha_1 e^{\eta x} + \tilde{\alpha}_1 e^{-\eta x} + \alpha_2 e^{i\mu x} + \tilde{\alpha}_2 e^{-i\mu x} \\
\phi_2^c(x) &= C_1 (\alpha_1 e^{\eta x} + \tilde{\alpha}_1 e^{-\eta x}) + C_2 (\alpha_2 e^{i\mu x} + \tilde{\alpha}_2 e^{-i\mu x})
\end{align*}$$  \tag{17}$$

Due to the symmetry conditions expressed by (1), the above can be simplified to:

$$\begin{align*}
\phi_1^c(x) &= \alpha_1 \cosh \eta x + \alpha_2 \cos \mu \\
\phi_2^c(x) &= \alpha_1 C_1 \cosh \eta x + \alpha_2 C_2 \cos \mu x
\end{align*}$$  \tag{18}$$

with $\alpha_1 = \alpha_1'$ and $\alpha_2 = \alpha_2'$.
Now (6) and (18) are the general solutions in the reflector and the core, and to determine the constants and the criticality the boundary and interface conditions have to be used.

3.4. Critical equation

Two of the constants are already determined with the symmetry so six constants and the eigenvalue (criticality) are to be determined. To fully determine the eigenvalue and the constants we need a normalization condition, which will be done by setting the fast flux to unity, in the centre of the core. Using (2) and (3) we arrive at a matrix equation $A^*\alpha = 0$ where the matrix $A$ is:

$$
\begin{pmatrix}
cosh\eta b & \cos\mu b & -\varepsilon_1 & -\frac{1}{\varepsilon_1} & 0 & 0 \\
C_2\cosh\eta x & C_1\cos\mu x & \varepsilon_1\kappa & \frac{\kappa}{\varepsilon_1} & -\varepsilon_2 & -\frac{1}{\varepsilon_2} \\
0 & 0 & -\left(\kappa_1 + \frac{1}{d_2}\right)\delta_1 & \left(\kappa_1 - \frac{1}{d_2}\right)\frac{\kappa}{\delta_1} & \left(\kappa_2 + \frac{1}{d_2}\right)\delta_2 & \left(\frac{1}{d_2} - \kappa_2\right)\frac{1}{\delta_2} \\
0 & 0 & \left(\kappa_1 + \frac{1}{d_1}\right)\delta_1 & \left(\kappa_1 - \frac{1}{d_1}\right)\frac{1}{\delta_1} & 0 & 0 \\
\eta\sinh\eta b & -\mu\sin\mu b & -\frac{\kappa_1 D_1^c\varepsilon_1}{D_1^c} & \frac{\kappa_1 D_1^r}{D_1^c\varepsilon_1} & 0 & 0 \\
C_2\eta\sinh\eta b & C_1\mu\sin\mu b & \frac{\kappa_1 D_2^r\varepsilon_1\kappa}{D_2^c} & -\frac{\kappa_1 D_2^r}{D_2^c\varepsilon_1} & -\frac{\kappa_2 D_2^r\varepsilon_2}{D_2^c} & \frac{\kappa_2 D_2^r}{D_2^c\varepsilon_2}
\end{pmatrix}
$$

Here the constants used to simplify the matrix are

$$
\varepsilon_1 = e^{(b-\tilde{b})B_1'}, \quad \varepsilon_2 = e^{(b-\tilde{b})B_2'}
\delta_1 = e^{(a-\tilde{b})B_1'}, \quad \delta_2 = e^{(a-\tilde{b})B_2'}
\kappa = \frac{-\Sigma_R}{(\kappa^2_2 - \kappa^2_1)D_2^r}
$$

$n$ and $u$ were defined in (14). The critical equation for determining $k_{\text{eff}}$ is:

$$
f(k) \equiv \det(A) = 0
$$

Using Mathematica one arrives at an equation for $k_{\text{eff}}$: 

---

-10-
The solution, which is physical, is the one with the largest \( k_{\text{eff}} \). See Fig. 2. By using that value in (14) the eigenvalues, \( \eta \) and \( \mu \) are determined. The structure of the critical equation plotted as a function of the inverse variable is displayed in Fig. 2. It shows that the \( k \)-function consists of infinitely many branches separated by the asymptotes:

\[
f(k_{\text{eff}}) = \left( \frac{\sum a_i c}{D_2^c} - \eta^2 \right) *(\eta \tanh b \eta + h_1) - \left( \mu \tan b \mu - \left( \frac{\sum a_i c}{D_2^c} \right) * h_1 2 + h_2 \right)
\]

\[
- \left( \mu^2 + \frac{\sum a_i c}{D_2^c} \right) *(h_1 - \mu \tan b \mu) \left( \eta \tanh b \eta + \left( \eta^2 - \frac{\sum a_i c}{D_2^c} \right) * h_1 2 + h_2 \right) = 0
\]

where \( \eta \equiv \eta(k_{\text{eff}}), \quad \mu \equiv \mu(k_{\text{eff}}) \) and \( h_1, h_2 \) and \( h_{12} \) are constants which consist of some \( \tan \) and \( \tanh \) functions and must be solved numerically. The constants in this equation are just some expressions involving other known terms as the extrapolation lengths and some exponential terms which are defined in (20).

The solution, which is physical, is the one with the largest \( k_{\text{eff}} \). See Fig. 2. By using that value in (14) the eigenvalues, \( \eta \) and \( \mu \) are determined. The structure of the critical equation plotted as a function of the inverse variable \( \lambda = 1/k \) is displayed in Fig. 2. It shows that the \( k \)-function consists of infinitely many branches separated by the asymptotes:

\[
\lambda_n = \frac{(D_1^c \mu_n^2 + (\sum a_i + \Sigma_R))(D_2^c \mu_n^2 + \Sigma_{a_i 2})}{\nu \sum f_{1,2} + (D_2^c \mu_n^2 + \Sigma_{a_i 2}) + \nu \sum f_{1,2} \Sigma_R}; \quad n = 0, 1, 2, \ldots
\]

A minimal bound, \( \lambda_{\text{min}} \), may be defined by \( \mu(\lambda_{\text{min}}) = 0 \), which gives

\[
\lambda_{\text{min}} = \frac{\nu \sum c f_{1,2} + \sum c + \nu \sum c_{f,2} \Sigma R}{2 (D_2^c \mu_n^2 + \Sigma_{a_i 2}) + \nu \sum f_{1,2} \Sigma_R}
\]
Then inserting them into the matrix equation and solving it gives the integrations constants. Since this gives us six constants that are linearly dependent we use the normalization condition \( \phi_i^c(x=0) = 1 \) to fully determine all six constants. The fast and thermal fluxes are now fully determined and are displayed in Fig. 3 with a core radius of 161.25 cm and a reflector radius of 279.5 cm. The flux from a numerical simulator [9], developed at our department is also plotted and as one sees the analytical solution agree very well with the solution from the simulator. This together with the reflector peaks of the thermal flux indicates the correctness of the solution. The size of the reactor is supposed to represent a real reactor. All the cross-sections are taken from SIMULATE-3. Although SIMULATE-3 started with a 3-D critical system the 1-D system is super-critical since it is less leakage in a 1-D system. The value for \( k_{\text{eff}} \) is 1.00146 from the simulator and the same for the analytical calculation done in this report. Although, there is a mismatch in the sixth decimal. Note that the fast flux is about ten times larger than the thermal.

\[
\lambda_{\text{min}} = \frac{(\Sigma_{a,1}^c + \Sigma_{R}^c)\Sigma_{a,2}^c}{(\nu\Sigma_{f,1}^c \Sigma_{a,2}^c + \nu\Sigma_{f,2}^c \Sigma_{R}^c)}
\]

Fig. 3. Fast and Thermal flux form semi-analytic calculations and from SIMULATE-3. \( k=1.00146 \) in both cases.

To really visualise the reflector peaks and the ratio between the fast and thermal fluxes they are plotted together in Fig. 4. Only the region close to the boundary between the core and reflector
is displayed. The peak of the thermal flux in the reflector arises from the slowing down of the fast neutrons to thermal ones that occurs in the reflector. Notice also that the system is critical in this figure compared to Fig. 3, this is done by rescaling the fission cross-sections and using the same parameters as in the super critical system.

3.5. Adjoint flux

What we want to investigate is how a vibration of the core-barrel effects the detector signal, i.e. perturb the thermal flux. To calculate this noise we need to calculate the adiabatic flux, and to normalize that we need the static adjoint flux. The adjoint flux and the direct flux have the same eigenvalue, same k, so we do not need to determine the criticality once more. Also the interface and boundary conditions are the same. The adjoint equations are similar to the direct ones, so the idea of solving is the same. The adjoint equations in the reflector are:

\[
\begin{align*}
D_1^r \frac{d^2}{dx^2} \phi_1^+(x) - (\Sigma_{a,1}^r + \Sigma_{s,1}^r) \phi_1^+(x) + \Sigma_{s,1}^r \phi_2^+(x) &= 0 \\
D_2^r \frac{d^2}{dx^2} \phi_2^+(x) - \Sigma_{a,2} \phi_2^+(x) &= 0
\end{align*}
\]

and in the core:

\[
\begin{align*}
D_1^c \frac{d^2}{dx^2} \phi_1^c(x) - (\Sigma_{a,1}^c + \Sigma_{s,1}^c) \phi_1^c(x) + \frac{1}{k} \nu \Sigma_{f,1}^c \phi_1^c(x) + \Sigma_{s,1}^c \phi_2^c(x) &= 0 \\
D_2^c \frac{d^2}{dx^2} \phi_2^c(x) - \Sigma_{a,2} \phi_2^c(x) + \frac{1}{k} \nu \Sigma_{f,2}^c \phi_1^c(x) &= 0
\end{align*}
\]

The solutions for the adjoint fluxes in the reflector are:
For the core the solution looks like

\[
\begin{align*}
\phi_2^+(x) &= \alpha_5^+ e^{\kappa_2(|x| - \hat{b})} + \alpha_6^+ e^{-\kappa_2(|x| - \hat{b})} \\
\phi_1^+(x) &= \alpha_3^+ e^{\kappa_1(|x| - \hat{b})} + \alpha_4^+ e^{-\kappa_1(|x| - \hat{b})} - \frac{\Sigma_R^+}{D_1^+ (\kappa_2^2 - \kappa_1^2)} \phi_1^+(x)
\end{align*}
\]

(27)

\[
\begin{align*}
\phi_2^+(x) &= \alpha_1^+ \cosh \eta x + \alpha_2^+ \cos \mu \\
\phi_1^+(x) &= \alpha_1^+ C_1^+ \cosh \eta x + \alpha_2^+ C_2^+ \cos \mu x
\end{align*}
\]

(28)

\[
\begin{align*}
C_1^+ &= \frac{\Sigma_R^c}{(\Sigma_1^c - D_1^c \eta^2)}, \quad C_2^+ &= \frac{\Sigma_R^c}{(\Sigma_1^c + D_1^c \mu^2)}
\end{align*}
\]

Compared with the direct solution, (6) and (18), we see that the fast and thermal flux in some sense have changed places. Though, the constants are somewhat different. To see the difference between the direct and adjoint fluxes they are plotted in Fig. 5. The system has a core radius of 20 cm and a reflector radius of 40 cm, but it is still critical. Showing this smaller system makes the difference between the direct and adjoint fluxes clearer.
4. The dynamic case

We now proceed with a dynamic model that describes the response of the originally static system to time-dependent perturbation. Assuming one average group of delayed neutron precursors, \( C \), in addition to two prompt neutron groups, we have for the slab reactor

\[
\begin{align*}
\frac{1}{v_1} \frac{\partial \phi_1}{\partial t} &= \frac{\partial}{\partial x} D_1 \frac{\partial \phi_1}{\partial x} - (\Sigma_{a,1} + \Sigma_{s,1}) \phi_1 + (1 - \beta)(v \Sigma_{f,1} \phi_1 + v \Sigma_{f,2} \phi_2) + \lambda C, \\
\frac{1}{v_2} \frac{\partial \phi_2}{\partial t} &= \frac{\partial}{\partial x} D_2 \frac{\partial \phi_2}{\partial x} - \Sigma_{a,2} \phi_2 + \Sigma_{s,1} \phi_1, \\
\frac{\partial C}{\partial t} &= -\lambda C + \beta (v \Sigma_{f,1} \phi_1 + v \Sigma_{f,2} \phi_2)
\end{align*}
\]

(29)

In contrast to the static system (5) and (8) now the unknown functions and cross sections are space and time dependent

\[
\phi_1 \equiv \phi_1(x, t); \quad D_1 \equiv D_1(x, t); \quad \Sigma_{a,1} \equiv \Sigma_{a,1}(x, t); \ldots
\]

\[
\phi_2 \equiv \phi_2(x, t); \quad D_2 \equiv D_2(x, t); \quad \Sigma_{a,2} \equiv \Sigma_{a,2}(x, t); \ldots
\]

\[
C \equiv C(x, t)
\]

(30)

We model the shell mode vibrations by letting the core boundary, \( b(t) \), oscillate around the static position, \( b \), at both sides of the core in a symmetrical manner:

\[
b(t) = b + \delta b(t); \quad \langle \delta b(t) \rangle = 0
\]

(31)

In addition, we assume the value of the cross sections in the core to be proportionally modified

\[
\Sigma_x^c(x, t) = \Sigma_{x, static} \frac{b}{b + \delta b(t)} \quad D_x^c(x, t) = D_{x, static} \left( \frac{b}{b + \delta b(t)} \right)^{-1}
\]

(32)

whereas the value of the cross sections in the reflector is assumed to be unaffected by the vibrations. It should be noted here that because of the moving core boundary all the cross sections must be regarded as time and space dependent even in the reflector.

Instead of solving equations (29) directly we are going to derive properties of the perturbed system by using the adiabatic approximation. To this order we start by factorising the unknown fluxes into an amplitude factor, \( P(t) \), and a shape function, \( \psi(x, t) \), as follows

\[
\begin{align*}
\phi_1(x, t) &= P(t) \cdot \psi_1(x, t), \\
\phi_2(x, t) &= P(t) \cdot \psi_2(x, t)
\end{align*}
\]

\[
P(0) = 1, \quad \psi_{1,2}(x, 0) = \phi_{1,2}^{static}(x)
\]

(33)

Since we have introduced three new quantities instead of the two unknown functions, \( \phi_1 \) and \( \phi_2 \), we need to impose an additional constraint that normally reads as \([10]\)

\[
\int_{-a}^{a} \left[ \frac{1}{v_1} \phi_1^+(x) \psi_1(x, t) + \frac{1}{v_2} \phi_2^+(x) \psi_2(x, t) \right] dx = const
\]

(34)

Here, \( \phi_{0,1}^+ \) denotes static adjoint fluxes. Usually, a derivation of the kinetic equations, details of which one can find in \([10]\), goes as follows. One puts the factorization (33) into (29), multiplies
each equation by the corresponding static adjoint flux, then subtracts the static equations, and finally, integrates over the reactor volume. This yields in the end for the amplitude factor, $P(t)$, the equations:

$$
\begin{align*}
\frac{dP(t)}{dt} &= \frac{\rho(t) - \beta}{\Lambda(t)} P(t) + \lambda c(t) \\
\frac{dc(t)}{dt} &= \frac{\beta}{\Lambda(t)} P(t) - \lambda c(t)
\end{align*}
$$

(35)

The new quantities that appear in (35) involve an arbitrary factor, $F(t)$, that is usually defined as

$$
F(t) \equiv \int_{-a}^{a} \phi_{0,1}^{+}(x) \cdot \left[ \nu \Sigma_{f,1} \psi_{1}(x, t) + \nu \Sigma_{f,2} \psi_{2}(x, t) \right] dx
$$

(36)

Then the prompt neutron generation time reads as

$$
\Lambda(t) = \frac{1}{F(t)} \int_{-a}^{a} \left[ \frac{1}{\nu_1} \phi_{0,1}^{+}(x) \psi_{1}(x, t) + \frac{1}{\nu_2} \phi_{0,2}^{+}(x) \psi_{2}(x, t) \right] dx
$$

(37)

The reactivity term becomes

$$
\rho(t) = \frac{1}{F(t)} \left\{ \int_{-a}^{a} \phi_{0,1}^{+} \cdot \left[ \Delta(\nu \Sigma_{f,1}) \psi_{1} + \Delta(\nu \Sigma_{f,2}) \psi_{2} \right] dx + \int_{-a}^{a} \Delta(\Sigma_{r}) \left[ \phi_{0,2}^{+} - \phi_{0,1}^{+} \right] \psi_{1} - \int_{-a}^{a} \left[ \Delta(\Sigma_{a,1}) \phi_{0,1}^{+} \psi_{1} + \Delta(\Sigma_{a,2}) \phi_{0,2}^{+} \psi_{2} \right] dx \right\}
$$

(38)

Here, $\Delta$ denotes the fluctuation in a corresponding cross section, for example

$$
\Delta(\nu \Sigma_{f,1}) \equiv \nu \Sigma_{f,1}(x, t) - \nu \Sigma_{f,1}(x)
$$

(39)

Finally, the precursor density, $c(t)$, reads as

$$
c(t) = \frac{1}{\Lambda(t) F(t)} \int_{-a}^{a} \phi_{0,1}^{+}(x) C(x, t) dx
$$

(40)

It should be noted here that $\beta$, which is involved in the kinetic equations (35), depends on time $t$ in the most general case. But our model assumes that both prompt fission and delayed neutrons are born in the fast group only. It follows from the exact definition, [10], that $\beta$ is time independent and equals to the static delayed neutron fraction under this assumption.

Our next step is to linearize the kinetic equations (35) in order to obtain the zero reactor transfer function for our two-group model. The standard linearization technique starts with splitting quantities into a steady part plus a small deviation:

$$
\rho(t) = \rho_0 + \delta \rho(t); \quad F(t) = F_0 + \delta F(t); \quad \Lambda(t) = \Lambda_0 + \delta \Lambda(t)
$$

(41)
One then puts this into the original equations (35), and neglects second-order terms. Here

\[
F_0 = \int_{-a}^{a} \phi_{0,1}^+(x) \cdot [v\Sigma_{f,1}\phi_{0,1}(x) + v\Sigma_{f,2}\phi_{0,2}(x)] dx
\]

and

\[
\Lambda_0 = \frac{1}{F_0} \int_{-a}^{a} \left[ \frac{1}{v_1}\phi_{0,1}^+(x)\phi_{0,1}(x) + \frac{1}{v_2}\phi_{0,2}^+(x)\phi_{0,2}(x) \right] dx
\]

In the end one arrives at the following formula in the frequency domain:

\[
\delta P(\omega) = G_0(\omega) \cdot \delta \rho(\omega)
\]  

Here, \( G_0(\omega) \) is a transfer function that is similar to the ordinary (one-group) zero reactor transfer function:

\[
G_0(\omega) = \frac{1}{i\omega \left( \Lambda_0 + \frac{\beta}{\lambda + i\omega} \right)}
\]  

with an energy and space averaged constant, \( \Lambda_0 \), defined in (42). In the plateau region, \( \lambda < \omega < \beta/\Lambda_0 \) (see Fig. 6), an approximate equality holds:

\[
|G_0(\omega)| \approx \frac{1}{\beta}
\]  

Equation (43) in conjunction with (45) allows us to derive both a very important and useful relationship in the plateau region, namely:

\[
\delta P(\omega) \approx \frac{1}{\beta} \delta \rho(\omega) \Rightarrow \delta P(t) \approx \frac{1}{\beta} \delta \rho(t)
\]

The importance of the above formula stems from the fact that \( \delta \rho(t) \) may be relatively easily evaluated by calculating a corresponding eigenvalue problem:

\[
\delta \rho(t) = 1 - \frac{1}{k(t)}
\]

Here \( k(t) \) depends on time \( t \) parametrically through the time-dependence of the corresponding cross sections. A straightforward linearization of (33) gives us

\[
\begin{align*}
\delta \phi_1(x, t) &= \delta P(t) \cdot \phi_{0,1}(x) + \delta \psi_1(x, t) \\
\delta \phi_2(x, t) &= \delta P(t) \cdot \phi_{0,2}(x) + \delta \psi_2(x, t)
\end{align*}
\]  

The first term on the right-hand-side is called the point reactor term whereas the second one is referred to as the space-dependent term.

The space-dependent terms may be relatively easily evaluated through the adiabatic approximation:
Here, $\psi_{1,2}^{ad}(x,t)$ are the positive eigenfunctions for the eigenvalue problem (8) with the cross sections corresponding to the time instance $t$ (i.e. the momentary core boundary $b(t)$). Eq. (48) combined with (49) and (46) gives, finally, the following approximation:

\[
\begin{align*}
\delta \psi_1(x,t) &\approx \delta \psi_1^{ad}(x,t) \equiv \psi_1(x,t) - \phi_{0,1}(x) \\
\delta \psi_2(x,t) &\approx \delta \psi_2^{ad}(x,t) \equiv \psi_2(x,t) - \phi_{0,2}(x)
\end{align*}
\]  

(49)

In our numerical calculations we use the following parameters

\[
\beta = 0.00535; \quad \lambda = 0.08510; \quad \Lambda_0 = 2.0776 \cdot 10^{-5}
\]

(51)

that correspond to a plateau region of $10^{-1} < \omega < 10^2$. The shell mode vibrations are expected to be around $125$ rad/s, which is within the plateau region.

Fig. 6. Zero power transfer function
5. Numerical work

Now we have everything we need to calculate the effect on the flux due to a vibration of the core-barrel. The vibrations are assumed to change the volume of the core but not its mass. Hence, the density of the material present, changes due to the larger (or smaller) size of the core and this will affect all cross sections as well as the diffusion constants.

The new cross sections and diffusion constants in the core are:

\[
\Sigma^c_x(x, t) = \Sigma^c_{x, static} \frac{b}{b + \delta b} \\
D^c(x, t) = D^c_{static} \left( \frac{b}{b + \delta b} \right)^{-1}
\]  (52)

Here \(\delta b\) is the change of the boundary between the core and the reflector. In the reflector on the other hand, the density is assumed to be the same so all cross sections and diffusion constants are the same. By this we assume that some water is removed (added) from the reflector when its size is smaller (larger). In other words we assume that it is impossible to compress the water. Now, it is possible to calculate a new \(k_{eff}\) and a new flux \(\Phi_{adiabatic}(x)\). And by using the new \(k_{eff}\), \(\rho(t)\) can be calculated. Then one can use (50) to calculate the new flux with its time-dependent perturbation. Out of this it is possible to determine the phase-behaviour between in-core and ex-core positions (detectors), due to this perturbation.

6. Results and Discussion

When investigating the phase behaviour one has to estimate the size of the core barrel vibration. We assume that it is in the order of \textit{mm} if the core has a size of several metres. The core used in SIMULATE-3, for calculation of the reactor parameters, is Ringhals 4 and it has a core radius of 152.0 cm and the outer radius of the reflector is 166.3 cm (the reflector is 14.3 cm thick). The small thickness of the reflector is due to the fact that SIMULATE-3 only uses one node for modelling the reflector. But on the other hand we can extend the reflector to a radius of 279.5 cm and the core radius is actually 161.25 cm in the 1-D model used by our simulator. Using this we calculated \(k_{eff} = 1.00146\) and in our analytical model described above we also received \(k_{eff} = 1.00146\), as one can see they match perfectly.

However, it is possible to choose any size of the system. One just has to adjust the fission cross sections so that the system becomes critical. To display the features of the system we choose a small system and a large unphysical movement of the core boundary. By using a core radius of 20 cm, a reflector radius of 40 cm and a displacement of the core of 2 cm we see that there is an out-of-phase behaviour (shaded area, Fig. 8) between the whole core and the ex-core detector. Since this change of the boundary, is quite large, the factor \(\rho/\beta\) is also large.
meaning that the point reactor term, i.e. the first term on right hand side in (50), will contribute to the perturbed flux together with the space-dependent term.

For the ex-core value we use the thermal flux at the outer edge of the reflector, whereas the in-core values are of course the thermal flux in the core. What one also can see is that the system becomes less critical, i.e. \( \rho \) is negative, for increasing size. On the other hand a decrease of the core increases the reactivity and the system is super-critical. The out-of-phase behaviour is still present in the whole core in both cases.

When we increase the system size and use a more realistic displacement of 1 mm it is still possible to notice the out-of-phase behaviour but it is not appearing as distinctly as for the smaller system. The reactivity change is really small, \( \sim 10^{-15} \), this means that the point reactor term is almost negligible, as expected, because a small change to a large system is not affecting
the reactivity that much. As one can see in Fig. 9 the out-of-phase (shaded areas) behaviour appears in the middle for both 1 mm larger and 1 mm smaller core.

Thus, by comparing the ex-core position (detector) with in-core positions (detectors) close to the centre of the reactor it should be possible to detect shell mode vibrations. In fact it should be possible by using only in-core positions (detectors), one close to the centre and one somewhere between the centre and the core-barrel. The perturbed thermal flux, $\delta\phi_2(x)$, in Fig. 10, clearly illustrates the phase behaviour. One can also see that the ex-core flux is increasing with increasing size and decreasing with decreasing size.

For a closer look at the interesting area around the boundary between the core and the reflector Fig. 11 displays this part of Fig. 9. Here it is possible to see that a position close to the core-barrel also shows an out-of-phase behaviour with respect to an ex-core position. The advantage with this position compared to the one in the centre of the reactor is that here the change of the
flux is much greater (see also Fig. 10) so it would be easier to detect vibrations. But on the other hand it is impossible to have a detector at this position in a real core.

One other way of describing the vibration would be to not change the cross sections and diffusion constants, instead just change the size of the core. Then the reactivity change is much larger, meaning a contribution from the point reactor term and that removes the out-of-phase behaviour from the centre of the core, Fig. 12.

Fig. 11. This figure is a zoom in, of the core boundary region, of Fig. 9. The dashed vertical line is the core boundary before the movement and the dotted one is after the movement.

Fig. 12. a) displays the static and perturbed flux for a 1 mm outward movement of the core boundary without any change of the cross sections and the diffusion constants. b) displays the thermal noise $\phi_2(x)$. 
7. Conclusion

We have solved the 2-group diffusion equation in a 2-region 1-D system and simulated a core barrel vibration by assuming a larger (smaller) core with higher (lower) density, change of the cross sections and diffusion constants. Then we applied reactor dynamics to see if it is possible to get an out-of-phase behaviour between ex-core and in-core positions. The results show that there is an out-of-phase behaviour between ex-core positions (detectors) and positions around the centre of the core and also positions (detectors) close to the boundary between the core and the reflector. The out of phase behaviour in the centre if the core disappears if the density is kept constant. It is also shown that the reactivity change of the system is almost negligible meaning that the point reactor term of the perturbation is small. Thus, it is the local part of the noise that is dominating.

8. Acknowledgements

We would like to thank Dr. Christophe Demazière for providing us with cross section data and other reactor parameters. We are also thankful for his calculations of the neutron noise with his neutron noise simulator, it gave us some numerical values to compare our semi-analytical solution with [9].
Core-barrel vibrations CTH-RF-173

References


[12] User manual for Matlab on the web
Appendix

The following values were calculated for cross sections and diffusion constants in 1-D with SIMULATE-3. Note that the absorption cross section for the fast group in the core is negative, this is due to the homogenization from 3-D to 1-D.

\[
\begin{align*}
\Sigma_{a,1}^c &= 0.0115 \text{ cm}^{-1} \\
\Sigma_{a,2}^c &= 0.1019 \text{ cm}^{-1} \\
\Sigma_R^c &= 0.0151 \text{ cm}^{-1} \\
\nu\Sigma_{f,1}^c &= 0.0057 \text{ cm}^{-1} \text{ for the core} \\
\nu\Sigma_{f,2}^c &= 0.1425 \text{ cm}^{-1} \\
D_1^c &= 1.4376 \text{ cm} \\
D_2^c &= 0.3723 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
\Sigma_{a,1}^r &= -0.0098 \text{ cm}^{-1} \\
\Sigma_{a,2}^r &= 0.0284 \text{ cm}^{-1} \\
\Sigma_R^r &= 0.0238 \text{ cm}^{-1} \text{ for the reflector} \\
D_1^r &= 1.3116 \text{ cm} \\
D_2^r &= 0.2624 \text{ cm}
\end{align*}
\]
Calculation of the neutron noise induced by shell-mode core-barrel vibrations in a 1-D 2-group 2-region slab reactor model

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Abstract

The subject of this work is the calculation of the in-core neutron noise, induced by the shell-mode vibrations of the core-barrel. The original motivation was to investigate whether an out-of-phase behaviour can exist between the in-core and ex-core (ex-vessel) detectors lying at the same azimuthal position. To this order a two-region two-group diffusion model was used in one dimension. The noise was calculated by representing the vibrations of the core-barrel by a model developed earlier to describe control rod vibrations. It was found that such an out-of-phase behaviour indeed exists, although only for in-core detector positions close to the core boundary. This behaviour is due to the local component of the noise, which is accounted for in a two-group treatment. The finding is in accordance with the experiment whose result prompted the present work. In addition to its effect on the phase, the local component also manifests itself by a large amplitude of the noise around the vibrating core boundary, i.e both in the core and the reflector. The appearance and the properties of the local component of the neutron noise for core-barrel vibrations is the main finding of this work. The results suggest that the efficiency of core-barrel vibrations can be enhanced if, in addition to the ex-core detectors, in-core detectors in the outermost fuel assemblies are used.

1. Introduction

The ex-core neutron noise, induced by core-barrel vibrations have long been used to diagnose both beam-mode and shell-mode vibrations [1]-[12]. The corresponding methods have undergone a quite long development stage, and became quite effective for the diagnostics of beam-mode (pendular) vibrations. By starting out with underlying models restricted to either unidirectional or isotropic vibrations, they have successively advanced to a stage of being able to treat arbitrary anisotropic 2-D random motions. This way it became possible to monitor a change in the preferred directions and the amplitude of the vibrations simultaneously. Even reconstruction of the 2-D random motion of the core barrel has been performed with success in some cases, although this latter has less direct diagnostic value.

The quantitative diagnostics of shell-mode vibrations, however, has not reached a similar state, at least not in Westinghouse-type reactors where there are four ex-core detectors with an equal 90° spacing. Due to the fact that the shell-mode vibrations, and hence also the neutron noise induced by such vibrations, exhibit the same symmetry against rotations with 90° as the detectors, the information content in all detectors is equivalent, i.e. all ex-core detectors carry the same information. (the situation is different at reactor constructions, such as the East-European VVER-440 reactors, where 3 ex-core detectors are used with a 120° spacing, and in
some Japanese reactors with 5 ex-core detectors [9]). Hence it is not possible to determine both the vibration amplitude and the direction from the detector signals. In particular it is not possible to find out if the reason for a change in the signal amplitude is due to a change in the vibration amplitude or to a change in the direction of the vibrations.

In connection with the study of in-core neutron noise induced by fluctuating system boundaries [13], originally considered for the description of the neutron noise induced by vibrating control rods, it was realized that core-barrel vibrations might lead also to in-core noise. For this reason the present authors started using the in-core detector signals for the analysis of core-barrel vibrations as part of a collaborative research project between the Ringhals power plant and the Department of Reactor Physics at Chalmers University of Technology ([14] - [17]). A compact solution for the radial and angular dependence for the in-core noise, induced by core-barrel vibrations, was given in [18]. The full confirmation of the theory, however, was hindered by the low number of in-core detectors (a maximum of five movable in-core detectors at a time). In particular, during the evaluation of the only measurement in Ringhals-3 when both in-core and ex-core detectors were available, in order to have a consistent interpretation, it was necessary to assume that ex-core and in-core detectors lying on the same azimuthal position exhibited opposite phase [18].

This latter statement, and the apparent contradiction that it implies, will be expounded here in some detail because it was this observation that prompted the present work. The theoretical results and experimental comparison are summarized in Fig. 1 below. It was shown in [18] that the angular and radial dependence of the in-core noise, induced by shell-mode vibrations at the vibration eigenfrequency \( \omega \), can be given in a 2-D polar co-ordinate system \( r = (r, \varphi) \) as

\[
\delta \phi(r, \varphi) = AJ_2(B(\omega)r)\cos2(\varphi - \chi)
\]

(1)

where \( B(\omega) \) is the dynamic buckling ([13]), \( J_2 \) a Bessel function and \( \chi \) is the angle of the vibration axis. As (1) shows, and as can be expected also by simple considerations, the induced noise has a rotational symmetry by 90°. In particular, the APSD, given from (1) as

![Fig. 1. The structure of in-core noise induced by shell-mode vibrations (a) and evaluation of measurement from Ringhals-3 (b)](image)
\[ APSD_{\delta \phi}(r, \varphi) = |AJ_2(B(\omega)r)|^2 \cos^2 2(\varphi - \chi) = \frac{1}{2} |AJ_2(B(\omega)r)|^2 [1 + \cos 4(\varphi - \chi)] \] (2)

has four maxima (along the vibration axes) and four minima (nodal lines) in between, as shown in Fig. 1a. As a function of the polar angle, the phase changes sign whenever the nodal line is crossed, i.e. there is an out-of-phase relationship at the two sides of the nodal lines, as is seen from (1).

If the model is applied equally to the in-core and ex-core noise with the motivation that the ex-core noise arises from leakage neutrons that are related to the neutron flux close to the boundary, then the interpretation of the measurement, shown in Fig. 1b becomes problematic. Basically, we wanted to determine only the direction of vibrations, by finding a suitable position of the vibration axes and the corresponding nodal lines that are consistent with the amplitudes of the APSDs and the phase relations. Due to the amplitudes of the APSDs (some small, some large, as seen in Fig. 1b), the direction of vibrations had to be positioned as shown in the Figure, indicated by the arrow lines. However, at the same time, the in-core detector B10 and the ex-core detector N44, at two different sides of the vibration axis but in between two nodal lines, showed an out-of-phase behaviour, which contradicts the above model.

To resolve this apparent contradiction, it was argued in [18] that there should be an extra out-of-phase factor between the in-core and ex-core detectors at the same azimuthal position. Naturally, the occurrence of such a phase factor cannot be explained within the simple theory that was used so far. However, in that theory a bare system was used. In real systems a reflector region is situated between the core and the ex-core (=ex-vessel) detectors, whose thickness is also changing with oscillations of the core. This difference between the real situation and the simple non-reflected model opens a possibility to find the out-of-phase factor in question.

The signal of the ex-core detectors is attributed to the variations of the attenuation of the outbound flux from the core, due to the variations in the reflector thickness. Hence one does not expect a different behaviour of the ex-core detector signals for a reflected system, as compared to a bare one. What regards the in-core noise, however, one cannot exclude the possibility that the core-barrel vibrations, if treated in a reflected system, would lead to another structure of the neutron noise than in a bare system. For instance, the outbound movement of the boundary in the non-reflected system means some multiplicative material added outside the static boundary, which then would lead to both an increase of reactivity and, in addition, to a local increase of the neutron flux. Both components would be in phase with the signal of the ex-core detectors. However, in a reflected system, an outbound movement of the core boundary means a decreasing reflector thickness, and, under certain conditions, also possibly a decrease of reactivity. Hence, it is not obvious whether such a movement will lead to the increase of the in-core flux or not, depending on the behaviour of the reactivity with the core motion.

In order to take into account the effect of the reflector, one needs to use two-group theory. The purpose of this work is to investigate the in-core noise induced by core-barrel vibrations in reflected systems by the extension of the model used earlier. As it turns out, this not only will necessitate the use of two-group theory, but also lends the possibility of a simplification of the treatment of the vibrating boundary. Namely, the vibrations of the free boundary required some sophisticated methods of treating boundary conditions at a moving free vacuum boundary, where, in diffusion theory, one cannot make use of the existence of the flux outside the boundary. In case of the vibrations of the core boundary in a reflected system, the oscillations of the boundary can be treated by adding or subtracting core material and subtracting or adding
reflector material at either side of the boundary, respectively. This can be treated by a particularly simple model that was used for the description of vibrating control rods in the past [20]-[21].

In order to make a first principal investigation of the possible change of the phase relationships by including a reflector, a one-dimensional model was used in this work. Combined with the simple way of treating the core-barrel vibrations as a noise source, the present model allowed a fully analytic solution. The dependence of the amplitude and phase of the noise throughout the whole core was readily calculated and studied. It turned out that, not surprisingly, in the two-group treatment a strong local component, characteristic of that of an absorber of variable strength, is present at and around the core boundary. The large amplitude of this component suggests that it is practical to use in-core detectors as close to the core boundary as possible in order to be able to make an effective core-barrel vibration diagnostics.

The present exercise was appended with, and compared to, the results of fully numerical solutions of the same problem by using a so-called noise simulator [22]. The noise simulator is a code for the calculation of the dynamic transfer function $G(r, r', \omega)$ of real, inhomogeneous cores in two-group theory, very much the same way as static ICFM codes calculate the static flux in real cores. In particular, it is compatible with the input deck of ICFM codes, most notably SIMULATE, which is used by most of the Swedish power utilities in core optimization and loading. The idea behind the development was to be able to perform spatial unfolding of localised or distributed noise sources, by inverting the basic formula of linear noise theory, expressing the noise as a convolution of the transfer function with the noise source. Such inversion or unfolding procedures were performed in the past exclusively by using simple transfer or Green’s functions for homogeneous systems, equivalent with the analytical model used in this paper. Although such models were used in a few singular cases with success in the past even in real cores, one cannot trust that such models would always work in reflected non-homogeneous cores. With the present noise simulator, any measurement taken in Swedish power plants can be evaluated for spatial unfolding of the noise source.

The simulator was used with success in locating the channel-type instability in the Forsmark-1 BWR, [23], and also for the explanation of a spatially dependent decay ratio in the same measurement, [24]. However, more testing of the applicability and the performance of the simulator was desirable. One special task in each application of the noise simulator is to find a proper representation of the noise source, since this is not an autonomous part of the simulator. While analytical models can use simplifications such as Dirac-delta functions and the like, in the noise simulator a numerical representation is to be found which is constrained with the discretisation of the numerical algorithm. The present problem appeared to be a suitable exercise for yet another application of the noise simulator for a new type of noise source. Thus the problem was solved by the use of the noise simulator. The amplitude and the phase across the core was calculated, and a good agreement was found with the results of the analytical method.

2. Calculation of the noise induced by shell-mode vibrations

A one-dimensional model of a reflected reactor is selected for this study with a homogeneous central core and outer reflectors placed symmetrically around the core (see Fig. 2). Two-group diffusion theory will be used with corresponding cross sections and diffusion constants. A list of those, together with the numerical values used in the quantitative
work is given in the Appendix. Quantities in the fast and thermal groups are labelled with a subscript 1 and 2, respectively. A superscript $c$ and $r$ indicates whether the quantity belongs to the core or the reflector. The same convention with notations is used also for the fast and thermal static flux in the core and in the reflector, and also for the noise. One averaged-group of delayed neutrons is used in the dynamic calculations. Vacuum boundary conditions are assumed at the outer boundary. A schematic view of the system is shown in Fig. 2.

To simplify the formalism, whenever practical, vector notations will be used, where the two components belong to the fast and thermal groups. E.g. the static flux in the core and the reflector can be written as

$$
\begin{align*}
\vec{\phi}^c(x) &= \begin{bmatrix} \phi^c_1(x) \\ \phi^c_2(x) \end{bmatrix}, -b \leq x \leq b \\
\vec{\phi}^r(x) &= \begin{bmatrix} \phi^r_1(x) \\ \phi^r_2(x) \end{bmatrix}, -a \leq x \leq -b, b \leq x \leq a
\end{align*}
$$

The static fluxes are given by the solution of the static equations, as well as by the boundary conditions at $x = \pm a$ and the interface conditions at $x = \pm b$. Instead of using all these interface and boundary conditions, it is easier to notice that the static flux is symmetrical around the origin. Hence it is simpler to seek a solution that is symmetric, i.e. satisfies

$$
\begin{align*}
\vec{\phi}^c(-x) &= \vec{\phi}^c(x), -b \leq x \leq b \\
\vec{\phi}^r(-x) &= \vec{\phi}^r(x), -a \leq x \leq -b, b \leq x \leq a
\end{align*}
$$

fulfills the interface conditions at $x = b$

$$
\begin{align*}
\vec{\phi}^c(x)|_{x = b} &= \vec{\phi}^r(x)|_{x = b}, \\
D^r_1 \frac{d}{dx} \phi^r_1(x)|_{x = b} &= D^c_1 \frac{d}{dx} \phi^c_1(x)|_{x = b}, \\
D^r_2 \frac{d}{dx} \phi^r_2(x)|_{x = b} &= D^c_2 \frac{d}{dx} \phi^c_2(x)|_{x = b}
\end{align*}
$$

and the vacuum boundary conditions at $x = a$,

$$
\vec{\phi}^c(x)|_{x = a} = 0.
$$
This way we need only consider the solution for \( x \geq 0 \). The solution to this standard problem is well-known. The advantage of this treatment is that due to the symmetry of the shell-mode vibrations in this 1-D model, and hence that of the induced neutron noise, the same simplification can be applied in the dynamic case.

The material data were derived from a real core of Ringhals-4 condensed into in form of 2-group cross sections, diffusion coefficients, velocities and one-group delayed neutron data. These were taken from a SIMULATE calculation, then converted into equivalent 1-D data by modifying the absorption cross sections to account for the higher leakage in 3-D than in the 1-D model used. These data were finally homogenized in the core and reflector regions, respectively in order to get a 2-region reactor. Since the original data correspond to a core loading in the critical state, these data should yield a \( k_{\text{eff}} \) close to unity in the analytical calculations. In order to have rigorous criticality in the analytical calculations, after the criticality calculations, the fission cross sections were re-adjusted as

\[
\nu \Sigma_{f,i}^c = \frac{1}{k_{\text{eff}}} \nu \Sigma_{f,i(\text{old})}^c \quad i = 1, 2
\]  

We turn now to the calculation of the noise, induced by the shell-mode vibrations of the core. These will be treated by a method, originally introduced for the treatment of control rod vibrations in a numerical work by Sanchis-Arnal et al. [19], and developed into an analytical tool under the name “\( \varepsilon/D \) method” by one of the present authors [20]. In that treatment, at the interface of the core and reflector at \( x = b \), a uniform description of the static cross sections over both the core and reflector can be given as

\[
\Sigma_{\alpha,i}(x) = \{1 - H(x-b)\} \Sigma_{\alpha,i}^c + H(x-b) \Sigma_{\alpha,i}^r
\]  

where \( \alpha = \{a, R, f\} \), i.e. it stands for the type of reaction (absorption, removal or fission), \( i = \{1, 2\} \) stand for the energy group index, and \( H(x) \) is the unit step function. A similar description is given at \( x = -b \).

The vibrations of the boundary are described by a time-dependent amplitude \( \varepsilon(t) \) around the interface, i.e. one will have \( b(t) = b + \varepsilon(t) \). Then, from (8), the time-dependent cross sections are given as

\[
\Sigma_{\alpha,i}(x, t) = \{1 - H(x-b-\varepsilon(t))\} \Sigma_{\alpha,i}^c + H(x-b-\varepsilon(t)) \Sigma_{\alpha,i}^r
\]  

Likewise, the shell-mode vibrations imply that the interface at \( -b \) will fluctuate as \( -b(t) = -b - \varepsilon(t) \). Then the perturbation \( \delta \Sigma_{\alpha,i}(x, t) \), defined as

\[
\Sigma_{\alpha,i}(x, t) = \Sigma_{\alpha,i}(x) + \delta \Sigma_{\alpha,i}(x, t)
\]  

can be obtained from (8)-(10) with a one-term Taylor expansion as

\[
\delta \Sigma_{\alpha,i}(x, t) = \varepsilon(t) \delta \Sigma_{\alpha,i}(b) [\delta(x-b) + \delta(x+b)]
\]  

where the notation

\[
\delta \Sigma_{\alpha,i}(b) \equiv \Sigma_{\alpha,i}^c - \Sigma_{\alpha,i}^r
\]  

was introduced. Eq. (11) shows that the perturbation of the cross sections obeys the same symmetry as the static cross sections, hence also the induced neutron noise will be symmetric.
around the core centre $1$. Therefore it is sufficient to calculate the induced neutron noise $\dot{\phi}(x, t)$ for $x \geq 0$ only. For the reasons of symmetry, the argument $b$ can be neglected in (12) in the notation of the perturbation.

From now on the treatment is rather standard, and can be described as follows. Starting with the time-dependent two-group diffusion equations for the noise in the core and the reflector, splitting up the fluxes and delayed neutron precursors to static values and fluctuations, neglecting second order terms and eliminating the delayed neutron precursors by a temporal Fourier-transform, one arrives at the noise equations in the frequency domain in the following form:

$$\hat{L}^c(x, \omega) \delta \phi^c = \varepsilon(\omega) \delta(x - b) \hat{S}(x) \phi^c$$  \hfill (13)

$$\hat{L}^r(x, \omega) \delta \phi^r = \varepsilon(\omega) \delta(x - b) \hat{S}(x) \phi^r$$  \hfill (14)

Here the following notations are used:

$$\hat{L}^c = \begin{bmatrix} D_1^c \frac{d^2}{dx^2} - \hat{\Sigma}_1^c & \nu \hat{\Sigma}_{f, 2}^c \\ \hat{\Sigma}_R^c & D_2^c \frac{d^2}{dx^2} - \hat{\Sigma}_{a, 2}^c \end{bmatrix} \quad \text{(core)}, \quad (15)$$

$$\hat{L}^r = \begin{bmatrix} D_1^r \frac{d^2}{dx^2} - \hat{\Sigma}_1^r & 0 \\ \hat{\Sigma}_R^r & D_2^r \frac{d^2}{dx^2} - \hat{\Sigma}_{a, 2}^r \end{bmatrix} \quad \text{(reflector)} \quad (16)$$

$$\hat{S} = \begin{bmatrix} \delta \hat{\Sigma}_1 & -\left(1 - \beta + \frac{\lambda \beta}{\lambda + i \omega}\right) \nu \delta \Sigma_{f, 2} \\ -\delta \hat{\Sigma}_R & \delta \Sigma_{a, 2} \end{bmatrix} \quad (17)$$

with the further following shorthand notations:

---

1. It might be interesting to note that for the case of a vibrating central control rod, one would have $\delta \Sigma_{\alpha, l}^{(-b)} = -\delta \Sigma_{\alpha, l}^{(+b)}$ in contrast to (11), which leads to an antisymmetric noise distribution in the core.
In order to make the solutions unique, one needs to specify the interface and boundary conditions to the above equations. The boundary conditions for the noise will be the same as for the static flux in (6), i.e.

\[
\delta \Sigma^c = \delta \Sigma^c_{a, 1} + \delta \Sigma^c_{a, 2} - \delta \Sigma^c_{f, 1} - \delta \Sigma^c_{f, 2}
\]

The interface conditions will, however, be modified, due to the Dirac-delta function character of the perturbation, as expressed in (13) and (14):

\[
\begin{align*}
\delta \Sigma^r_1 &= \delta \Sigma^r_{a, 1} + \delta \Sigma^r_{a, 2} - \frac{i\omega}{v_2} \\
\delta \Sigma^c_1 &= \left( \Sigma^c_{a, 1} + \Sigma^c_{R, 1} \right) - \left( 1 - \beta + \frac{\lambda \beta}{\lambda + i\omega} \right) v \delta \Sigma^c_{f, 1} \\
\delta \Sigma^r_2 &= \delta \Sigma^r_{a, 2} + \frac{i\omega}{v_2} \\
\delta \Sigma^c_2 &= \Sigma^c_{a, 2} + \frac{i\omega}{v_2} \\
\end{align*}
\]

In order to make the solutions unique, one needs to specify the interface and boundary conditions to the above equations. The boundary conditions for the noise will be the same as for the static flux in (6), i.e.

\[
\delta \phi^c(x) \big|_{x = a} = 0
\]

The interface conditions will, however, be modified, due to the Dirac-delta function character of the perturbation, as expressed in (13) and (14):

\[
\begin{align*}
D_1^r \frac{\partial}{\partial x} \delta \phi^r_1(x, \omega) \big|_{x = b} - D^c_1 \frac{\partial}{\partial x} \delta \phi^c_1(x, \omega) \big|_{x = b} &= \epsilon(\omega)(S_{11} \phi_1(b) + S_{12} \phi_2(b)) \\
D_2^r \frac{\partial}{\partial x} \delta \phi^r_2(x, \omega) \big|_{x = b} - D^c_2 \frac{\partial}{\partial x} \delta \phi^c_2(x, \omega) \big|_{x = b} &= \epsilon(\omega)(S_{21} \phi_1(b) + S_{22} \phi_2(b))
\end{align*}
\]

On the right hand side of the last two equations in (20) we did not indicate if the flux is taken from the core or the reflector; the reason is that, as the first equation also expresses, they are equal to each other at the interface.

A comparison of the static boundary conditions (5) and (6) with the dynamic ones, (19) and (20) shows, that the number of conditions, and that of the regions separated by interface conditions, is the same for the static and the dynamic cases. The reason for this is that the perturbation is acting exactly on the interface. Hence, the degree of difficulty of solving the dynamic equations is the same as that of the static equations. In effect the dynamic task is even simpler in the sense that due to the inhomogeneous terms on the r.h.s. of the last two equations in (20), and the non-vanishing determinant of the matrix for the unknown flux coefficients (given shortly below), the dynamic problem is not an eigenvalue problem, rather it is the solution of a directly invertible non-singular matrix equation.
To show the above, one writes the solutions of the equations (13)-(14), with taking the symmetry also in consideration, in the following form:

\[
\frac{\delta \phi}{\delta \phi^c}(x, \omega) = A_3 \left[ \frac{1}{\tilde{C}_k(\omega)} \right] \frac{\sinh(\tilde{k}_1(|x| - a))}{\sinh(\tilde{k}_1(b - a))} + A_4 \left[ \frac{1}{\tilde{C}_\lambda(\omega)} \right] \frac{\sinh(\tilde{k}_2(|x| - a))}{\sinh(\tilde{k}_2(b - a))}
\]

\[
\frac{\delta \phi^c}{\delta \phi}(x, \omega) = A_1 \left[ \frac{1}{\tilde{C}_\mu(\omega)} \right] \cos(\tilde{\mu}x) + A_2 \left[ \frac{1}{\tilde{C}_\eta(\omega)} \right] \frac{\cosh(\tilde{\eta}x)}{\cosh(\tilde{\eta}b)}
\]

where

\[
\tilde{k}_1 = \frac{\Sigma^r_1}{\sqrt{D_1^r}}, \\
\tilde{k}_2 = \frac{\Sigma^r_{a,2}}{\sqrt{D_2^r}}, \\
\tilde{C}_k = \frac{\Sigma^c_R}{D_2^c(\tilde{k}_2^2 - \tilde{k}_1^2)}
\]

and

\[
\tilde{\eta} = \frac{\sqrt{(D_1^c \Sigma^c_{a,2} + D_2^c \Sigma^c_1) + \sqrt{(D_1^c \Sigma^c_{a,2} - D_2^c \Sigma^c_1)^2 + 4 \sqrt{\Sigma^c_{f,2} \Sigma^c_R D_1^c D_2^c}}}}{2D_1^c D_2^c} \\
\tilde{\mu} = \frac{\sqrt{(D_1^c \Sigma^c_{a,2} - D_2^c \Sigma^c_1) + \sqrt{(D_1^c \Sigma^c_{a,2} - D_2^c \Sigma^c_1)^2 + 4 \sqrt{\Sigma^c_{f,2} \Sigma^c_R D_1^c D_2^c} - (D_1^c \Sigma^c_{a,2} + D_2^c \Sigma^c_1)}}}{2D_1^c D_2^c} \\
\tilde{C}_\eta = \frac{\Sigma^c_R}{\Sigma^c_{a,2} - D_2^c \eta^2} \\
\tilde{C}_\mu = \frac{\Sigma^c_R}{\Sigma^c_{a,2} + D_2^c \mu^2}
\]

The unknown constants $\tilde{A}_1 - \tilde{A}_4$ can be determined from the equation that results from putting (21) into (20) and rearranging to obtain
where $\hat{M}$ is given as

$$
\begin{bmatrix}
\hat{A}_1 \\
\hat{A}_2 \\
\hat{A}_3 \\
\hat{A}_4
\end{bmatrix} = \varepsilon(\omega) \begin{bmatrix}
0 \\
0 \\
S_{11}(b) + S_{12}(b) + S_{21}(b) + S_{22}(b)
\end{bmatrix}
$$

or

$$
\hat{M} \cdot \hat{A} = \varepsilon(\omega) \hat{F}
$$

(25)

The solution of (25) is given as

$$
\hat{A} = \varepsilon(\omega) \hat{M}^{-1} \hat{F}
$$

(27)

Substituting the solution of (27) back to (21) yields the formal solution of the problem, which can then be numerically evaluated.

### 3. Alternative calculation of the noise

As mentioned earlier, the noise was also calculated by the dynamic code, developed at the Department ([22]) and also referred to as the “noise simulator”. Although this simulator was primarily designed for handling 2-D cores, a 1-D version was also developed in the framework of this study so that the 1-D system described previously could be easily modelled. The fact that the system consists of a homogeneous core and an (obviously homogenous) reflector meant that the requirements on the node resolution were rather mild. In order to be able to represent small amplitude vibrations of the core, a node size of 0.5375 cm was selected. This parameter was solely determined by the fact that the spatial extent of a perturbation cannot be smaller than one node, hence the small node size. This node resolution can only be achieved in 1-D systems (i.e. the node size in 2-D systems cannot be smaller than typically 8 cm due to excessive computational times).

The absolute value and the phase of the noise was calculated over the core with using the same static parameters as in the analytical model (see the Appendix) for a vibration of 0.5375 cm amplitude. The noise simulator also calculates the static flux by solving the static eigenvalue equation. It is an important aspect of any noise calculation that the system is exactly critical (i.e.
to an accuracy of a large number of decimal points) by applying the same spatial discretisation algorithm as the one used in the dynamic calculations, otherwise the asymptotic properties, such as the dominance of the point kinetic term at low frequencies, will not be reconstructed. Hence also the correct spatial behaviour of the noise will be incorrect. Because of this requirement, the noise simulator and the analytical method, although starting with the same static data set, need to make the eigenvalue calculation separately and modify the fission cross sections according to (7) each with their own $k_{\text{eff}}$ value. This also gives an opportunity to compare the $k_{\text{eff}}$ of the analytical model and the simulator. In the present calculations one obtained $k_{\text{eff}} = 1.0014629$ in the analytical model, and $k_{\text{eff}} = 1.0014594$ with the noise simulator.

Finally, in some cases comparisons were made with the calculations made for the same problem by using the adiabatic approximation [25]. In the adiabatic approximation the noise is calculated as a point kinetic (or reactivity) term, and the adiabatic fluctuation of the so-called shape function $\psi(x, t)$ which is defined in the flux factorisation, leading to the well-known reactor kinetic approximations [26]. That is, the noise is written in the form

$$\delta \phi_i(x, t) = \delta P(t) \cdot \phi_{0, i}(x) + \delta \psi_i(x, t) \quad i = 1, 2$$

(28)

where the first term on the right-hand-side is the point kinetic term with $\phi_{0, i}(x)$ being the static flux, whereas the second one is referred to as the space-dependent term.

In the adiabatic approximation the functions $\delta \psi_i(x, t)$ are determined from static calculations, as the difference between the static eigenfunctions corresponding to the perturbed and unperturbed state. The perturbed state corresponds to a static case when the core-reflector interface is at $x = b + \varepsilon$, whereas the unperturbed case is when $\varepsilon = 0$. Hence

$$\delta \psi_i(x, \varepsilon) \approx \delta \psi_i^{ad}(x, \varepsilon) \equiv \psi_i(x, \varepsilon) - \phi_{0, i}(x)$$

(29)

where $\psi_i(x, \varepsilon)$ are the static (normalised) shape functions corresponding to the boundary being at $x = b + \varepsilon$. The superscript $ad$ stands for the adiabatic approximation. By letting $\varepsilon$ depend parametrically on time, $\varepsilon = \varepsilon(t)$, one can write $\delta \psi_i(x, \varepsilon(t)) = \delta \psi_i(x, t)$. Since in the calculations only terms linear in $\varepsilon$ were kept, one can easily switch to the frequency domain since all time dependence in the second term on the r.h.s. of (28) is contained in $\varepsilon(t)$:

$$\delta \psi_i(x, t) \approx \frac{\partial \delta \psi_i(x, \varepsilon)}{\partial \varepsilon} \bigg|_{\varepsilon = 0}$$

(30)

In the concrete calculations by Sunde and Arzhanov, [25], the further simplification of the plateau frequency approximation was used, in which one assumes

$$G_0(\omega) \approx \frac{1}{\beta}$$

(31)

This leads finally to the form used in the numerical work as

$$\delta \phi_i(x, \omega) = \frac{1}{\beta} \delta \rho(\omega) \cdot \phi_{0, i}(x) + \delta \psi_i^{ad}(x, \omega)$$

(32)

Due to the application of the plateau frequency approximation, the frequency dependence of both terms on the r.h.s. is contained only in $\varepsilon(\omega)$, which means that $\delta \phi_i(x, \omega) / \varepsilon(\omega)$ will be a real number. That is, the phase will take the values of either zero or $\pm \pi$ only. This is a clear limitation of the adiabatic approximation with the plateau frequency approximation, when
investigating the behaviour of the phase.

One advantage of the adiabatic approximation, which constituted a good part of the motivation for using it, is the fact that it easily allows to account for the dilution of the core material with the “stretching” of the system during shell-mode vibrations. In the real case, unlike in the simple analytical model we assumed here, due to material conservation, the distortion of the core means a re-distribution of the material. That is, along the directions of the core expansion the material density will decrease, and vice versa. In the simple one-dimensional case, this means that the macroscopic cross sections in the core will scale as

$$\frac{b}{b + \varepsilon}$$

with a change of the boundary position by $\varepsilon$. Such an effect is very easy to incorporate in the adiabatic approximation, both in the reactivity and in the space-dependent term. On the other hand, in the full analytical model, this would mean the appearance of a noise source which, in addition to the Dirac-delta one at the core-reflector interface, has also a term uniformly distributed in the core. Such a case would be an order of magnitude more difficult to handle in the analytical model. Indeed, such calculations are much easier made in the noise simulator, or in the adiabatic approximation. We, however, disregard the dilution effect and only use the perturbation as defined at the beginning of this section. The calculations by Sunde and Arzhanov ([25]) showed that the dilution effect has only moderate influence on the solutions in the core, and a negligible effect what regards the local component, which is our main concern in this paper.

In all quantitative work, for sake of comparison, the calculated values of $\frac{\delta\phi_i(x, \omega)}{\varepsilon(\omega)}$ were calculated and are shown in the figures.

4. Numerical work

Before turning to the comparison between the different methods of calculating the noise, we first investigate the amplitude and phase of the noise as calculated in the analytical model for the system specified in the Appendix. The dimensions of the critical system are $b = 161.25$ cm and $a = 279.5$ cm. The result of such a calculation with the frequency of the shell-mode vibration set to 20 Hz, is shown in Fig. 3. The figure shows a large local component in the amplitude of both the fast and the thermal noise, and in the phase of the thermal noise. The phase of the fast noise, on the other hand, remains smooth even at the boundary. The amplitude raises quite markedly for both the thermal and the fast flux close to the interface core/reflect.

The phase behaviour of the thermal noise, (and also the fine structure of the amplitude of the thermal noise, shown in Fig. 4) close to the interface is rather intricate, and deserves a more detailed analysis. It is seen that in most parts of the core the phase is negative, which is natural, but also relatively close to zero except close to the boundary. It is quite constant in the reflector, while it decreases in the core with increasing distance from the core boundary. This latter behaviour is also self-explaining, it just shows the physical effect of increasing phase lag with increasing distance between perturbation and response.

In order to understand the opposite phase behaviour of the thermal noise close to the core-

...
Fig. 3. Absolute value and phase of fast and thermal noise

Fig. 4. Absolute value and phase of the thermal noise at the interface between the core and the reflector

\[ \alpha \equiv \frac{\delta \rho}{\delta \varepsilon} \bigg|_{\varepsilon = 0} \] (33)
This can be calculated from either a simple perturbation formula using the perturbation as defined by (11), or from a static eigenvalue calculation, performed in the adiabatic approximation. These of course need to give the same value in first order of $\varepsilon$. With the material and geometrical data of the system treated here, one obtains

$$\alpha = 4.95 > 0 \text{ pcm/cm},$$

(34)
i.e. the point kinetic term of the noise is in-phase with the perturbation. Based on the calculations made in the adiabatic approximation (Ref. 25), where also the shape function is calculated separately, one can see that except close to the interface where the local component dominates, the noise is rather well described by the point kinetic term, i.e. the traditional space-dependent term is small. Hence the phase is close to zero in those points.

It remains to explain why a perturbation that has a positive reactivity effect leads to a decreasing thermal flux (i.e. a noise with opposite phase) just in the very neighbourhood of the perturbation itself. This is however also understandable. An increase of $\varepsilon$ actually means adding a layer of core material at the boundary and replacing the reflector material there. The core material means increased production of fissions and hence fast neutrons, which is the ultimate reason for the increase in reactivity; but this is achieved at the price of increased absorption of thermal neutrons. This is the reason of the out-of-phase behaviour of the local thermal noise. The same facts explain also the fine spatial structure of the amplitude of the thermal noise close to the boundary (Fig. 4) where the close to zero value of the minimum of the amplitude corresponds to the crossing of its (otherwise dominating) real part from positive to negative. It is also seen that, in accordance with the above, the phase of the fast noise shows no local out-of-phase behaviour, it remains smooth and close to zero everywhere.

One can also check now how the present results support the possibility of an out-of-phase behaviour between an ex-core (ex-vessel) detector and an in-core detector, as described in the introduction and in [18]. Using vacuum boundary conditions at the outer edge of the reflector, an ex-vessel detector cannot be directly modelled in diffusion theory. It can be best approximated in the present model with a detector inside the reflector, but close to its outer boundary. The data shown in Fig. 2 illustrate that an out-of-phase behaviour can indeed exist between such an ex-core detector and an in-core one, if the in-core detector is close to the core boundary.

A comparison with the measurement that prompted the recent work, shown in Fig. 1b, supports the validity of the results. Namely, the in-core detector in question was in the centre position of an outermost fuel assembly, even if not exactly on the boundary. Unfortunately, the signals from the other in-core detectors had a low APSD at the shell-mode vibration frequency and supplied a non-reliable phase value, so a more thorough checking of the calculation results is not possible for the time being.

Of course the model used here contains several simplification compared to reality, and the severity of the constraint this fact puts on the applicability of the results to a concrete case is not clear. One neglected effect, already mentioned, is the re-arrangement (dilution) of the core density with the shell-mode vibrations. It was shown in the adiabatic calculations ([25]) that the dilution effect decreases the reactivity coefficient of the vibrations very significantly, to the extent that the magnitude of the ordinary (non-local) space dependent component becomes dominating in the core. In such a case an out-of-phase behaviour can exist even between an ex-vessel detector and an in-core detector, which is not based on the local component, and hence the in-core detector does not need to be close to the core boundary. A final difference between
the models and reality is that the detector, being situated within a fuel assembly, vibrates
together with the core, and hence the noise it measures is not the same as the one calculated in
all the models used here. This effect actually could be calculated in the adiabatic approximation,
which could be an interesting exercise to do. Finally, a more realistic description of the core-
barrel vibrations requires a 2-D description. In two dimensions, the space-dependent component
of the noise is, as a rule, enhanced as compared to the point kinetic one, and this could also alter
some of the conclusions of the present work. However, in this paper our main purpose was to
perform a conceptual study and understanding of the basic phenomena, rather than trying to
exactly reconstruct certain measurements. The present investigations will be extended to 2-D
and alleviating some of the restrictions mentioned above, in later work.

It is interesting to compare the results of the analytical model with those of the noise simulator. Such a comparison is shown, now only for the thermal noise, in Fig. 5. It is seen that

![Fig. 5. Absolute value and phase of the thermal noise from both the analytical solution and the numerical simulator.](image)

they agree with each other very well. The relative weight of the amplitude of the local components in the two models show some disagreement, which is due to the fact that in the
Dirac-delta function representation and in the finite node size perturbation of the cross sections,
the internal magnitude of the perturbation of the various cross sections will not be exactly the
same in the two models. The phase curves, on the other hand, agree very well. This serves as a
further confirmation of the validity of the results.

Also, a comparison with the calculations from the adiabatic approximation can be made,
and it is shown in Fig. 6. The figure illustrates well that while the amplitudes, as obtained from
the two models, agree qualitatively and quantitatively well, the phase of the noise, in the
adiabatic approximation, is reproduced only in a very coarse way (as a binary variable taking the values 0 or \(-\pi\)). Hence the phase is zero in this approximation in most parts of the reflector and the core, except close to the core-reflector interface. The opposite-phase behaviour around the core-reflector interface is though correctly reproduced also by the adiabatic approximation.

5. Conclusions

The present work constitutes the first calculation of the in-core and ex-core neutron noise, induced by shell-mode core-barrel vibrations, in a two-group treatment. Although the model used has several limitations, such as a one-dimensional treatment and no accounting for the change in material density during vibrations, certain aspects of the induced noise, most notably the strong local component at the core-reflector interface, was reconstructed correctly. Some measurements with in-core detectors close to the core boundary would be useful for checking the applicability of the results, and indeed such measurements are planned. Extension of the calculations to two dimensions constitutes no conceptual problem, as well as the accounting for the effect of the vibration together with the core of the in-core detectors itself in the adiabatic approximation can be performed without difficulty. Such extensions are planned in future work.

References


Appendix

The following values of cross-sections and diffusion coefficients were obtained from static SIMULATE-3 calculations in 3-D corresponding to the Ringhals-4 PWR (cycle 16, burnup 8.767 GWd/THM) and then homogenized from 3-D to 1-D. These data were finally homogenized for the core and the reflector regions, respectively. Note that the absorption cross section for the fast group in the core is negative, this is due to the homogenization from 3-D to 1-D which preserves the leakage rate in the directions not accounted for in the 1-D model.

\[
\begin{align*}
\Sigma_{a,1}^c &= 0.0115 \text{ cm}^{-1} \\
\Sigma_{a,2}^c &= 0.1019 \text{ cm}^{-1} \\
\Sigma_R^c &= 0.0151 \text{ cm}^{-1} \\
\nu\Sigma_{f,1}^c &= 0.0057 \text{ cm}^{-1} \\
\nu\Sigma_{f,2}^c &= 0.1425 \text{ cm}^{-1} \\
D_1^c &= 1.4376 \text{ cm} \\
D_2^c &= 0.3723 \text{ cm} \\
v_1 &= 1.82304 \cdot 10^7 \text{ cm/s} \\
v_2 &= 4.13067 \cdot 10^5 \text{ cm/s} \\
\end{align*}
\]

for the core (35)

\[
\begin{align*}
\Sigma_{a,1}^r &= -0.0098 \text{ cm}^{-1} \\
\Sigma_{a,2}^r &= 0.0284 \text{ cm}^{-1} \\
\Sigma_R^r &= 0.0238 \text{ cm}^{-1} \\
D_1^r &= 1.3116 \text{ cm} \\
D_2^r &= 0.2624 \text{ cm} \\
\end{align*}
\]

for the reflector (36)

\[
\begin{align*}
\beta &= 0.00535 \\
\lambda &= 0.08510 \\
\end{align*}
\]

for the precursors (37)